HEAD-DRIVEN SIMULATION OF WATER SUPPLY NETWORKS

M. Tabesh

*Department of Civil Engineering, University of Tehran
P.O. Box 14155-6457,Tehran, Iran, mtabesh@shafagh.ut.ac.ir*

T. T. Tanyimboh and R. Burrows

*Department of Civil Engineering, University of Liverpool
Liverpool L69 3BX, England*

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Abstract Up to now most of the existing water supply network analyses have been based on demand-driven simulation models. These models assume that nodal outflows are fixed and are always available. However, this method of simulation neglects the pressure-dependent nature of demand that is characterized by changes in actual nodal outflows particularly during critical events like major mechanical or hydraulic failures including local excessive demands. A novel approach is presented herein for head-driven simulation of water distribution networks. The methodology is based on the Newton-Raphson method and incorporates, directly, the relationship between nodal outflows and pressures. Through several examples, the applicability and advantages of this new formulation are demonstrated including accuracy and computational efficiency.

Key Words Water Supply Networks, Head-Driven Simulation, Demand-Driven Simulation, Pressure-Outflow Relationship, Newton-Raphson Method

چکیده در حال حاضر اکثر مدلهایی که شبکه‌های آبرسانی را تحلیل می‌نمایند از روشی مبتنی بر تقاضا استفاده می‌کنند. در این مدلها فرض می‌شود که تقاضا در گره ها همیشه ثابت و قابل دسترس می‌باشد. این روش شبیه سازی، وابستگی مصرف با فشار که از عوامل مؤثر تغییر میزان جریان خروجی در گره ها بخصوص در حین شرایط بحرانی است را در نظر نمی‌گیرد. شرایط بحرانی در مواقع خرابی (شکست) مکانیکی اجزاء و متعلقات شبکه و یا افزایش موضعی تقاضا ایجاد می شود و باعث افت فشار و آشفتگی هیدرولیکی در شبکه می‌گردد. این مقاله با در نظر گرفتن رابطه‌ای منطقی بین جریان خروجی و فشار در گره ها، روش جدیدی برای تحلیل هیدرولیکی مبتنی بر فشار در شبکه های آبرسانی ارایه می‌نماید. در این روش رابطه دبی فشار در گره ها به طور مستقیم در معادلات حاکم وارد گردیده و دسته معادلات هیدرولیکی به روش نیوتن رافسون حل می‌شوند. سپس از طریق چند مثال، واقعی بودن و فواید روش جدید (همچون دقت و کفایت محاسباتی) ارائه می گردد. براساس نتایج بدست آمده، این روش بخوبی قادر است اثرات ناشی از شرایط بحرانی که باعث کاهش فشار در گره ها می‌گردد را منعکس نموده و میزان واقعی جریان خروجی گره ها که در عالم واقع بدست مصرف کننده می رسد را مشخص کند.

INTRODUCTION

In the recent past, several papers have appeared on the subject of analysis and optimal design of water supply networks consisting of a number of sources, pipes, valves, pumps and reservoirs. These analyses usually have as their objective the need for an efficient technique to compute the pressure and flows in a defined network at defined levels of computational accuracy, given the characteristics of the network such as the pipe lengths, diameters and friction factors as well as the hydraulic characteristics of all ancillary plant such as pumps and valves.

Most network simulation models used in current engineering practice are based on the conventional Demand Driven Simulation Method (DDSM). They assume that nodal outflows are fixed and are satisfied regardless of network pressures. The assumption simplifies the mathematical solution of the problem but is not always appropriate because it is clear that the amount of outflow at nodal outlets depends on network pressures. If the pressure falls below a minimum required level (due to some critical events such as mechanical and hydraulic failures or excess demand), the flow will be significantly reduced. Although some nodes may be able to satisfy their demands, others may meet the demand partially while the rest may fail and may not provide any water at all. The assumption of fixed nodal consumptions is therefore valid only under normal conditions when the pressures can be expected to be adequate to satisfy the stipulated demands. If the operation of the system is simulated under pressure-critical conditions, the relationship between pressure and outflow should, therefore, be taken into account if the simulation results are to be realistic [1-7]. This kind of analysis in which the relationship between nodal outflow and pressure is explicitly considered is referred to herein as Head Driven Simulation Method (HDSM).

The terms 'outflow' and 'demand' should be clearly distinguished. Demand is the quantity of water required at the nodal outlet but outflow is the quantity, which the network actually yields, this is influenced by the hydraulic characteristics of the network as a whole including the outflows at other nodes [5-8].

From the early 1980s different researchers have referred to the importance and necessity of considering the pressure dependency of nodal consumption in water distribution systems modeling from different points of view [2-4,6,9,11]. For example, [12] stated that reduced service (i.e. 0 < nodal outflow < demand) should be recognized and accounted for somehow and any shortfall in flow should be reflected by network reliability measures. Also, [13] pointed out that in some developing countries where the water distribution systems operate intermittently, the lack of adequate pressure leads to substantially less discharge than the requirement (demand) and very short duration of supply. It is, therefore, necessary to develop a network analysis methodology that automatically takes into account the variation of nodal outflows with pressure.

