NATURAL CONVECTION HEAT TRANSFER FROM HORIZONTAL CYLINDERS IN A VERTICAL ARRAY CONFINED BETWEEN PARALLEL WALLS

S. Kazemzadeh Hannani, M. S. Sadeghipour and M. Nazaktabar

Department of Mechanical Engineering, Sharif University of Technology
P.O. Box 11365-9567, Tehran, Iran
hannani@sina.sharif.ac.ir - msadeghis@yahoo.com - nazaktabar7@yahoo.com

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Abstract Laminar natural convection from an array of horizontal isothermal cylinders confined between two vertical walls, at low Rayleigh numbers, is investigated by theoretical and numerical methods. The height of the walls is kept constant, however, number of the cylinders and their spacing, the distance between the walls and Rayleigh number have been varied. The optimal spacing (confining walls) and the maximum Nusselt number predicted theoretically are validated by means of numerical simulations. It has been shown that with increasing the number of cylinders or their spacing the optimal spacing will increase. In addition, increasing the Ra number decreases the optimal spacing of the walls.

Key Words Natural Convection, Array of Cylinders, Theoretical, Numerical

چکیده در این تحقیق انتقال حرارت جابجایی آزاد از یک ردیف عمودی از استوانه های افقی محدود به دیوارهای عایق برای عدد رایلی کوچکتر از ۱۰۰۰ مورد بررسی قرار گرفته است. ارتفاع دیوارها ثابت فرض شده است. تعداد استوانه ها، فاصله استوانه ها از هم، فاصله دیوارها از هم و عدد رایلی متغیر می‌باشند. در ابتدا فاصله بهینه دیوارهای محدود کننده برای بیشترین مقدار انتقال حرارت جابجایی آزاد به روش تحلیلی پیش بینی شده است. در پایان به کمک حل عددی نیز درستی نتایج بدست آمده به روش تحلیلی نشان داده شده است. بررسیها نشان می‌دهد با افزایش تعداد استوانه ها و فاصله استوانه ها از هم، فاصله بهینه دیوارهای محدود کننده افزایش می‌یابد در حالی که با افزایش عدد رایلی فاصله بهینه کاهش می‌یابد.

1. INTRODUCTION

Natural convection is still a problem of many engineering applications. Heat transfer from different geometries has been studied and, due to the low heat transfer coefficients, techniques have been developed to enhance the rate of heat transfer.

One of the problems of this group that has received a good attention in recent years, and has applications in such areas as electronic cooling and design of condensers for the household refrigerators, is natural convection from a single horizontal cylinder or arrays of horizontal cylinders. Effects of confining walls on the rate of heat transfer from a single cylinder and arrays of cylinders have been investigated extensively in recent years.

Marsters [1] was the first one who studied the effects of adiabatic confining walls on the rate of free convection heat transfer from a horizontal isothermal cylinder. He used both experimental and theoretical methods. His experimental results cover a vast range of Rayleigh numbers. He studied the effects of changes in the height and the spacing of the walls, on the Nusselt number. He did not observe any optimum wall spacing for the maximum Nusselt number.

Sadeghipour and Kazemzadeh Hannani [2] studied the transient natural convection from a confined isothermal cylinder, numerically. They observed an optimum wall distance to cylinder diameter ratio for the maximum Nusselt number.

Tokura et al. [3] studied the effects of confining walls on natural convection from arrays of horizontal cylinders, experimentally. They reported an optimum spacing for the confining walls that maximized the heat transfer from the cylinders. They considered high Rayleigh numbers ($Ra \cong 10^5$).

Sadeghipour and Asheghi [4] investigated the steady state free convection heat transfer from horizontal isothermal cylinders in vertical array of two to eight without any confining walls, at low Rayleigh numbers, experimentally. Results show that there is an optimum separation distance for the best overall convection heat transfer of each array.

The other investigation was the theoretical, numerical and experimental work of Bejan et al. [5]. They determined the optimal spacing between horizontal cylinders in vertical arrays under laminar natural convection, such that the total heat transfer between the arrays of cylinders and the ambient was maximized. The volume occupied by the array was fixed.

Recently, Sadeghipour and Pedram Razi [6] studied the steady state natural convection from an isothermal horizontal cylinder confined between two adiabatic vertical walls, for low Rayleigh numbers. They observed an optimum wall distance for the maximum heat transfer, using the idea of intersection of asymptotes [5].

In the present investigation, natural convection heat transfer from arrays of horizontal isothermal cylinders confined by two symmetrically placed vertical adiabatic walls is studied (figure 1). Theoretical and numerical approaches are employed to determine the optimum spacing for the confining walls. The optimal spacing is important particularly because of its obvious implications on the design of condensers for the household refrigerators and electronic packaging.

