A FUZZY EXPERT SYSTEM FOR PREDICTING THE PERFORMANCE OF SWITCHED RELUCTANCE MOTOR

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(Received: November 28, 1999 - Accepted in Revised Form: November 2, 2000)

Abstract In this paper a fuzzy expert system for predicting the performance of a switched reluctance motor has been developed. The design vector consists of design parameters, and output performance variables are efficiency and torque ripple. An accurate analysis program based on Improved Magnetic Equivalent Circuit (IMEC) method has been used to generate the input-output data. These input-output data is used to produce the initial fuzzy rules for predicting the performance of Switched Reluctance Motor (SRM). The initial set of fuzzy rules with triangular membership functions has been devised using a table look-up scheme. The initial fuzzy rules have been optimized to a set of fuzzy rules with Gaussian membership functions using gradient descent training scheme. The performance prediction results for a 6/8, 4kw, SR motor shows good agreement with the results obtained from IMEC method or Finite Element (FE) analysis. The developed fuzzy expert system can be used for fast prediction of motor performance in the optimal design process or on-line control schemes of SR motor.

Key Words Fuzzy Expert System, SR Motor, Performance Prediction

چکیده در این مقاله یک سیستم فازی خبره برای تخمین رفتار موتور سوئیچ رلکتانس ارائه شده است. با بهره گیری از روش آنالیز دقیق IMEC اطلاعات مربوط به رفتار موتور نسبت به تغییر پارامترهای طراحی بدست می آید. بازده و ریپل گشتاور موتور به عنوان خروجی سیستم در نظر گرفته می شوند. سیس یک سری قوانین فازی به فرم IF-Then و با در نظر گرفتن توابع مثلثی شکل حاصل می‌شود. این توابع عضویت با استفاده از روش گرادیان به فرم گوسی بگونه ای بهینه می گردند که خطای خروجی سیتم فازی و خروجی واقعی از یک خطای از پیش تعیین شده (ε) کوچکتر گردد. مقایسه نتایج حاصل‌ه از عملکرد سیستم فازی خبره با روشهای آنالیز دقیق موتور مانند ،روش مدار معادل مغناطیسی بهبود یافته (IMEC) یا روش اجزاء محدود (FEM) نشان دهنده عملکرد مناسب این سیستم می باشد. .سیستم خبره ارائه شده می تواند در روشهای طراحی بهینه یا کنترل موتور سوئیچ رلکتانس بکار رود.

INTRODUCTION

Fuzzy expert systems are used to formulate human knowledge. Human knowledge about a particular engineering problem may be classified into two categories, conscious knowledge and sub-conscious knowledge. By conscious knowledge, we mean the knowledge that can be explicitly expressed in words, and subconscious knowledge refer to the situations where the human experts know what to do but can not express exactly in words how to do it. For conscious knowledge, the human expert knowledge can be expressed in terms of fuzzy If-Then rules and put them into fuzzy systems. For subconscious knowledge, when the expert is demonstrating, he can be viewed as a black box, which can be expressed into inputs and outputs; i.e., a set of input-output data pairs can be collected. In this way, the subconscious knowledge is transformed into a set of input-output pairs. Hence, the important task is to construct the fuzzy system form input-output pairs.

A reluctance motor is a variable reluctance motor in which the torque is produced by the tendency of rotor to move into aligned position with stator poles of excited phase where the winding inductance is maximized. SR motor and it's design are relatively new in the field of electrical machines and there is not as much experience and publication in this area as for the classical machines. Because of the highly saturated operating condition and the doubly salient structure of the motor, accurate analysis of SR motor is difficult and time consuming. Optimal design process of motor needs the information of motor performance. The information can be obtained from an expert designer or from sensitivity analysis of the motor parameters. The information can be formulated into a fuzzy expert system.