Some researchers have previously looked for an explicit relationship between nodal head and outflow [1,4,5,9,14-20], but this paper does not delve into the issue. Rather, the aim of this paper is to present an analysis algorithm in which a realistic pressure-outflow relationship is incorporated directly into the main set of non-linear constitutive equations. Through a number of examples, the accuracy of the results and the computational efficiency of the new methodology are discussed. The results suggest that the proposed HDSM can simulate networks with insufficient pressure in a realistic way (unlike DDSM) without any significant loss of computational efficiency (compared to DDSM). Furthermore, unlike the cumbersome nature of previous HDSM methods [10,21] the present formulation is easy to implement.

NODAL PRESSURE-OUTFLOW RELATIONSHIPS

During the last decade, several formulas have been suggested to describe the pressure dependency of nodal consumption (outflow). A comprehensive comparison of these relationships can be seen in [6]. He concluded that a parabolic relationship (no flow at minimum head to required flow at desirable head) was sufficiently representative of the hydraulic performance of networks. In this section, first, the parabolic head-outflow relationship is presented. Then more recently published relationships are discussed.

The parabolic head-outflow relationship is shown graphically in Figure 1 and can be expressed as [6,8,13,20]:

$$Q_j^{avl} = Q_j^{req}; \text{ if } H_j \geq H_j^{des} \tag{1a}$$

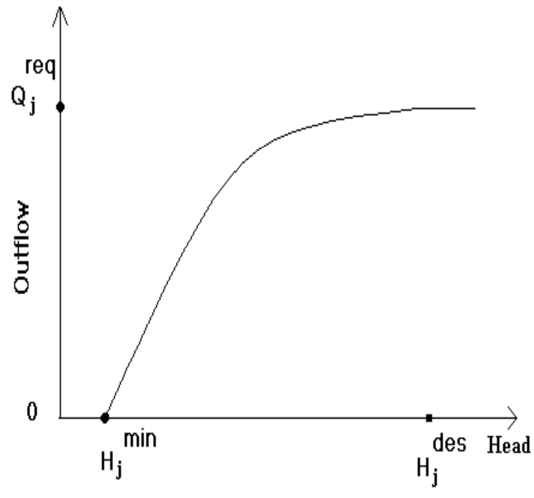

Figure 1. Head-outflow relationship [4].

$$Q_j^{avl} = Q_j^{req} \left(\frac{H_j - H_j^{min}}{H_j^{des} - H_j^{min}} \right)^{(\frac{1}{n})}; \text{ if } H_j^{min} \prec H_j \prec H_j^{des} \quad (1b)$$

$$Q_j^{avl} = 0; \text{ if } H_j \leq H_j^{min} \quad (1c)$$

where Q_j^{avl} and Q_j^{req} are available outflow and demand at node j, respectively. H_j^{des} is the desired head /pressure to satisfy the demand. H_j is the available head and H_j^{min} the minimum head at node j. n is an exponent usually between about 1.5 and 2 [6,8,13,20,21]. Herein a value of 2 is applied for n.

The minimum head (H_j^{min}) below which no flow can be discharged may be taken as the minimum outlet level in the locality served by the node. In the absence of field data it may be set equal to ground elevation. The desired head (H_j^{des}) below which the nodal demand cannot be totally satisfied might typically be about 14 to 15 m or more [22,23]. Under certain circumstances, the absolute minimum desired pressure is suggested to be 7 m [24,25]. The desired head can also be calculated through the following equation [8,21]

$$H_j^{des} = H_j^{min} + K_j (Q_j^{req})^n \quad (2)$$

in which K_j is an empirical resistance factor for node j.

Considering the required and absolute minimum pressures as 14 and 7 m, respectively, [15] used a fuzzy relationship between the nodal availability (equivalent to Q_j^{avl}/Q_j^{req}) and head similar to the cumulative normal distribution to represent the variation of available outflow for residual pressures below the desired value. However, their lower limit of nodal availability seems to be questionable because when pressure reaches the absolute minimum value, 50% of demand is still available. This would appear to be because of their consideration of an absolute minimum pressure required for proper system operation, which results from hydraulic constraints on the operation of fire fighting equipment [6]. However, as fire-fighting operations would be infrequent, their formulation would appear to be a special case, which, consequently, overestimates the values of available outflow. (Also see [23]).