This study is conducted in two steps. In the first step, a theory is developed to show the existence of an optimum spacing for the confining walls and to reveal the proper dimensionless groups. In the second step, natural convection is modeled numerically to validate the theoretical results and to optimize the dimensions for the maximum rate of heat transfer.

2. THEORETICAL APPROACH

In this investigation the idea of intersection of asymptotes was utilized to show the existence of an optimum spacing for maximum rate of heat transfer. This technique was first introduced by Bejan [7,8] and was used by Bejan et al. [5] and by Sadeghipour and Pedram Razi [6]. Using this technique, proper dimensionless groups needed to correlate the optimum spacing can be determined more accurately.

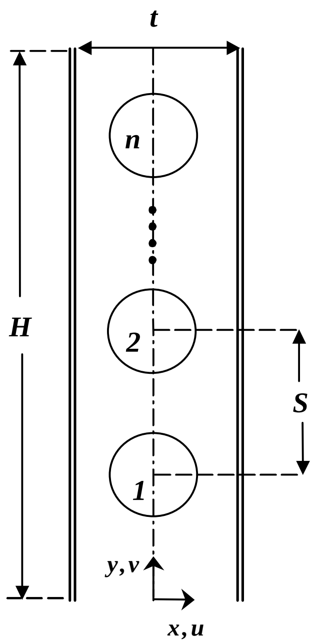

Figure 1. Configuration and the coordinate system.

In the case when the distance between the walls is small, the mass flow rate through the wall region increases with the separation distance between the walls. This is because of lower average velocity, causing less pressure drop due to friction, in larger ducts. In this case, Nu number increases with wall distance (Figure 2 curve 1). Conversely, in the case that the wall spacing is large, the mass flow rate

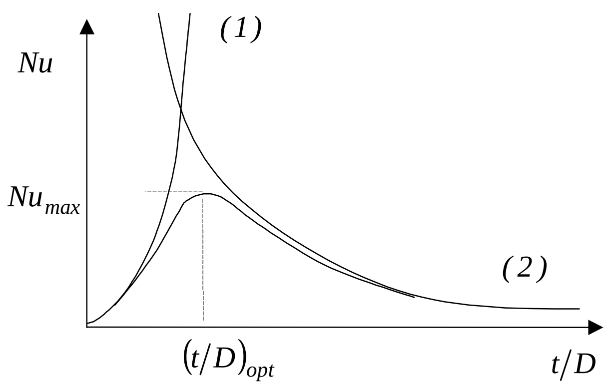

Figure 2. Variation of Nusselt number with the ratio t/D .

variation with wall separation distance is not significant. However, in this case, the maximum velocity at the centerline increases and moves towards the cylinder surface. Decreasing the distance between the walls will increase this maximum velocity, leading to an increase in Nu number (figure 2, curve 2). We can conclude from figure 2, then, that the intersection of the two asymptotic cases will give a rough estimate of the optimal spacing.

Case I: The Limit $t/D \rightarrow 1$ (Small Values of t/D)

a) n is Large and $S/D \rightarrow 1$ When the number of cylinders is large and they almost touch, the temperature of the coolant leaving the wall region is essentially the same as that of the cylinders, T_W. The heat transfer from the array to the coolant (ambient) is, therefore, equal to the enthalpy gained by the coolant, which can be expressed by Equation 1:

$$q = \dot{m} C_P \left(T_W - T_\infty \right) \tag{1}$$

Let us assume that a straight channel can model the walls confining the array of cylinders. Noting that the width of the flow varies between a minimum value $(t-D)$ and a maximum value (t), and following Bejan [5], the averaged volume thickness of the equivalent channel can be defined as:

$$\bar{t} = \frac{t.H - n\pi D^2/4}{H} \tag{2}$$

If $\bar{t}$ is sufficiently small, the flow rate through the channel of cross sectional area $\bar{t} \times 1$ and length H is proportional to the pressure difference between inlet and outlet. The pressure difference can be written as, $\Delta P = \rho g H \beta \left(T_W - T_\infty \right)$, or as the hydrostatic pressure difference between the inlet and outlet sections, which are at T_∞ and T_W, respectively. The mean velocity of the flow, U, can be approximated using the Hagen-Poiseuille solution for flow between two parallel plates.

$$U = \frac{(\bar{t})^2 \, \Delta P}{12 \mu H} \tag{3}$$

The total mass flow rate through the channel can be written now as:

$$\dot{m} = \left(\frac{\bar{t}^3 \, \Delta P \, l}{12 \upsilon H} \right) \tag{4}$$

Combining Equations 1 and 4, the total heat transfer can be expressed as:

$$q \cong \frac{l}{12 D^3} \left[t - \frac{n\pi D^2}{4H} \right]^3 k \, \Delta T_W \, Ra \tag{5}$$

If the height of the walls is much greater than the cylinders diameter, then $\frac{n\pi D^2}{4H}$ will be much smaller than t, distance between the two walls. Hence, when t is small compared to H, the heat transfer increases as $\left(\bar{t}^3\right)$, similar to the results of Bejan et al. [5].

