A FAST METHOD FOR CALCULATION OF TRANSFORMERS LEAKAGE REACTANCE USING ENERGY TECHNIQUE

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Abstract Energy technique procedure for computing the leakage reactance in transformers is presented. This method is very efficient compared with the use of flux element and image technique and is also remarkably accurate. Examples of calculated leakage inductances and the short circuit impedance are given for illustration. For validation, the results are compared with the results obtained using practical tests for 3 types of a small single-phase transformer, 3-phase distribution transformers and a high voltage test transformer were studied.

Key Words Leakage Reactance, Transformer, Electromagnetic Energy, Inductance

چکیده امپدانس اتصال کوتاه، یک پارامتر مهم در طراحی و بهره برداری از ترانسفورماتور است. در این مقاله با بکارگیری انرژی الکترومغناطیسی، روش موثری برای محاسبه اندوکتانس پراکندگی ترانسفورماتور ارائه شده است. این روش در مقایسه با سایر روشها از جمله المان شار و روش تصاویر موثرتر و دقیقتر است. این موضوع از طریق شبیه سازی تایید می شود. برای نشان دادن دقت روش محاسبه انرژی، نتیجه محاسبات با نتایج آزمایشهای اتصال کوتاه برای یک ترانسفورماتور تک فاز و ترانسفورماتورهای توزیع جفت فاز و یک ترانسفورماتور فشار قوی مقایسه شده است.

1. INTRODUCTION

Determination of transformer leakage reactance using magnetic cores has long been an area of interest to engineers involved in the design of power and distribution transformers. This is required for predicting the performance of transformers before actual assembly of the transformers. A method has been presented for estimating the leakage reactance by flux tube in order to be included in an electric circuit model of transformer [1].

Computer-based numerical solution techniques using finite elements analysis, boundary element method and boundary-integral method are accurate and form an important part of the design procedures. However, they require rather elaborate computer resources and a somewhat lengthy setup before a solution can be obtained. Also a closed-form solution often provides more insight about critical physical parameters than a computer-based numerical solution.

Finite-element method is used for calculation of leakage inductances of electric machineries nowadays, but it needs special software and ample time [2-4]. A superposition of 2-D analysis results for approximate determination of the flux density distribution is presented in Reference 3. The FEM is also used to determine the leakage inductance of synchronous machines [2].

Dowell has also presented an analytical theory to calculate the ohmic losses of transformers but there has not been involved the reactance [5].

The reverse design method is also applied to partial core transformers; but the method is used for special type of transformers [6]. The windings should be wrapped around the core with primary winding inside the secondary winding and the yokes and limbs that usually form the rest of the core in full-core transformers, were not presented [6].

In this paper a closed form solution technique applicable to the leakage reactance calculations for transformers is presented. An emphasis is on the development of a simple method to characterize the leakage reactance of the transformers.

Leakage reactance calculations play an important role in designing geometry of transformers. The design parameters may be varied as such that the required short circuit impedance is determined. A 2D representation proves to be satisfactory in determining the leakage reactance. Final expressions are developed on a per-unit-of-length basis for the third dimension. Certain assumptions have been made in this calculation. End effects introduced by the terminations in 3D configurations are not evaluated here.

There are different techniques for the leakage reactance evaluation in transformers. The most common technique is the use of the flux leakage elements and estimation of the flux in different parts of the transformer [1,7-11]. The images technique can also be used. The base of this method is considering the image of every turn of the winding where the magnetic potential vector [12,13] is employed to compute the mutual and leakage inductances. Although the technique is effective, the computation result depends on the current of the image conductor [12].

This paper presents a novel technique for calculation of the leakage inductance in different parts of the transformer using the electromagnetic stored energy.

2. COMPUTATION USING FLUX ELEMENT TECHNIQUE

In order to compute the leakage reactance analytically, some approximation is required to achieve a closed-loop solution [1]. The assumptions are:

1. The leakage flux distribution in the winding and the space between the windings must be in the direction of the winding axial.
2. The leakage flux is uniformly distributed along the length of the windings.
3. The leakage flux in the space of two windings is divided equally between them.