The response of fuzzy expert system is fast and relatively accurate. Hence, it can be used for optimal design of SR motor in optimizing processes, which are based on genetic algorithms, immune systems, and or neural networks and need numerous performance evaluations of motor performance during the optimization process.

DESIGN OF FUZZY SYSTEM

Methods for constructing fuzzy systems from input-output pairs exist. A number of such methods are as follows:

- Table Look-Up Scheme [1,2].
- Gradient Descent Training [1,3,4,5,6].
- Recursive Least Squares [1,7,8,9].

In the table look-up scheme the membership functions are fixed in the first step and do not depend on the input-output pairs; that is, the membership functions are not optimized according to the input-output pairs. However, in the gradient descent training, the gradient descent algorithm is used to determine the parameters of the membership functions in such a way that a certain criterion is optimized. The gradient descent algorithm minimizes a certain criterion, which accounts for the error of only one input-output pair. In this way, the training algorithm updates the parameters to match one input-output pair at a time.

The recursive least squares method is a training algorithm that minimizes the summation of the matching errors for all the input-output pairs.

In this paper, a set of fuzzy rules has been devised for a SR motor by using a table look-up scheme using the information obtained from an accurate analysis program. Then, using gradient descent training optimizes the fuzzy rules. This fuzzy expert system is used to predict the performance of the SR motor.

Design the Fuzzy System Using A Table Look-Up Scheme Suppose that the following input-output pairs are given from IMEC analysis program. This information has been obtained for the range of design parameter variations within the feasible limits [1].

$$(x_o^p; y_o^p) \quad p=1,2,\ldots,N \tag{1}$$

where:

$$x_o^p \in U \subset R^n \qquad y_0{}^p \in V \subset R$$

$$U=[\alpha_1,\beta_1]\times[\alpha_2,\beta_2]\times\ldots.\times[\alpha_n,\beta_n]$$

$$V=[\alpha_y,\beta_y]$$

The objective is to design a fuzzy system, F(x), based on the N input-output pairs. F(x) is defined as follows:

$$F(x)=\frac{\sum_{L=1}^{M}\bar{y}^{L}[\Pi_{i=1}^{n}\mu_{A_i^L}(x_i)]}{\sum_{L=1}^{M}[\Pi_{i=1}^{n}\mu_{A_i^L}(x_i)]} \tag{2}$$

Where, $\mu_{A_i^L}(x_i)$ is the i'th membership value of the i'th fuzzy set in the precedence predicate and $\bar{y}^L$ is the center of fuzzy set in the consequence predicate for i'th rule.

Now, the following five-step table look up scheme is proposed to design the fuzzy system:

Step 1: Define fuzzy sets to cover the input and output spaces

For each $[\alpha_i,\beta_i], i=1,2,...,n$ defines N_i fuzzy sets $A_i^j(j=1,2,...,N_i)$, which are required to be complete in $[\alpha_i,\beta_i]$; i.e. for any $x_i\in[\alpha_i,\beta_i]$, there exists A_i^j such that $\mu_{A_i^j}(x_i)\neq 0$. Similarly, define N_y fuzzy sets $B^j, j=1,2,...,N_y$, which are complete in $[\alpha_y,\beta_y]$.

Step 2: Generate one rule from one input-output pair.

First, for each input-output pair, $(x_{01}^p,...,x_{0n}^p;y_0^p)$, determine the membership values of $x_{0i}^p\ (i=1,2,..,n)$ in fuzzy sets $A_i^j\ (j=1,2,...,N_i)$ and the membership value of y_0^p in fuzzy sets $B^L(L=1,2,...,N_p)$. Then, for each input variable $x_i(i=1,2,...,n)$, determine the fuzzy set in which x_{oi}^p has the largest membership value, that is, determine A_i^{j*} such that $\mu_{A_i^{j*}}(x_{o_i}^p)\geq\mu_{A_i^j}(x_{o_i}^p)$ for j = 1,2, ..., N_I similarly, determine B^{L*} such that

$$\mu_{B^{L*}}(y_0^p)\geq\mu_{B^i}(y_0^p)\ for L=1,2,...,N_y.$$

Finally, obtain a fuzzy IF-THEN rule as:
If x_1 is A_1^{j*} and, …., and x_n is A_n^{j*}, THEN y is B^{L*}.