Using the Newton's cooling law, $q = \bar{h}(n\pi Dl)\Delta T_W$, the Nusselt number is defined as:

$$\overline{Nu} = \frac{\bar{h} D}{k} = \frac{1}{12 n\pi} \left[\frac{t}{D} - \frac{n\pi D}{H} \right]^3 Ra_D \tag{6}$$

where $\Delta T_W = T_w - T_\infty$

We conclude from (6) that Nu increases with t/D and $1/n$.

b) n Is Small and $S/D \rightarrow \infty$ When the separation distance, t, and the number of cylinders, n, are small, and there is large cylinder to cylinder spacing, we cannot assume that the outlet temperature of fluid is equal to the cylinders temperature, T_W. In this situation Marster's [1] integral method can be employed to develop a theoretical solution for the heat transfer behavior of the confined cylinders. The governing continuity, momentum and energy equations presented, in integral form, are as follows.

Continuity Equation For the conservation of mass between inlet and outlet, we can write:

$$\dot{m} = \rho_1 u_1 t = t \int_{-1/2}^{1/2} \rho_2 u_2 \, d\hat{x} \tag{7}$$

where $\hat{x} = \frac{x}{t}$

Momentum Equation The momentum equation, which is a balance between the buoyancy force, the chimney effects and the friction forces on the walls and on the cylinders, is written as:

$$\begin{aligned}&(P_1 - P_2)t - \int_0^H \tau \, dy - C_D \frac{1}{2} \rho_1 u_1^2 D \\ &- g\rho_1 \int_0^H \int_{-\frac{t}{2}}^{\frac{t}{2}} \frac{\rho}{\rho_1} dx\, dy = \int_{-\frac{t}{2}}^{\frac{t}{2}} \rho_2 u_2^2 \, dx - \dot{m} u_1\end{aligned} \tag{8}$$

(Chimney effect + Friction force + Drag force + Buoyancy force = Momentum change)

The inlet and outlet pressures can be expressed as:

$$P_1 = P_\infty - \frac{1}{2} \rho_1 u_1^2 \tag{9}$$

$$P_2 = P_\infty - \rho_1 g H \tag{10}$$

Introducing Equations 9 and 10 into 8, and defining $\hat{y} = \frac{y}{H}$, the momentum Equation 8 leads to:

$$\begin{aligned}&\frac{gH}{u_1^2}\left(1 - \int_0^1 \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\rho}{\rho_1} d\hat{x}\, d\hat{y}\right) - \frac{1}{2}\left(1 + \frac{C_D D}{t}\right) - \\ &\frac{H}{t}\int_0^1 \frac{\tau(\hat{y})}{\rho_1 u_1^2} d\hat{y} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{\rho_2 u_2^2}{\rho_1 u_1^2} - 1\right) d\hat{x}\end{aligned} \tag{11}$$

In Equation 11, ρ is defined as:

$$\rho = \rho_1 (1 - \beta \Delta T) \tag{12}$$

where, $\Delta T = T - T_\infty$

In the limit if $t/D \to 1$, $S/D \to \infty$ and "n" is small, (i.e. the confining walls are tall enough), neglecting the inertia and drag forces against the buoyancy force, the friction force of walls will balance the buoyancy force, therefore, from Equation 11 we have:

$$\frac{Gr_D}{Re_D^2} \frac{H}{D} C_1 \cong \frac{H}{t} \int_0^1 \frac{f}{2} \frac{\rho u^2}{\rho_1 u_1^2} d\hat{x} \tag{13}$$

The shear stress at the wall is defined as:

$$\tau(\hat{y}) = \frac{1}{2} f \rho u^2 \tag{14}$$

where, for flow between two parallel plates, the friction factor is given as $f = \frac{24}{Re_{D_h}}$, and $D_h = 2t$.