The leakage flux for each winding for a two-winding transformer, based on the above assumptions is [1]:

$$\lambda = \mu_0 N^2 I L_{mt} (\frac{d}{3} + \frac{s}{2}) / L_C \tag{1}$$

Using the following equation:

$$X = 2\pi \frac{f\lambda}{I} \tag{2}$$

and reflecting leakage reactance between HV windings to the primary side yields:

$$X = X_1 + (N_1/N_2)^2 X_2 \tag{3}$$

Equation 3 will be as follows:

$$X_{12} = 2\pi f \mu_\circ \frac{N_2^2}{L_c} (L_{mt_1} (\frac{d_1}{3} + \frac{s_2}{2}) + L_{mt_2} (\frac{d_2}{3} + \frac{s_2}{2})) \tag{4}$$

If it is assumed that $L_{mt1} = L_{mt2}$ (meaning that the length of each turn of HV and LV windings are equal). Equation 3 can be simplified as follows:

$$X = 2\pi f \mu_0 \frac{N_1^2}{L_C} L_{mt1} (\frac{d_1 + d_2}{3} + s) \tag{5}$$

This is the conventional equation used in References 8, 9 and 10.

3.COMPUTATION USING IMAGE METHOD

This method is based on considering an Image conductor to the core for each loop, because the surface of the core is an equipotent for the magnetic scalar potential, so the core surface will be a mirror for magnetic field [12].

Using the magnetic vector potential for a circular filament for two conductors, the leakage inductance is [12]:

$$L_{leakage12} = f\ r_1, r_2, A(r_1), A_{image}(r_1), A(r_1\text{-}r_2),$$

$$A_{image}(r_1\text{-}r_2), i_1, i_2, i_{1image}, i_{2image} \tag{6}$$

Where r1 and r2 are the radiuses of conductors, A and A_{image} are the magnetic vector potentials with respect to actual and image conductors that are dependent on elliptic integral of first and second kinds [12,13].

The current of image conductor has a value different from the real conductor and it should be adjusted for each image conductor. So the result of the leakage inductance changes according to the default of image conductor currents.

There are several recommendations for this parameter to evaluate the best result of Image Method but there is not a fixed rule for it and the error from calculation and test results may not be optimized.

4. COMPUTATION USING ENERGY TECHNIQUE

The electromagnetic energy stored in the windings and the space between them can be used to calculate the inductance between the windings and the leakage inductance.

The previous assumptions are considered here in order to obtain a closed-form solution. Consider the path F_1 in Figure 1, mmf for the path having distance x from the beginning of LV winding is [7]:

$$F_x = N_1 I_1 \frac{x}{d_1} \tag{7}$$

Rising x from 0 to d_1 increases the magnetic field intensity and approaches its peak value N_1I_1/L_c. A volume differential from LV winding as shown in Figure 1, with height Lc, thickness dx and radius $r+s_1+x$ is considered. The electromagnetic energy stored in this element is [14]:

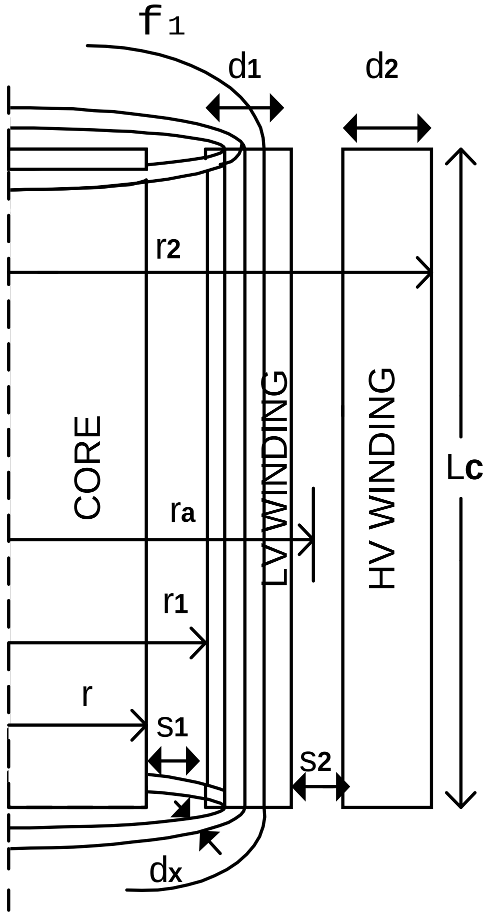


Figure 1. The volume element of LV winding for energy computation.