Later, [16] proposed an expected served demand concept, which took into account both insufficient heads and flows at individual nodes in the network. The relative effectiveness of nodal head (equivalent to Q_j^{avl}/Q_j^{req}), termed nodal hydraulic availability therein, was defined as a non-decreasing smooth function of head, taking values between zero and one, the values being zero below minimum head level and one above the desired head level. Their approach, therefore, further refined the availability concept in that at H_j^{min} availability was zero and at H_j^{des}, one. Furthermore, the nodal hydraulic availability during reduced service mode in which H_j is not fully satisfactory was defined as a differentiable function of head, H, as follows

$$\frac{Q_j^{avl}}{Q_j^{req}} = \frac{\int_{H_j^{min}}^{H_j} (H - H_j^{min})(H_j^{des} - H)dH}{\int_{H_j^{min}}^{H_j^{des}} (H - H_j^{min})(H_j^{des} - H)dH}; \text{ if } H_j^{min} \prec H_j \prec H_j^{des} \quad (3)$$

Although the above equation can be applied to any network, it is not as straightforward as Equation 1b to use and more computational effort is needed for its evaluation.

Herein, Equation 1 is used to represent the head-outflow relationship because it is a continuous function with realistic upper and lower bounds for outflow. Also, it can represent the behavior of the system reasonably.

REVIEW OF ALGORITHMS FOR PRESSURE-DRIVEN NETWORK ANALYSIS

In the review of algorithms to analyze the hydraulic equations of the system including pressure dependency of demand, different approaches can be seen in the literature [20,21] used a two-phase formulation. Thus, using a conventional demand-driven simulation the head value at each node was obtained. Then the head-outflow relationship of Equation 1 was used to calculate the outflows for those nodes with head value less than desired ones [20,21]. In addition, the iterative scheme of [21] repeats the above procedure until there would be no significant changes in nodal outflows or pressures between successive iterations.

Calculating nodal heads by DDSM, a corrected nodal outflow was obtained in [13] by the Newton-Raphson univariate iterative formula, i.e.

$$Q_j = Q_j^{avl} - \frac{H_j^{min} + K_j (Q_j^{avl})^n - H_j}{nK_j (Q_j^{avl})^{n-1}} \quad (4)$$

in which Q_j is the updated outflow for nodes with less than fully satisfactory pressures, H_j, in the range $H_j^{min} < H_j < H_j^{des}$ and Q_j^{avl} represents the previous value of nodal outflow. Also, for nodes with $H_j < H_j^{min}$, $Q_j = 0$ and for nodes with $H_j > H_j^{des}$, Q_j = demand.

Although the analysis incorporated the pressure dependency of demand, it was also a two-phase approach. The disadvantage of the method is that there is an intermediate step between iterations in which nodal pressures are checked and modified outflows calculated.

The algorithm of [16] to calculate the available outflows at each node was based on an optimization procedure which maximized the sum of the available outflows over all demand nodes. The head-outflow relationship was approximated by means of the nodal hydraulic availability approach of Equation 3 while the other hydraulic characteristics of the system were considered as constraints. The disadvantage of the approach is that it involves the solution of a difficult-to-solve non-linear programming problem, which is computationally expensive.

To improve the weaknesses of the above-mentioned approaches, a fully integrated algorithm is clearly needed to carry out the pressure-dependent network analysis. Such an algorithm is presented in the next section.

STEADY STATE HEAD-DRIVEN ANALYSIS OF WATER SUPPLY NETWORKS

The governing equations for flow in water supply networks can be set up by considering the basic physical laws, i.e. the equations of continuity applied at each node and conservation of energy applied to each loop or path.

Different methods of computation have been developed (e.g. Hardy-Cross, Newton-Raphson and Linear Theory) and many computer programs have been produced to solve the conventional network analysis problem [17,26,28,29,30,31].

In comparison with other solution methods, the Newton-Raphson method has good convergence characteristics [17,32,33]. Herein, a Newton-Raphson-based method has been chosen and the pressure dependency of demand incorporated in the system of equations as shown shortly.

The continuity equation for each node j, j = 1, ..., NJ, may be written as

$$\sum_{i:H_i<H_j} Q_{ij} - \sum_{i:H_i>H_j} Q_{ij} = Q_j^{avl} \quad (5)$$

where Q_{ij} is the flow in pipe ij and NJ is the number of the nodes in the network. Using the Hazen-Williams equation for flow in a pipe, Equation 5 becomes

$$F_j \equiv \sum_{i:H_i<H_j} \left(\frac{H_i - H_j}{K_{ij}}\right)^{\left(\frac{1}{n}\right)} - \sum_{i:H_i>H_j} \left(\frac{H_i - H_j}{K_{ij}}\right)^{\left(\frac{1}{n}\right)} - Q_j^{avl} = 0 \quad (6)$$

in which n = 1.852 and F_j represents the continuity equation for node j. H_i and H_j are piezometric heads at nodes i and j, respectively. K_{ij} is a resistance coefficient for pipe ij and Q_j^{avl} is the outflow of node j. The values of K_{ij} can be obtained as follows

$$K_{ij} = \frac{10.675 \cdot L_{ij}}{CHW_{ij}^{1.852} \cdot D_{ij}^{4.87}} \quad (7)$$

where L_{ij} is length of pipe ij (m), CHW_{ij} is the Hazen-Williams coefficient for pipe ij and D_{ij} is diameter of pipe ij (m).