After rearranging Equation 13 we have:

$$Re_D \cong Gr_D (t/D)^2 \frac{C_1}{6C_2} \tag{15}$$

where, $C_1 = \int_0^1 \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\Delta T}{\Delta T_W} d\hat{x} d\hat{y}$ and $C_2 = \int_0^1 \frac{\rho u_2}{\rho_1 u_1} d\hat{y}$.

Energy Equation The energy equation is a balance between the heat transfer from the cylinders and changes in the flow energy between the inlet and outlet. This equation is, then, written as:

$$\begin{aligned}\dot{Q} = tC_P &\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_2 u_2 T_2 \, d\hat{x} - \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_1 u_1 T_1 \, d\hat{x}\right] + \frac{t}{2} \times \\ &\left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_2 (u_2)^3 \, d\hat{x} - \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_1 (u_1)^3 \, d\hat{x}\right] - gt \left[\int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_2 u_2 Y_2 \, d\hat{x} - \int_{-\frac{1}{2}}^{\frac{1}{2}} \rho_1 u_1 Y_1 \, d\hat{x}\right]\end{aligned} \tag{16}$$

where, $\dot{Q} = n\pi D \bar{h} \Delta T_W$.

Knowing that $y_1 - y_2 = H$, Equation 16 can be rearranged as:

$$\frac{\overline{Nu}}{\Pr \mathrm{Re}_D} n\pi \left(\frac{D}{t}\right) = \frac{T_1}{\Delta T_W} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{\rho_2 u_2 T_2}{\rho_1 u_1 T_1} - 1\right) d\hat{x} + \frac{u_1^2}{2 C_P \Delta T_W} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{\rho_2 u_2^3}{\rho_1 u_1^3} - 1\right) d\hat{x} + \frac{g H}{C_P \Delta T_W} \tag{17}$$

(Total heat transfer = Enthalpy difference + Changes in kinetic and potential energies)

In the energy Equation 17, we neglect the kinetic and potential energy effects with respect to the enthalpy gained by the coolant, because, the velocity change and the mass of the fluid are very small in free convection. Therefore, Equation 17 simplifies to:

$$\overline{Nu} = \frac{t}{nD} Re_D \, Pr \frac{T_\infty}{\Delta T_W} C_3 \tag{18}$$

where, $C_3 = \frac{1}{\pi} \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{\rho_2 u_2 T_2}{\rho_1 u_1 T_1} - 1\right) d\hat{x}$

Substituting Equation 15 for Re_D in the energy Equation 18, leads to:

$$\overline{Nu} = \frac{C_4}{n} Ra_D (t/D)^3 \tag{19}$$

where, $C_4 = \frac{C_1 C_3}{6 C_2} \times \frac{T_\infty}{\Delta T_W}$.

What is obvious from Equation 19 is the high dependence of Nusselt number on t/D. Equation 19 is very much similar to what is given by Bejan [5]. Also, for $H/D \to \infty$, Equation 6 takes a form similar to Equation 19. Equation 19 shows that the Nusselt number decreases with the number of cylinders. Note that as the coolant passes over the cylinders, its temperature approaches to the temperature of cylinders.

From Equation 19, we conclude that Nu increases with $(t/D)^3$ and decreases with n.

Case II: The Limit $t/D \to \infty$ (Large Values of t/D) As distance between the confining walls is increased, their effect on the rate of heat transfer from the cylinders vanishes, gradually. For $t/D \to \infty$ the solution of this problem should eventually approach that of heat transfer from an array of cylinders in free space. Therefore, the experimental results of Sadeghipour and Asheghi [4] can be used. The results of ref. [4] predict Nusselt number for any array with the number of cylinders in the range of experiments. Nusselt number for arrays of the horizontal isothermal cylinders is given as [4]:

$$\overline{Nu} = \left[0.823 + Exp\left(-1.5(S/D)^{0.05n}\right)\right] Ra^{\frac{1}{4}} \tag{20}$$

$500 \le Ra \le 700$, $3.5 \le S/D \le 27.5$, $2 \le n \le 8$.