$$dw_x = \frac{1}{2}\mu_\circ H_x^2 dv \tag{8}$$

and the total energy is:

$$W_1 = \int_0^{d_1} dw_x = \frac{\pi\mu_\circ N_1^2 I_1^2}{Lcd_1^2}\int_0^{d_1}(r+s_1+x)x^2 dx = \frac{\pi\mu_\circ N_1^2 I_1^2}{Lc}\left(\frac{r+s_1}{3}+\frac{d_1}{4}\right)d_1 \tag{9}$$

Similarly the stored energy in HV winding can be determined:

$$W_2 = \frac{\pi\mu_\circ N_2^2 I_2^2}{Lc}\left(\frac{r+s_1+s_2+d_1+d_2}{3} - \frac{d_2}{4}\right)d_2 \qquad (10)$$

With a constant magnetic field intensity between the windings, W_a the electromagnetic energy stored between them is:

$$W_a = \mu_0 H_a^2 \frac{V_a}{2} \qquad (11)$$

Hence:

$$W_a = \frac{\pi\mu_\circ N_2^2 I_2^2}{Lc}\left(r + s_1 + d_1 + \frac{s_2}{2}\right)s_2 \qquad (12)$$

The stored energy for this two-winding transformer is:

$$W = W_1 + W_a + W_2 = \frac{1}{2} Leq I^2 \qquad (13)$$

Using Equations 9-13, the inductance will be as follows:

$$\text{Leq}_{12} = \frac{2\pi\mu_\circ \text{N}_2^2}{\text{Lc}}\left[\left(\frac{\text{r}_1}{3} + \frac{\text{d}_1}{4}\right)\text{d}_1 + \left(\frac{\text{r}_2}{3} - \frac{\text{d}_2}{4}\right)\text{d}_2 + \text{r}_a\text{s}_1\right] \qquad (14)$$

r_1 , r_2 and r_a are defined as follows:

$$r_1 = r + s_1$$
$$r_2 = r + s_1 + d_1 + s_2 + d_2 \qquad (15)$$

$$r_a = r + s_1 + d_1 + \frac{s_2}{2} \qquad (16)$$

A notable point is that if term ($r_1/3+d_1/4$) is substituted by $r_{ave1}/3$ and term ($r_2/3-d_2/4$) in replaced by $r_{ave2}/3$, L_{eq} will be:

$$\text{Xeq}_{12} = 2\pi\text{f}\,\frac{2\pi\mu_\circ\text{N}_2^2}{\text{Lc}}\left(\frac{\text{r}_{\text{ave}_1}\text{d}_1 + \text{r}_{\text{ave}_2}\text{d}_2}{3} + \text{r}_a\text{s}_2\right) \qquad (17)$$

The following simplification is applied:

$$\frac{L_{mt_1} + L_{mt_2}}{2} = ra(2\pi)$$
$$L_{mt_1} = 2\pi r_{ave_1}, L_{mt_2} = 2\pi r_{ave_2} \qquad (18)$$

Equation 17 is converted into Equation 5. It means that the flux element method is an approximation of the energy method.

5. SIMULATION

5.1 A Single-Phase Transformer A small single-phase transformer with specifications given in Table 1 is simulated [12]. The image method result of this reference is used to compare the accuracy of different methods.

Using the previous results of the transformer from Reference 12, the result of different methods can be compared. As shown in Table 2, the error using the energy method is lower than that of the

TABLE 1. Specifications of the Proposed Single-Phase Transformer. Dimensions are in mm.

Power	2kVA
N_1/N_2	118/118
Voltage	110 v
L_c	198
S_1	5
S_2	17
d1	5
d2	5
r	60

TABLE 2. Computations Results Using Different Methods.

Method	Inductance (mH)	Error in respect to the test (%)
Data sheet	0.5475	21.68
Flux method	0.4673	10.51
Image method	0.4609	4.44
Energy method	0.430	2.42
Test results	0.45	***

TABLE 3. Specifications of the Proposed Transformers.

Power kVA	N1	N2	D1 mm	D2 mm	D3 mm	V/N	Zsc %
25	116	5224	105	110	135	2.30	4.13
50	88	3962	111	116	161	3.03	4.09
100	68	3062	122	127	177	3.92	4.25

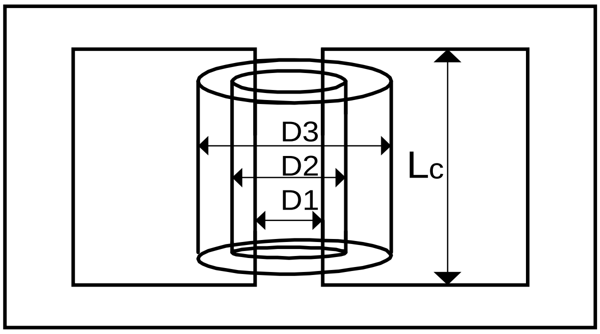
image method. Also the flux element method has larger error compared to the other two methods.