Step 3: Assign a degree to each rule generated in Step 2.

Since the number of input-output pairs is usually large and for each pair, a fuzzy rule is generated, we may have contradicting rules, i.e., rules with the same IF parts but different THEN parts.

To resolve the conflict, a degree will be assigned to each generated rule in Step 2 and only the rule with maximum degree from a conflicting group is kept. In this way, the conflicting rules will be discarded, and the number of rules is greatly reduced. The degree of a rule is defined as follows:

$$\text{D(rule)}=\prod_{i=1}^{n}\mu_{A_i^{j*}}(x_{0i}^p).\mu_{B^{L*}}(y_0^p) \tag{3}$$

Step 4: Create the fuzzy rule base.

Step 5: Construct the fuzzy expert system based on the fuzzy rule base.

Two numbers bound the number of rules in the final fuzzy rule base: N, the number of input-output pairs, and $\Pi_{i=1}^{n}N_i$, the number of all possible combinations of the fuzzy sets defined for the input variables.

The fuzzy rule base generated by this method may not be complete. So we may fill in the empty boxes by interpolating the existing rules.

Design of Fuzzy system using Gradient Descent Training In this section, a procedure is presented for a fuzzy system in which the membership functions are chosen to optimize a certain criterion [1].

First, it is assumed that the designed fuzzy system is of the following form [1]:

$$\text{F(x)}=\frac{\sum_{L=1}^{M}\bar{y}^{L}[\Pi_{i=1}^{n}\exp(-(\frac{x_i-\bar{x}_i^L}{\sigma_i^L})^2)]}{\sum_{L=1}^{M}[\Pi_{i=1}^{n}\exp(-(\frac{x_i-\bar{x}_i^L}{\sigma_i^L})^2)]} \tag{4}$$

Where M is the fixed number of fuzzy rules, and $\bar{y}^L, \bar{x}_i^L$ and σ_i^L are free parameters.

The fuzzy system can be represented by F(x) given in Equation 4 as a feed forward network. Specifically, the mapping from the input $x \in U \subset R^n$ to the output F(x) $\in V \subset R$ can be implemented as follows:

The input x is passed through a product Gaussian operator to become z^L

$$z^L = \Pi_{i=1}^{n} \exp(-(\frac{x_i - \bar{x}_i^L}{\sigma_i^L})^2) \qquad (5)$$

z^L is passed through a summation operator and a weighted summation operator to obtain b and a.

$$b = \sum_{L=1}^{M} z^L \qquad (6)$$

$$a = \sum_{L}^{M} \bar{y}^L z^L \qquad (7)$$

The output of the fuzzy system is computed as

F(x)=a/b (8)

The six-step gradient descent-training scheme to design the fuzzy system is proposed as follows:

Step 1: Choose the determination and initial parameter setting.

Choose the fuzzy system in the form of Equation 4 and determine M. Specify the initial parameters $\bar{y}^L(0), \bar{x}_i^L(0)$ and $\sigma_i^L(0)$. The initial parameters may be determined from triangular membership functions that are obtained from table look-up scheme. Also M is chosen from the same scheme.

Step 2: For a given input, calculate the output of the fuzzy system.

For a given input-output pair:

$(x_o^p; y_o^p)$ p=1, 2,…, N

and at the q'th stage of training, q = 0, 1, 2, ... present x_0^p to the input layer of the fuzzy system and compute the output from Equation 5, 6, 7 and 8.

Step 3: Update the parameters.

Use the following training algorithm to update the parameters.