In this case, neglecting the inertia and the wall friction, the buoyancy force should balance the drag force on the cylinder and Equation 11, which is also valid for the limiting case of, $t/D \to \infty$, can be written as:

$$C_1 \frac{Gr_D}{Re_D^2} \frac{H}{D} = \frac{1}{2} C_D \frac{D}{t} \tag{21}$$

Proper value for C_D is proposed in [9] as:

$$C_{D1} = f(Re_D) = 5.48 \, Re_D^{-0.25} \tag{22}$$

For "n" cylinders in an array, the average drag coefficient of the array can be approximated as:

$$C_{Dn} = f(Re_D) = 5.48 n \, Re_D^{-0.25} \tag{23}$$

Introducing Equation 23 into Equation 21, leads to:

$$Ra = Pr \, Gr_D = \frac{2.74}{C_1} n \frac{D}{t} \frac{D}{H} Pr Re_D^{1.75} \tag{24}$$

For $t/D \to \infty$, the Nusselt number in the Equation 18 should approach a constant value ($\overline{Nu} \cong 3 \text{ to } 5$). Note that Equation 18 is also valid for $t/D \to \infty$. Therefore, Equation 18 can be approximated as:

$$Re_D = \left(\frac{t}{D}\right)^{-1} \frac{nC'}{Pr \dfrac{T_\infty}{\Delta T_W} C_3} \tag{25}$$

where, $C' = \overline{Nu} \cong 3 \text{ to } 5$.

Substituting Re_D from Equation 25 into Equation 24 results:

$$Ra_D = C_5 \frac{n^{2.75}}{Pr^{0.75}} \left(\frac{H}{D}\right)^{-1} \left(\frac{t}{D}\right)^{-2.75} \tag{26}$$

where, $C_5 = \dfrac{2.74}{C_1} \left(\dfrac{C' \Delta T_w}{C_3 T_\infty}\right)^{1.75}$

Finally, combining Equations 26 and 20, leads to:

$$\overline{Nu} = f(S/D,n) C_5^{\,0.25} \frac{n^{0.69}}{Pr^{0.19}} \left(\frac{H}{D}\right)^{-0.25} \left(\frac{t}{D}\right)^{-0.69} \tag{27}$$

where, $f(S/D,n) = \left[0.823 + Exp\left(-1.5(S/D)^{0.05n}\right)\right]$

Equation 27 shows that, for the limiting case $t/D \to \infty$, Nusselt number is inversely proportional to "t". On the other hand Nu increases with n.

3. THE OPTIMUM WALL DISTANCE

Inspecting the results obtained for the two cases "I" and "II", represented by Equations 6, 19 and 27, we observe that for case "I", Nu increases with t/D, however, for case "II", Nu is inversely proportional to t/D. Therefore, the results for these two limiting cases intersect at a point where the rate of heat transfer from the array is maximum.

Case a Relation for the optimum distance for the case when number of cylinders is large and cylinder to cylinder spacing is small can be obtained by intersecting Equations 6 and 27:

$$\left[\left(\frac{t}{D}\right)_{opt} - \frac{n\pi D}{H}\right]^3 =$$

$$12\pi f(S/D,n) C_5^{\,0.25} \frac{n^{1.69}}{\Pr^{0.187}} \left(\frac{H}{D}\right)^{-0.25} \left(\frac{t}{D}\right)_{opt}^{-0.69} Ra^{-1} \tag{28}$$

Case b On the other hand, when the number of cylinders is small and cylinder to cylinder spacing is large, the optimum distance between the confining walls can be obtained by intersecting Equations 19 and 27:

$$\left(\frac{t}{D}\right)_{opt} =$$

$$\left(C_4 C_5^{\,0.25}\right)^{0.271} \left[f(S/D,n)\right]^{0.271} \frac{n^{0.46}}{\Pr^{0.05}} \left(\frac{H}{D}\right)^{-0.068} Ra^{-0.271} \tag{29}$$

Equation 29 shows that, $(t/D)_{opt}$ decreases as Ra increases. Therefore, for large Rayleigh numbers the optimum wall spacing can hardly be identified experimentally. In addition, when Pr or (H/D) increases $(t/D)_{opt}$ decreases. Inversely, $(t/D)_{opt}$ will increase when number of cylinders "n" increases, because the drag force on the cylinder increases and temperature of coolant approaches to that of the cylinder "T_W" at the top of the array.

4. NUMERICAL SOLUTION

The governing equations for free convection heat transfer using Boussinesq approximation are the following:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \tag{30}$$

$$\tilde{u}\,\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\,\frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{P}}{\partial \tilde{x}} + Pr\,\nabla^2 \tilde{u} + Bo\,\tilde{T} \tag{31}$$

$$\tilde{u}\,\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\,\frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\rho}\frac{\partial \tilde{P}}{\partial \tilde{y}} + Pr\,\nabla^2 \tilde{v} \tag{32}$$

$$\tilde{u}\,\frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v}\,\frac{\partial \tilde{T}}{\partial \tilde{y}} = \nabla^2 \tilde{T} \tag{33}$$