Other network components (e.g. pumps, valves, reservoirs, etc.) can be included in Equation 6 in a similar way. The head-flow relationships for some of these components are indicated briefly as follows.

1) Pump The head flow relationship of a pump may be typically approximated by a parabolic curve as follows

$$H_p = H_j - H_i = AQ_{ij}^2 + BQ_{ij} + C \quad (8)$$

in which A, B and C are constants (usually set by the manufacturer). H_p is the head lift of the pump and Q_{ij} is the flow delivered by pump. H_i and H_j are the heads at the upstream and downstream nodes of the pump, respectively.

2) Non-Return Valve (NRV) The head-flow relationship for a pipe fitted with a non-return valve which allows flow in one direction only is

$$Q_{ij} = (\frac{H_i - H_j}{K_{ij}})^{(\frac{1}{n})}; \text{ if } H_i \succ H_j \tag{9a}$$

$$Q_{ij} = 0; \text{ if } H_i \leq H_j \tag{9b}$$

in which Q_{ij} is the flow in the pipe having the NRV and n = 1.852 (see Equations 6-7).

3) Flow Control Valve (FCV) The head-flow relationship for a pipe having an FCV is

$$Q_{ij} = K_v (\frac{|H_i - H_j|}{K_{ij}})^{(\frac{1}{n})} \text{sgn}(H_i - H_j) \tag{10}$$

where $0 \leq K_v \leq 1$ is a continuous valve control parameter.

4) Pressure Reducing Valve (PRV) A pressure reducing valve produces a constant outlet pressure for a range of higher inlet pressures. For a PRV the head flow relationship is given as

$$Q_{ij} = (\frac{H_{PRV} - H_j}{K_{ij}})^{(\frac{1}{n})}; \text{ if } H_j \leq H_{PRV} \leq H_i \tag{11a}$$

$$Q_{ij} = (\frac{H_i - H_j}{K_{ij}})^{(\frac{1}{n})}; \text{ if } H_j \leq H_i \prec H_{PRV} \tag{11b}$$

$$Q_{ij} = 0; \text{ if } H_j \geq H_{PRV} \tag{11c}$$

in which H_{PRV} is the pressure reducing valve setting corresponding to the constant outlet head.

INCORPORATION OF PRESSURE DEPENDENT OUTFLOWS IN THE GOVERNING EQUATIONS

As concluded earlier from the various head-outflow relationships proposed in the literature, the approach [20] has been chosen as a good representation of the pressure dependency of nodal outflows. Because outflow is a function of head, it seems to be more reasonable that this dependency be accounted for in the main set of hydraulic equations throughout the analysis procedure.

The head-dependent outflow term can be added to the continuity equations of the system as follows, giving in general NJ equations with NJ unknowns.

$$F_j \equiv \sum_{i=1}^{NJ_j} (\frac{|H_i - H_j|}{K_{ij}})^{0.54} \text{sgn}(H_i - H_j) + Q_j^{req} (\frac{H_j - H_j^{min}}{H_j^{des} - H_j^{min}})^{0.5} = 0 \tag{12}$$

in which $H_j^{min} \leq H_j \leq H_j^{des}$, NJ_j is the number of nodes directly connected to node j and

$$\text{sgn}(H_i - H_j) = 1; \text{ if } H_i \succ H_j \tag{13a}$$

$$\text{sgn}(H_i - H_j) = 0; \text{ if } H_i = H_j \tag{13b}$$

$$\text{sgn}(H_i - H_j) = -1; \text{ if } H_i \prec H_j \tag{13c}$$

From Equations 1, the second term of Equation 12 is equal to Q_j^{req}, if $H_j = H_j^{des}$ and it is equal to zero when $H_j = H_j^{min}$. Based on the Newton-Raphson method and choosing the nodal piezometric heads as unknown parameters, Equation 12 would be solved by the following iterative scheme:

$$J^m \underline{\Delta H}^m = \underline{F}(\underline{H}^m) \tag{14a}$$

$$\underline{H}^{m+1} = \underline{H}^m - \underline{\Delta H}^m \tag{14b}$$

in which $\underline{H}$ is the vector of unknown heads, the matrix J is the Jacobian of the set of equations, $\underline{\Delta H}$ is the vector of the respective changes in nodal heads and $\underline{F}$ is the vector of the respective values of the nodal continuity expressions, i.e. F_j, j = 1, ..., NJ. The iteration number is denoted by m.

The elements of the Jacobian matrix for each nodal equation are given by

$$\frac{\partial F_j}{\partial H_i} = 0.54 (\frac{|H_i - H_j|^{-0.46}}{K_{ij}^{0.54}}); \quad \forall j, \forall i : i \neq j \tag{15a}$$

$$\frac{\partial F_j}{\partial H_j} = -0.54 \sum_{i=1, i\neq j}^{NJ_j} (\frac{|H_i - H_j|^{-0.46}}{K_{ij}^{0.54}}) + \frac{0.5 Q_j^{req}}{(H_j^{des} - H_j^{min})} (\frac{H_j - H_j^{min}}{H_j^{des} - H_j^{min}})^{-0.5}; \quad \forall j, \forall i : i \neq j \tag{15b}$$

The second term of Equation 15b would be applicable when $H_j^{min} \leq H_j \leq H_j^{des}$ and it is zero otherwise, since Q_j^{avl} is zero when head drops below H_j^{min} and Q_j^{avl} = demand (a constant) when the head exceeds H_j^{des}.