$\bar{y}^L(q+1), \bar{x}_i^L(q+1) and \sigma_i^L(q+1)$

$$e^p = \frac{1}{2}[f(x_0^p) - y_0^p]^2 \qquad (9)$$

e^p is the matching error and

$$\bar{y}^L(q+1) = \bar{y}^L(q) - \alpha \frac{\partial e}{\partial \bar{y}^L}\Big|_q = \bar{y}^L(q) - \alpha \frac{f-y}{b} z^L$$

L=1, 2, …, M (10)

α is the constant step size

$$\bar{x}_i(q+1) = \bar{x}_i^L(q) - \alpha \frac{\partial e}{\partial \bar{x}_i}\Big|_q = \bar{x}_i^L(q) - \alpha \frac{f-y}{b}(\bar{y}^L(q) - f) z^L \frac{2(x_{0i}^p - \bar{x}_i^L(q))}{\sigma_i^{L2}(q)}$$

i=1,2,…n

L=1,2,…M (11)

$$\sigma_i^L(q+1) = \sigma_i^L(q) - \alpha \frac{\partial e}{\partial \sigma_i^L}\Big|_q = \sigma_i^L(q) - \alpha \frac{f-y}{b}(\bar{y}^L(q) - f) z^L \frac{2(x_{0i}^p - \bar{x}_i(q))^2}{\sigma_i^{L3}(q)} z^L$$

i=1,2,…n L=1,2,…M (12)

The training algorithm 10, 11 and 12 perform an error back-propagation procedure.

Step 4: Repeat by going to Step 2 with q=q+1 until the error $\left|f - y_0^p\right|$ is less than a pre-specified value or until the q equals a pre-specified number.

Step 5: Repeat by going to Step 2 with p=p+1.

Step 6: If desirable and feasible, set p = 1 and do Steps 2-5 again until the designed fuzzy system is satisfactory.

In this paper the aim is to obtain motor design parameters, so, this step is feasible.

The choice of the initial parameters is crucial to the success of the algorithm. However, the initial parameters in this study are chosen from table-look up scheme, which are close to optimal solution. So, the algorithm has a good chance to converge to the optimal solution.

SWITCHED RELUCTANCE MOTOR

A SR motor is a variable reluctance motor that is designed to convert energy efficiently. The motor is double salient, and it is essential for the machine operation that the number of rotor and stator poles be different. Torque is produced by the tendency of the rotor poles to align with the poles of the excited stator phase. Torque is independent of the direction of phase current, giving rise to the possibility of unipolar phase current in which only one main switching device may be required per phase. A SRM is of very simple structure: its rotor is brushless and has no winding of any kind. The motor is singly excited from stator windings, which are concentric coils wound in series on diagonally opposite stator poles. Both rotor and stator are made of laminated iron.

In spite of the simple structure of the motor, because of the highly saturated operating condition and the doubly salient structure of the motor, the accurate analysis of SR motor is very difficult and conserving. Several methods are reported for the analysis of SR motors, such as: Finite Element Method (FEM), [10,11], Magnetic Equivalent Circuit (MEC), [12,13], and piecewise linear model [14]. FEM is applied for accurate prediction of the machine parameters and performances. This method requires complicated modeling and large computational time, which is prohibitive for optimal design of SR motors. Improved MEC is the other method that is used for modeling and analysis of SR motors, which is based on the magnetic field distribution in the several parts of the motor. The IMEC modeling [13], results show good agreement with the results obtained from FEM, however, the modeling is less complicated and the computational time in IMEC is much lower. For optimal design of SR motor a large number of performance evaluation are required and even with IMEC method, the computational time would be very large. Thus, in this study a fuzzy expert system for performance prediction of the SR motor has been developed. The fuzzy expert system can be used as a very fast performance prediction tool in optimal design programs.