The dimensionless parameters are defined as:

$$\tilde{P} = \frac{P}{\rho U^{*2}}, \qquad \tilde{T} = \frac{T - T_\infty}{T_W - T_\infty}, \qquad U^* = \frac{\alpha}{D},$$

$$Bo = \frac{g\beta(T_W - T_\infty)D^3}{\alpha^2}, \qquad Pr = \frac{\nu}{\alpha}, \qquad \tilde{x} = \frac{x}{D},$$

$$\tilde{y} = \frac{y}{D}, \; \tilde{u} = \frac{u}{U^*}, \; \tilde{v} = \frac{v}{U^*}$$

The boundary conditions are defined as follows:

a - Inlet: $\tilde{u} = \frac{\partial \tilde{v}}{\partial \tilde{y}} = \tilde{T} = 0$

b - Outlet: $\frac{\partial \tilde{u}}{\partial \tilde{y}} = \frac{\partial \tilde{v}}{\partial \tilde{y}} = \frac{\partial \tilde{T}}{\partial \tilde{y}} = 0$

c - Confining walls (adiabatic): $\tilde{u} = \tilde{v} = \frac{\partial \tilde{T}}{\partial \tilde{x}} = 0$

d - Symmetry plane: $\tilde{u} = \frac{\partial \tilde{v}}{\partial \tilde{x}} = \frac{\partial \tilde{T}}{\partial \tilde{x}} = 0$

e - On the cylinder: $\tilde{u} = \tilde{v} = 0, \; \tilde{T} = 1$

The problem is solved for wall distance to cylinder diameter ratios of $(t/D = 2.5, 3, 4, ..., 8, 12)$, cylinder to cylinder spacing to cylinder diameter ratios of $(S/D = 7, 21)$, different number of cylinders (2, 3, 5, 7) and Rayleigh numbers (Ra=300, 600, 1000), using a finite element method. Linear quadrilateral elements for velocity and temperature are employed. Pressure is assumed constant in each element. A penalty function has been employed to eliminate the pressure term at element level [10].

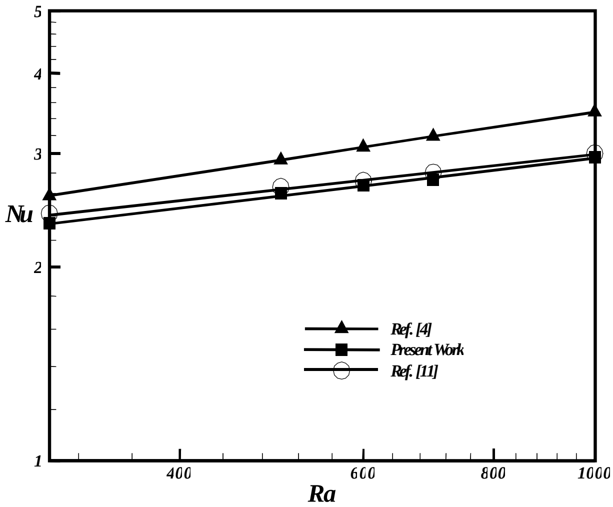

Figure 3. Variation of Nu with Ra for a single unconfined cylinder.

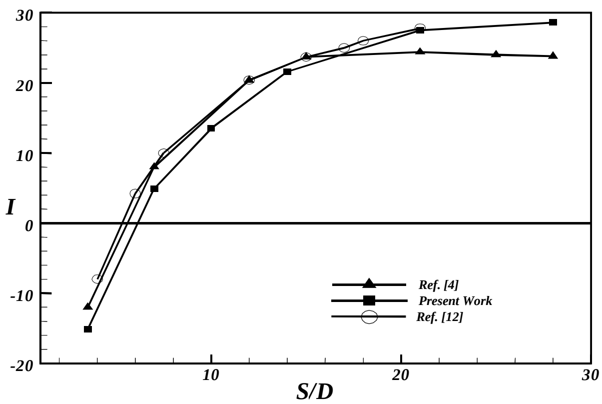

Figure 4. Heat transfer enhancement for upper cylinder, n=2 ($I = \frac{Nu_2 - Nu_1}{Nu_1}$).

5. RESULTS AND DISCUSSIONS

Numerical solutions were generated for Pr=0.7. Figure 3 shows the comparison of the present work with the results obtained by Sadeghipour and Asheghi [4] and Badr [11]. Present results agree very well with those from the numerical solution of Badr (with less than 2% difference). However, comparing the results with the experimental solution of Sadeghipour and Asheghi indicated a difference of 12 to 15%. In figure 4, the results of the present numerical solution are compared to the existing literature for the case of two parallel cylinders. In this figure, notation I represents the heat transfer enhancement of the upper cylinder, due to the presence of the lower cylinder. Nu_1 and Nu_2 denote the Nusselt number for lower and upper cylinders, respectively. The difference between the experimental and numerical results can be considered acceptable compared to the discrepencies between the experimental results of [4] and [12].