As can be seen, the incorporation of the pressure-dependent demand term in the main set of equations does not lead to any new equations or unknowns, so the basic structure of the Jacobian remains unchanged. It can, therefore, be expected that the computational characteristics of the solution methodology will not be highly affected.

To improve the computational efficiency (faster convergence), some modifications have been made in the Newton-Raphson method herein. First, instead of using Equation 14b, an approximation to the value of z^*, i.e. the value of z which minimizes the single variable function $f(\underline{H}^m - z\underline{\Delta H}^m)$ is found (see [34]). The new point in the sequence is now given by

$$\underline{H}^{m+1} = \underline{H}^m - z' \underline{\Delta H}^m \quad (16)$$

in which $z' \cong z^*$. The above procedure ensures that $\underline{H}^{m+1}$ is a better approximation to the solution than $\underline{H}^m$. Second, to avoid head oscillations for some demand nodes, a modification is made by averaging the computed values of head obtained at the (m)th and (m-1)th iterations.

The proposed algorithm can be summarized as follows [6].

1) Guess initial heads, H_j^0, for all nodes other than fixed head nodes.

2) Solve the system of non-linear equations, Equations 12 - 13.

3) Determine improved estimates of nodal heads using Equation 14 - 16.

4) Repeat steps 2 and 3 until the convergence criteria is satisfied.

5) Calculate available nodal outflows, Q_j^{avl}.

A Fortran computer program has been developed based on the above algorithm. This has been implemented on a PC with a 75 MHz pentium processor and 8 Mbyte RAM. In addition to the normal operating condition, the program developed for HDSM is capable of simulating failure of any component. Using only the data for the fully connected network, the program can automatically simulate the consequences, in terms of available flow, of the failure of up to any two-network components. The accuracy of the results and efficiency of the above methodology is illustrated by the following examples. The tolerances used in the examples were 0.001 m for nodal heads.

BENCHMARK SOLUTIONS

The first example is taken from [21] and the layout of the network is shown in Figure 2. The lengths and Hazen-Williams coefficients for all pipes are 1000 m and 130, respectively. The diameters for pipes 1 through 4 are 400, 350, 300 and 300 mm, respectively. The node resistance coefficient, K_j, and available flow exponent, n, are equal to 360 (s^2/m^5) and 2, respectively. The node data of the network along with the HDSM analysis results are presented in Table 1. The DDSM results are also shown for comparison.

It can be seen from the head-driven simulation (HDSM) results in Table 1 that the network is pressure deficient as the demand of node 4 of 3 m^3/min is only partially satisfied, the actual outflow being 0.381 m^3/min. To check the accuracy of the results of the proposed formulation and to demonstrate the effects of variations in the source head on available outflows, the source head for this network has been varied from 85 to 110.89 m, and the available outflow at each node based on HDSM can be observed in Table 2. As can be seen, these values are essentially the same as the results of [21] and, therefore, confirm the accuracy of the present formulation. The results in Table 2 demonstrate the reliability of the model in terms of

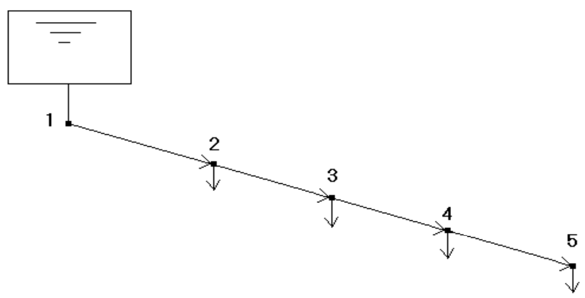

Figure 2. Simple network of Example 1 [11].

TABLE 1. Nodal Data and Results for the Network of Example 1 (Figure 2).

Node	Input Data			Output results			
				DDSM		HDSM	
	H_j^{min} (m)	H_j^{des} (m)	Q_j^{req} (m^3/min)	H_j (m)	Q_j^{avl} (m^3/min)	H_j (m)	Q_j^{avl} (m^3/min)
1[a]	-	100.0[b]	11.0	100.000	11.000	100.000	8.381
2	90.0	90.4	-2.0	95.131	-2.000	97.053	-2.000
3	88.0	88.4	-2.0	88.698	-2.000	93.647	-2.000
4	90.0	90.9	-3.0	80.139	-3.000	90.015	-0.381
5	85.0	86.6	-4.0	77.103	-4.000	86.982	-4.000

a Source b Available Source Head

TABLE 2. Available Nodal Outflows for Different Source Head Values in Example 1.