Improved Magnetic Equivalent Circuit (IMEC) Method The IMEC method is a numerical method for the analysis of nonlinear magnetic fields in electromagnetic devices. The method is based on the lumped representation of magnetic material with a series of permeance elements. Each element is a flux tube in which the flux does not cross the walls of the tube, just as electrical resistance represents a current tube. The Kirchhoff's current and voltage laws of electrical circuits are valid in magnetic circuits using the approximate form of Gauss and Faraday's laws. Discretization of the flux into flux tubes and defining them as permeance elements generates a dual circuit with ampere-turn sources and permeance elements, which can be solved by electrical circuit matrix methods.

However, the accuracy of the analysis by MEC method is based on the assumptions that the magnetic flux should not cross the walls of the tubes and should be homogeneously distributed in the cross-section of the tube. These conditions are not satisfied in saturated magnetic regions, if they represented by a single permeance element since the flux lines are not evenly distributed and have sharp gradients.

The flux lines curve sharply at the corner of the tooth and in the nearby air gap. It is in this region where proper modeling of the field has great impact on the accuracy of final results. The use of a proper circuit with enough elements to reflect all the flux tubes in the air gap and tooth tips is the basis of the Improved MEC (IMEC) method. To keep the proper definition of the permeance, the flux in each element should be homogeneous. Thus, the flux tubes should be modeled by enough elements of appropriate geometry in order to follow the flux lines. On the other hand, the regions with homogeneous and parallel flux lines can be modeled as single permeance elements and the regions without flux can be left out. Figure 1 shows the simplified circuit for conventional MEC analysis in which each region has been modeled with only one element, and Figure 2 shows the more detailed circuit in which the choice of the permeance elements follows the above-mentioned guidelines.

It should be mentioned that in calculating the permeance of each element, the geometric shape of the related flux tube has to be considered properly.

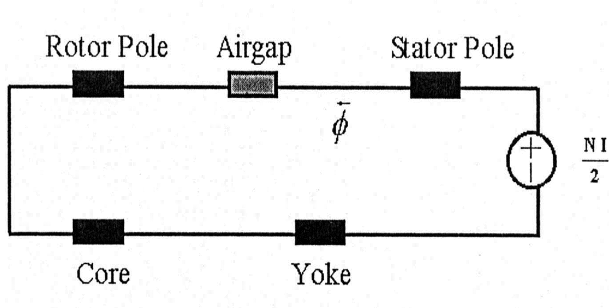

Figure 1. Conventional MEC model.

In the IMEC model, the pole tips of the rotor, stator and air gap between them are modeled by nine elements. Each three elements in series follow one flux tube from stator to motor. The rotor core is divided into four elements in order to follow the flux lines properly.

To obtain the static characteristic of the motor, the rotor position should be changed in small decrements to cover half an inductance cycle. For a changing rotor position, the flux pattern in the poles and air gap changes; hence, a new IMEC model should be defined. However, we are examining different flux patterns in this study. The circuit elements at each position will be calculated automatically, given the overlap area between rotor and stator poles. Then the circuit will be solved by electrical circuit matrix methods.

The flux linkage can be calculated directly, and the phase inductance is obtained afterwards. In order to obtain the torque accurately and avoid the errors from numerical differentiation, an analytical method has been applied within the program. The process involves analytical differentiation of the permeance elements to calculate the torque directly.

Design Sensitivity Analysis of SR Motor To

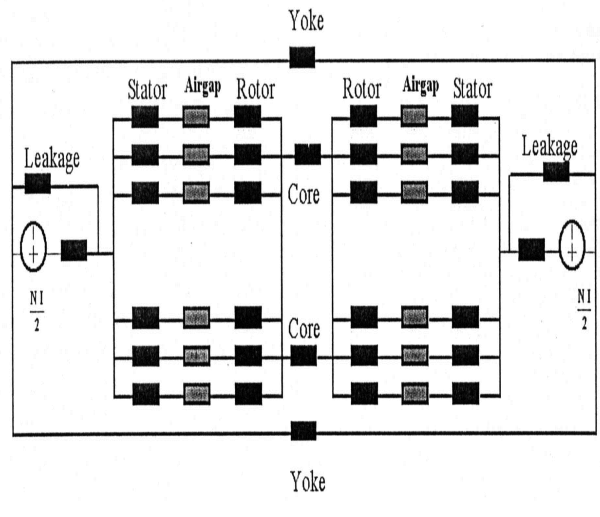

Figure 2. IMEC model.