For the configuration and geometry of the present study, calculations are conducted using five different mesh systems when the cylinder circumference is divided to 64, 128, 192, 256, and 320 parts, respectively. Figures 5 and 6 show the velocity and temperature profiles at y=3D for different mesh systems, for the case of n=3 and $S/D=7$. The solution for N=192 parts can be considered mesh-independent.

In figures 7a and 7b, variation of the Nusselt

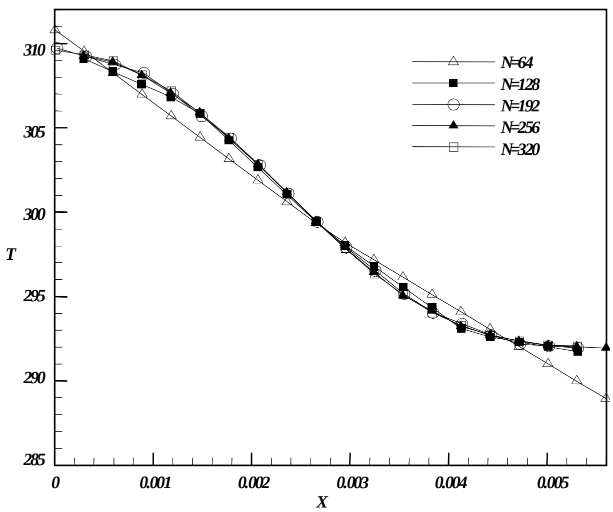

Figure 5. Temperature profile at y=3D for different mesh systems, Case: $S/D=7$ and n=3.

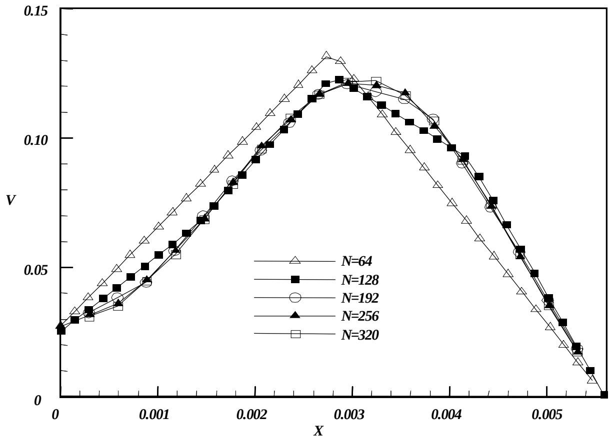

Figure 6. Velocity profile at y=3D for different mesh systems, Case: $S/D=7$ and n=3.

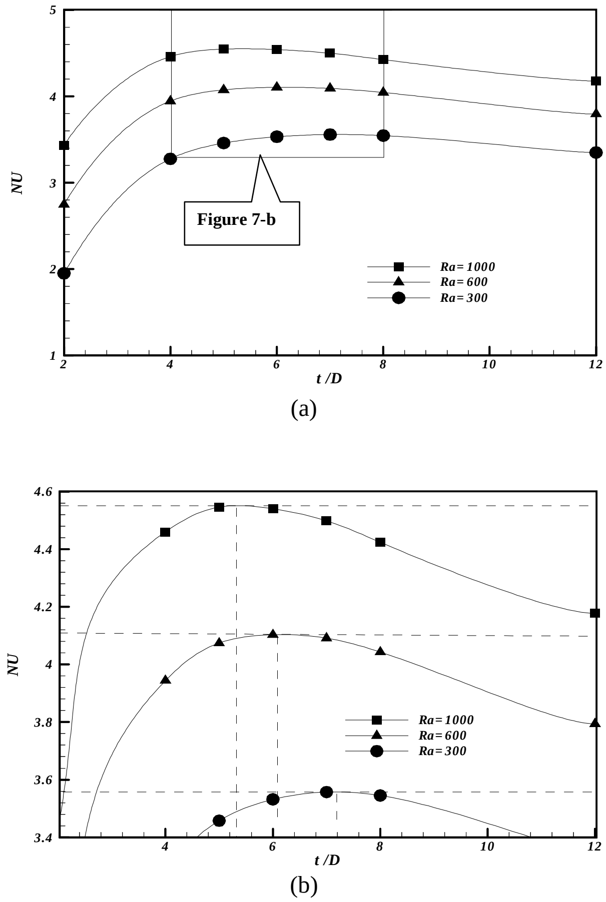

Figure 7. Variation of the Nusselt number with the ratio t/D for different Rayleigh numbers (Ra=300, 600 and 1000), when n=7 and S/D =21.