5.2 Three-Phase Distribution Transformers

Three types of three-phase distribution transformers with voltages 20/0.4 kV and connection group YZn5 with specification given in Table 3 are considered. Figure 2 shows the dimensions of the 25,50 and 100kVA transformers.

The last column of Table 3 shows the result of short-circuit test of transformers in the factory. The specifications and the test results (last column) in Table 3 are measured for more than 100 distribution transformers.

The short-circuit impedance has been calculated using the above-mentioned methods. The results for these transformers have been summarized in Table 4. The result shows that energy method has the best accuracy compared to other methods for all of these distribution transformers.



D_1 : diameter of core, D2: diameter of primary winding
D3: diameter of secondary winding

Figure 2. A simple geometrical schematic Dimensions of the transformer.

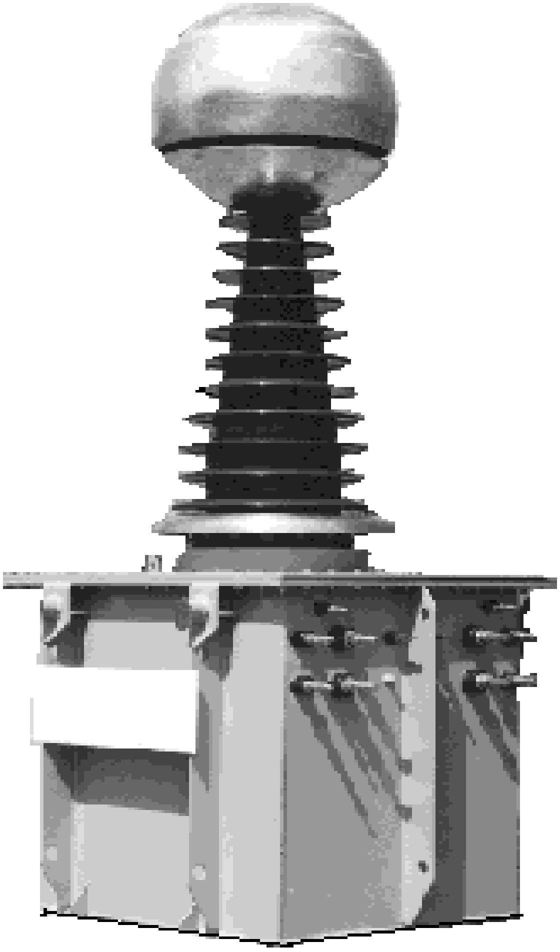


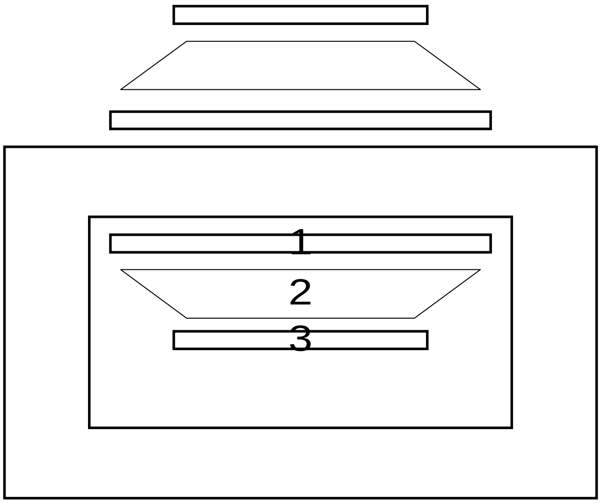
Figure 3. A 250 kV, 500 kVA test transformer designed and manufactured in Iran.

5.3 A Single-phase High Voltage Test Transformer

The result of simulation of a 500 kVA, 250kV test transformer, illustrated in Figure 3 and designed by the authors and manufactured in Iran, is shown in Table 5. The schematic presentation of windings are in Figure 4.

TABLE 4. Computation Results for Transformers.