Source Head (m)	Available outflow (m^3/min) at indicated node:				Total Supply to the Network (m^3/min)
	2	3	4	5	
85.00	-0.000	-0.000	-0.000	-0.000	0.000
	(-0.000)[a]	(-0.000)	(-0.000)	(-0.000)	(0.000)
88.87	-0.000	-0.000	-0.000	-2.424	2.424
	(-0.000)	(-0.000)	(-0.000)	(-2.420)	(2.420)
90.88	-0.000	-1.790	-0.000	-2.560	4.350
	(-0.000)	(-1.787)	(-0.000)	(-2.553)	(4.340)
91.96	-1.621	-2.000	-0.000	-2.592	6.214
	(-1.616)	(-2.000)	(-0.000)	(-2.586)	(6.202)
92.33	-2.000	-2.000	-0.000	-2.645	6.645
	(-2.000)	(-2.000)	(-0.000)	(-2.629)	(6.629)
98.50	-2.000	-2.000	-0.000	-4.000	8.000
	(-2.000)	(-2.000)	(-0.000)	(-4.000)	(8.000)
98.84	-2.000	-2.000	-0.000	-4.000	8.000
	(-2.000)	(-2.000)	(-0.000)	(-4.000)	(8.000)
110.89	-2.000	-2.000	-3.000	-4.000	11.000
	(-2.000)	(-2.000)	(-3.000)	(-4.000)	(11.000)

a Indicates Results of [21]

its ability to produce the correct results where pressures/outflows are less than fully satisfactory. As expected, when pressures are fully satisfactory both HDSM and DDSM give identical results.

Example 2 considers the applicability of the HDSM to more complicated networks, including ancillary components (e.g. pumps, reservoirs, etc.). Figure 3 shows the layout of this sample network that is taken from [13]. The Hazen-Williams coefficient is 120 for all pipes and H_j^{min} = 25.908 m for all nodes. The equation of each pump is represented by $H_p = -3823.64\ Q_p^2 + 27.172\ Q_p + 6.819$, where H_p is the head provided by the pump. Other features of the network are given in Table 3.

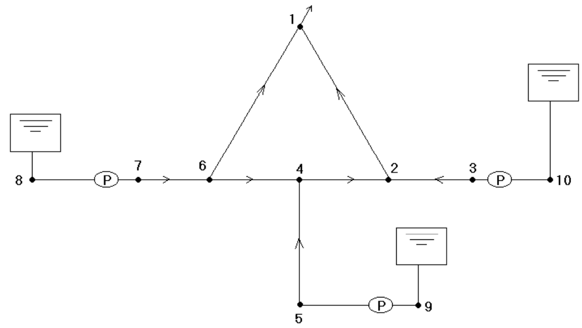

Figure 3. Layout of Example 2 [35].

To assess the effects of component failures on the hydraulic performance of the system, the HDSM is used assuming that only one link (pipe/pump) fails in each case. Table 4 shows values of inflow from the three sources and available flow at the demand node. It can be seen that in all cases of pipe failure, demand node 1 is in reduced service mode with all available outflows being less than the required demand of 0.0566 m^3/s. It can, therefore, be seen that HDSM can simulate the effects of mechanical failures on the hydraulic performance of the system.

TABLE 3. Pipe Data for Figure 3.

Pipe	Length (m)	Diameter (mm)
2-1	609.6	203.2
3-2	304.8	152.4
10-3	pump	-
4-2	304.8	203.2
5-4	304.8	152.4
9-5	pump	-
6-4	304.8	152.4
8-7	pump	-
7-6	304.8	203.2
6-1	609.6	152.4

The last example, Example 3, shows the application of the HDSM to a small real-world network taken from [6]. Figure 4 and Table 5, respectively, show the layout and physical characteristics of this branched system. In this network, the values of the minimum nodal heads

TABLE 4. Actual Nodal Inflows and Outflow for the Network of Figure 3 (m^3/sec).

Pipe failed	Node			
	1[b]	8[c]	9[c]	10[c]
None	-0.05660 (-0.05663)[d]	0.02373 (0.02381)	0.01127 (0.01127)	0.02160 (0.02155)
2-1	-0.02563	0.01481	0.00000	0.01082
6-1	-0.04331	0.01354	0.00850	0.02127
3-2	-0.04670	0.02874	0.01796	0.00000
4-2	-0.04689	0.01945	0.00000	0.02744
10-3[a]	-0.04670	0.02874	0.01796	0.00000
5-4	-0.05302	0.02766	0.00000	0.02436
6-4	-0.05623	0.02033	0.01339	0.02251
9-5[a]	-0.05202	0.02766	0.00000	0.02436
7-6	-0.04546	0.00000	0.01938	0.02608
8-7[a]	-0.04546	0.00000	0.01938	0.02608
	Available nodal outflow/inflow (m^3/sec)			

a Indicates Pipe Including Pump b Demand Node c Source Node d Indicates Results of [35]