design the fuzzy expert performance predictor for SR motor, a set of input-output information pairs is needed. These information pairs can be obtained from sensitivity analysis of the motor parameters.

In this section, for each set of design parameters, the performance characteristics of SR motor are obtained using IMEC method, which is a fast and relatively accurate analysis method. The design vector includes eleven parameters and output performance variables are efficiency and torque ripple. The design vector is defined as:

$$X_d^T = [\beta_s \ \ \beta_r \ \ h_r \ \ h_s \ \ y_r \ \ y_s \ D_r \ D_s \ L_s \ \ g \ \ Nph/2] \tag{13}$$

Where:

β_s : Stator pole arc
β_r : Rotor pole arc
h_r : Rotor pole height
h_s : Stator pole height
y_r : Rotor yoke thickness
y_s : Stator yoke thickness
D_r : Rotor diameter
D_s : Stator diameter
L_s : Stack length
g : Air gap length
Nph/2 : Number of winding turns per pole

The parameters are shown on the motor cross section in Figure 3.

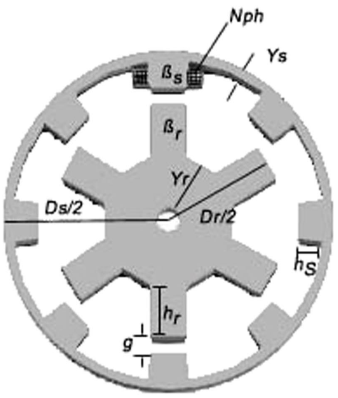

Figure 3. Design parameters of SR motor.

The feasible regions for various parameters are discussed in references 14 and 15 thoroughly. Designer can also enforce application limitation and restrictions.

Figure 4 shows the flowchart diagram, that is used for designed the fuzzy expert system predictor of SR motor performances.

The important sensitivity curves for a 4kw,8/6,250v SR motor is shown in Figure 5 to 19. These curves obtained by IMEC method. Using this information, a set of fuzzy rules will be devised for prediction of SR motor performances. The number of rules for each parameter is based on the sensitivity of performance variation to the parameter variation. The number of fuzzy sets chosen for each parameter is given in Table 1.

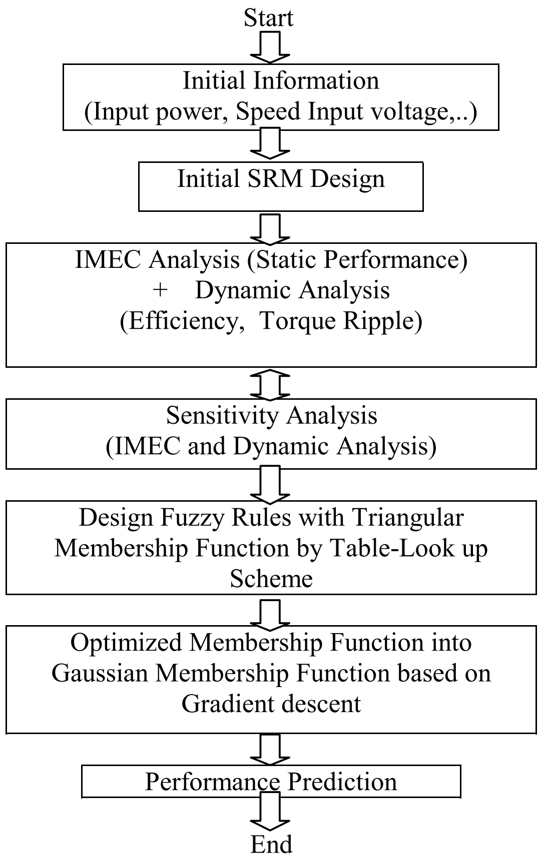

Figure 4. Design of the fuzzy expert system for predicting the SR motor performances.