Power kVA	Method	Impedance (%)	Error (%)
25	Equation (5)	6.56	58.7
	Flux Method	4.37	5.7
	Image Method	4.30	4.03
	Energy Method	4.27	3.45
	Test result	4.13	****
50	Equation (5)	5.10	24.68
	Flux Method	4.33	5.97
	ImageMethod	4.23	3.47
	Energy Method	4.20	2.81
	Test result	4.09	****
100	Equation (5)	5.086	19.71
	Flux Method	4.416	3.94
	Image Method	4.358	2.57
	Energy Method	4.340	2.16
	Test result	4.248	****



1-LV Winding
2- HV Winding
3-Coupling Winding

Figure 4 . Schematic of windings of the test transformer.

It is noticeable that the transformer has 3 windings: low voltage, high voltage and coupling (for energy transmission to upper steps).

6. CONCLUSIONS

Different analytical methods for the leakage inductance of transformer calculation have been compared. It has been shown that the energy method is the most accurate one. Although the image method is accurate and convenient, it depends on the current of the image conductors. These results are compared by practical measurements for three types of transformers: small single-phase transformer, three-phase distribution transformers and a high voltage test transformer.

So by calculating stored electromagnetic energy in windings and distance between them leakage reactance will be calculated simply and accurately comparing to test result.

7. APPENDIX

Calculation of Leakage Reactance of Test Transformer The reactance between windings calculated two by two and the detail of calculation is given here. The result shows that energy method is also reliable in this case.

Because of trapezoid shape of HV winding of test transformer according to Figure 5, the formula of 10 will be changed:

$$W_2 = \pi \mu_\circ \frac{N_2^2 I_2^2}{dw_2} [\frac{1}{2}(r_2 + 3kLC_2)(LC_1 + LC_2)$$

$$-2LC_2(r_2 + 1.5kLC_2) - \frac{k}{3}(LC_1^2 + LC_2^2 + LC_1 LC_2) +$$

$$\frac{(r_2 + kLC_2)LC_2^2}{LC_1 - LC_2} \ln \frac{LC_1}{LC_2}] \qquad \text{(A1)}$$

TABLE 5. Result of Methods for the Test Transformer.

	LV – HV	LV-Couple	HV-Couple
Flux Met.	6.12	22.5	6.69
Image Met.	6.05	22.6	6.45
Energy Met.	5.83	22.81	6.32
Test Result	5.85	22.74	6.30

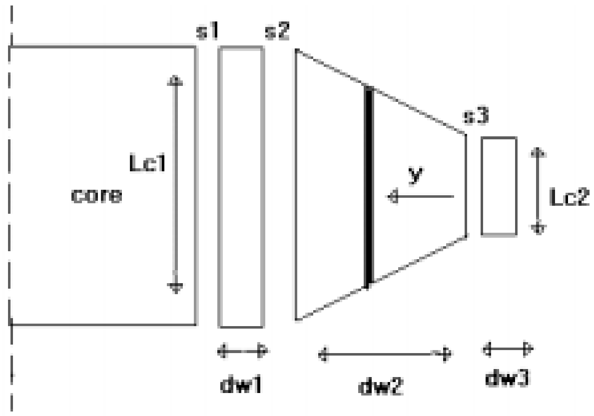


Figure 5. Dimensions of windings of the test transformer.

Considering Leakage reactance between LV and HV:

$$Xeq_{12} = \frac{4\pi^2 . f\mu_\circ N_2^2}{L C_1}\left\{(\frac{r_1}{3}+\frac{dw_1}{4})\,dw_1 + ras_2 + \right.$$

$$\frac{LC_1}{dw_2}\left[\frac{1}{2}(r_2+3kLC_2)(LC_1+LC_2)-2LC_2(r_2+1.5\,kLC_2)\right.$$

$$\left.\left.-\frac{k}{3}(LC_1^2+LC_2^2+LC_1LC_2+\frac{(r_2+k\,LC_2)LC_2^2}{LC_1-LC_2}\ln\frac{LC_1}{LC_2}\right]\right\}$$

(A2)

Where k is a constant.

8. LIST OF SYMBOLS

λ	Leakage flux
N	No. of turn of winding
I	current
L_{mt}	Mean length of one turn of winding
L_c	length of window of core
d	Width of winding
s_1	Distance between core and LV windings
s, s_2	Distance between LV and HV windings
r_{ave1}, r_{ave2}	Mean radius of HV and LV winding respectively
X	Reactance of transformer
F	Frequency of current of windings
W	Electromagnetic energy stored in active part
L_{eq}	Equivalent inductance of transformer
H_x	Magnetic field intensity in a distance x of first layer of primary winding

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