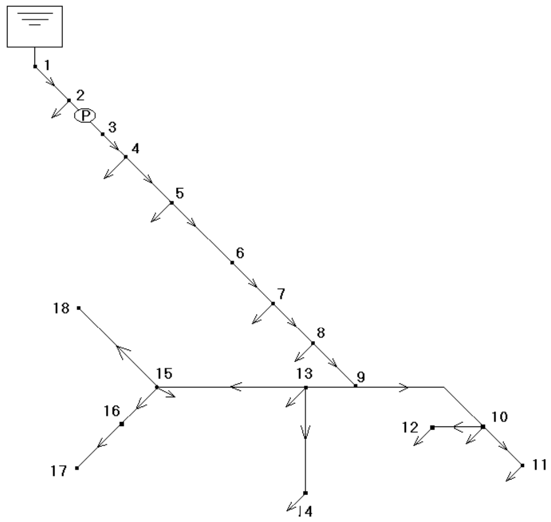

Figure 4. Layout of Example 3 (a real world case study).

are considered to be equal to the respective nodal ground levels and values of desired head are, somewhat optimistically, taken to be 7 m for each node. The hydraulic characteristics of the pump was represented by: $H_p = -11478.421Q_p^2 - 13822.773Q_p + 51.647$. For peak demand time (9:00 a.m.), the network was analysed by HDSM and the results are presented in Table 6. Values of available outflow at the nodes are identical to the respective demands except for nodes 2 and 14. It can be observed that the available head at node 2 is greater than the minimum but less than the desired head and so the demand is only partially satisfied. Also, at node 14 the available head is less than the assumed minimum head of 7 m and so the outflow is zero. This means that there could potentially be a shortfall in supply at nodes 2 and 14 during periods of high demand. These results suggest that any program based on DDSM cannot be relied upon to reproduce the real situation when available heads are not adequate. It can, therefore, be said that in comparison with DDSM, HDSM is better able to simulate the actual performance of the system and lead to more accurate and realistic results in terms of nodal head and flow.

DISCUSSION

The computational efficiency of the HDSM can be assessed in terms of the number of iterations required in the achievement of a solution to a chosen accuracy together with an overall accuracy measure for successive iterations. One such measure is the Euclidian norm defined as

$$\|\underline{\Delta H}\| = [\sum_{j=1}^{NJ} (\Delta H_j)^2]^{1/2} \qquad (17)$$

where ││ indicates the Euclidian norm.

Figure 5 illustrates the rapid convergence of the HDSM. Because the norm only measures the magnitude of the changes in head for all nodes, it may also be useful to examine the changes in head for successive iterations at some critical nodes. The critical node is taken as the node with the largest discrepancy between demand and available flow at the end of solution procedure. Figure 6 represents the variations of available heads at critical nodes against number of iterations. It can be seen that convergence of the solution using HDSM is good, bearing in mind that Figure 6 depicts conditions at the most pressure-critical nodes of the respective networks.

In Table 7 the number of iterations and computer run time for all the networks are presented. From

TABLE 5. Input Data for the Real Network of Figure 4.

Link	Diameter (mm)	CHW	Length (m)
1-2	76	100	1
2-3	pump	-	-
3-4	150	140	40
4-5	180	140	455
5-6	100	43	755
6-7	125	130	65
7-8	125	130	160
8-9	100	130	10
9-10	76	100	215
9-13	125	130	155
10-11	76	100	75
10-12	76	5	150
13-14	20	50	115
13-15	125	2	655
15-16	76	100	40
15-18	125	130	390
16-17	80	120	210

TABLE 6. HDSM Results for the Network of Figure 4 with Peak Demands.

Node	H_j^{min} (m)	H_j^{des} (m)	Q_j^{req} (l/s)	H_j (m)	Q_j^{avl} (l/s)
1 (Source)	84.3	86.0	2.130	86.000	1.780
2	84.3	91.3	-0.020	85.992	-0.010
3	84.3	91.3	-0.000	109.987	-0.000
4	84.0	91.0	-0.020	109.976	-0.020
5	72.0	79.0	-0.320	109.938	-0.032
6	83.0	90.0	-0.000	102.779	-0.000
7	82.8	89.8	-0.280	102.567	-0.280
8	82.6	89.6	-0.100	101.431	-0.100
9	82.6	89.6	-0.140	101.267	-0.140
10	84.0	91.0	-0.070	97.785	-0.070
11	87.0	94.0	-0.200	97.126	-0.200
12	86.0	93.0	-0.210	94.299	-0.210
13	83.5	90.5	-0.320	10.105	-0.320
14	89.5	96.5	-0.340	41.113	-0.000
15	63.9	70.9	-0.110	96.554	-0.110
16	63.8	70.8	-0.000	96.554	-0.000
17	61.6	68.6	-0.000	96.554	-0.000
18	64.6	71.6	-0.000	96.554	-0.000

the table it can be seen that in these particular examples with the same initial values of nodal heads, number of iterations and computational time of HDSM are close to the DDSM results. However, incorporation of the head-outflow relationship into the main set of non-linear equations may lead to a little increase in the number of iterations and computational time for the HDSM. Therefore, it can be said that, in general the HDSM represents the behavior of the physical system more realistically. Also, its computational efficiency is good, without significant difference with the DDSM.