TABLE 1. Maximum Nu and Optimum Wall Spacing for Different Arrays, Ra=300.

	$(t/D)_{opt}$		$(\overline{Nu})_{max}$	
N	$(S/D)=7$	$(S/D)=21$	$(S/D)=7$	$(S/D)=21$
2	3.1	5.5	3.35	3.58
3	3.4	6.1	3.10	3.64
4	5.4	6.6	2.82	3.66
5	6.4	7.2	2.60	3.56

TABLE 2. Maximum Nu and Optimum Wall Spacing for Different Arrays, Ra=600.

	$(t/D)_{opt}$		$(\overline{Nu})_{max}$	
N	$(S/D)=7$	$(S/D)=21$	$(S/D)=7$	$(S/D)=21$
2	2.9	4.5	3.90	4.15
3	3.1	5.2	3.60	4.22
4	3.4	5.8	3.25	4.30
5	5.1	6.1	3.02	4.10

TABLE 3. Maximum Nu and Optimum Wall Spacing for Different Arrays, Ra=1000.

	$(t/D)_{opt}$		$(\overline{Nu})_{max}$	
N	$(S/D)=7$	$(S/D)=21$	$(S/D)=7$	$(S/D)=21$
2	2.6	3.1	4.40	4.64
3	2.8	4.5	4.15	4.70
4	3.0	5.15	3.7	4.73
5	3.1	5.3	3.42	4.55

number with the ratio t/D for different Rayleigh numbers (Ra=300, 600 and 1000), when n=7 and S/D=21 are shown. As can be seen from these figures, the variations of Nu with t/D for the two extreme cases of $t/D \to 1$ and $t/D \to \infty$ are in good agreement with Equations 6, 19 and 27. In other words, the Nu curves versus t/D shows a sharp variation for the limiting case of $t/D \to 1$, while for the extreme case of $t/D \to \infty$, the variation of Nu is rather smooth. These behaviors are represented by the two terms $(t/D)^3$ and $(t/D)^{-0.65}$ predicted by Equations 6 and 19, and (27), respectively. It can be seen from figures 7a and 7b that a range of t/D will provide nearly the optimum heat transfer. This is interesting as the exact location of the maximum point may not be necessary from a practical point of view. However, to obtain optimum rate of heat transfer the walls should be positioned within the range.

The optimal wall spacing and the maximum average Nusselt numbers for different array of cylinders and different Rayleigh numbers (Ra=300, 600, 1000) are shown in tables 1, 2 and 3. The significant point is that increasing the number of cylinders or the cylinder to cylinder spacing or decreasing the Rayleigh number will increase the optimal spacing of confining walls. All the results obtained from the numerical solution are consistent with those predicted by the theoretical analyses and given in Equations 28 and 29.

6. CONCLUSION

The results of this study reveal that there exists a distance between the confining walls for which the Nusselt number is maximum. By increasing the number of cylinders or their spacing, or, decreasing the Rayleigh number the optimal spacing will increase. Moreover, by increasing Rayleigh numbers, cylinder to cylinder spacing and number of cylinders (if their spacing "S" is large), $\overline{Nu}$ will increase more than 40%. If it is intended to achieve this increase, for the case with no confining walls, it can be realized by increasing the cylinder to cylinder spacing. However, this would not be a favorable design option, because of the space limitation.

7. NOMENCLATURE

C_D	Drag coefficient for the cylinders
C_P	Thermal capacitance
D	Diameter of the cylinders
f	Friction factor
g	Gravitational acceleration
Gr_D	Grashof number
H	Height of the walls
h	Heat transfer coefficient
k	Thermal conductivity of air
l	Length of the cylinders (=1)
$\dot{m}$	Mass flow rate
Nu	Nusselt number
$\overline{Nu}$	Average Nusselt number
P	Pressure
Pr	Prandtl number
$\dot{Q}$	Total rate of heat transfer
Q_{conv}	Heat transfer by convection
Ra_D	Rayleigh number
Re_D	Reynolds number
t	Wall spacing
$\bar{t}$	Equivalent wall spacing
T	Temperature
x, y	Cartesian coordinates
u, v	Velocity components

Greek Letters

α	Thermal diffusivity
β	Coefficient. of volumetric thermal expansion
τ	Shear stress on the wall
υ	Kinematic viscosity
ρ	Density

Subscripts

1	Inlet condition
2	Outlet condition
∞	Ambient
w	Wall condition

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