SUMMARY AND CONCLUSIONS

A new methodology for pressure-driven analysis of water supply networks has been developed and its capability examined through a number of

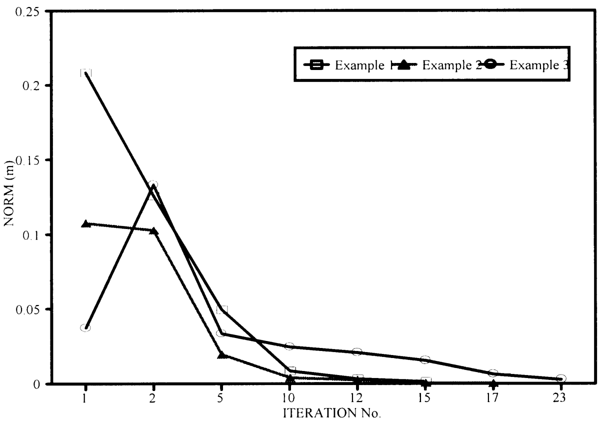

Figure 5. Convergence histories for Examples 1-3 using HDSM

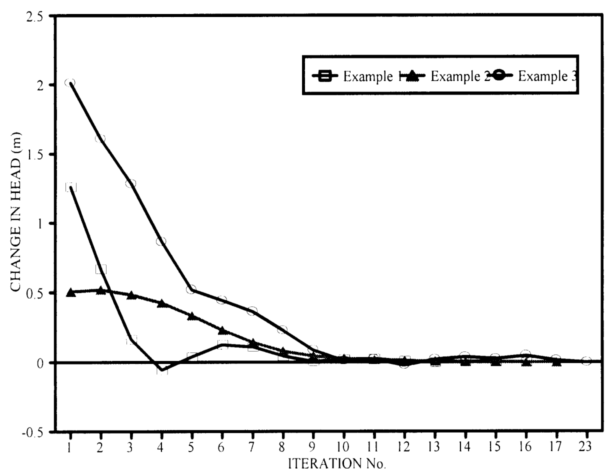

Figure 6. Changes in available head at Critical Nodes for Examples 1-3 using HDSM.

TABLE 7. Summary of Computational Efficiency for Fully-Connected Network of Examples 1-3 (with All Components Available).

Example	Type of Analysis	No. of iterations	CPU time (sec)
1[a]	DDSM HDSM	5 16	0.113 0.164
2	DDSM HDSM	17 17	0.275 0.275
3	DDSM HDSM	23 23	0.172 0.172

examples. It was observed that the HDSM works both for simple and realistic networks. However, unlike other formulations for pressure-driven simulation [13,21] the methodology presented herein does not require a separate step in which nodal outflows are adjusted at the end of each iteration. The proposed procedure explicitly incorporates a realistic head-outflow relationship in the continuity equations.

As observed by [6,21] the present method is equivalent to demand-driven simulation when flows and pressures are adequate such that designated demands are fully satisfied. In typical water supply applications this would usually be representative of normal operating conditions. However, under critical operating conditions e.g. pump failure, pressure-driven analysis (HDSM) can simulate the partial flow delivery realistically, whilst DDSM can only indicate that a supply problem will arise.

Finally, regarding computational efficiency, it has been observed that convergence of the iterative solution using HDSM compares favorably to an efficient DDSM implemented herein both in terms of CPU time and number of iterations. It would appear, therefore, that the proposed methodology has the potential to produce hydraulically more realistic results without any significant loss of computational efficiency compared to DDSM.

NOMENCLATURE

A,B,C = pump characteristics curve coefficients
CHW_{ij} = Hazen-Williams coefficient of pipe ij
D_{ij} = diameter of pipe ij
F_j = continuity function for node j
H_j = available pressure head at node j
H_j^{des} = desired pressure head at node j
H_j^{min} = minimum pressure head at node j
H_p = lift head across pump
H_{PRV} = outlet head of pressure reducing valve
J = Jacobian matrix
K_{ij} = resistance coefficient of pipe ij
K_j = resistance coefficient of node j
K_v = continuous valve control parameter
L_{ij} = length of pipe ij
n = an exponent
NJ = number of nodes
NJ_j = number of nodes directly feeding and fed by node j
Q_{ij} = flow in pipe ij
Q_j^{avl} = available outflow at node j
Q_j^{req} = required outflow (i.e. demand) at node j
Q_p = flow delivered by pump
z = step size
$|\ |$ = Euclidian norm

Subscripts

p = pump
PRV = pressure reducing valve
v = valve

Superscripts

* = optimum value
avl = available
des = desired
min = minimum
req = required

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