TABLE 1. The Number of Fuzzy Sets for Each Parameter in Optimized Fuzzy Expert System.

Parameter	Minimum	Maximum	Number of fuzzy sets with Gaussian membership function	Parameter	Minimum	Maximum	Number of fuzzy sets with Gaussian membership function
β_s	17 (deg.)	27(deg.)	6	D_r	82.6(mm)	102.6(mm)	12
β_r	18(deg.)	30(deg.)	8	D_s	176.9(mm)	204.9(mm)	6
h_r	9.3(mm)	21.3(mm)	4	L_s	96.4(mm)	282.2(mm)	15
h_s	21.2(mm)	33.2(mm)	6	g	0.22(mm)	0.36(mm)	9
y_r	9.3(mm)	21.3(mm)	4	Nph/2	18	36	11
y_s	8.4(mm)	20.8(mm)	6				

TABLE 2. Performance Results of SR Motor.

Method	% Efficiency	% Torque Ripple (%T.R)	% Error (w.r.t. FEM)	
			% Eff	% T.R
Fuzzy predictor with triangular membership function	86.2	22.5	5.6	59.6
Optimized Fuzzy predictor with Gaussian membership function	92	15.5	0.66	9.9
IMEC Method	91	15.1	0.44	7.1
FE Method	91.4	14.1		

CONSTRUCTING THE OPTIMAL FUZZY RULES FOR PREDICTING THE SR MOTOR PERFORMANCES

Using the information from sensitivity analysis, a set of fuzzy rules will be devised. The i'th rule of this fuzzy system is as follows:

$$\text{Ri: If } x_1 \text{ is } A_1 \text{ and } ..x_{11} \text{ is } A_{11} \text{ Then } y_1 \text{ is } B_1 \text{ and } y_2 \text{ is } B_2 \quad \text{for } I = 1{:}M \qquad (14)$$

TABLE 3. Design Parameters for SR Motor.

Design Parameters for SR motor										
$\beta_s{}^\circ$	$\beta_r{}^\circ$	$h_r\,(mm)$	$y_r(mm)$	$h_s\,(mm)$	$y_s(mm)$	$D_s(mm)$	$D_r\,(mm)$	$L_s\,(mm)$	$N_{ph}/2$	g(mm)
21	23	17.5	12.1	29	13.9	177.5	91.1	150	20	0.3

Where x1, x2, … and x11 are the design SR motor parameters that obtained in Equation 13 and y1 and y2 are the efficiency and torque ripple. Based on the high or low sensitivity of performance variable to design parameters and output performances, the number of fuzzy membership functions defined for each parameter could be high or low. The fuzzy rules with triangular shape membership functions are designed by table-look up scheme described before. Setting the rules for all parameters, we have a fuzzy rule base with a set of IF-THEN rules. Using the gradient descent, the fuzzy rules with triangular membership function can be optimized to Gaussian membership functions based on the method described. By using this method, 63 rules are obtained for predicting the performance of SR motor. The results of performance prediction of optimized fuzzy rule base for the 4 kW, 8/6 motor are given in Table 2, which is compared with the results obtained by fuzzy expert system with triangular shape membership function, IMEC method and FEM analysis methods. The results show good accuracy compared with results obtained by accurate methods, however, the computational time is very low compared to FE and IMEC method. The parameters of the SR motor are given in Table 3.

CONCLUSION

A fuzzy expert system has been developed based on the table-look up scheme and optimized by gradient descent method for performance prediction of a SR motor

The performance prediction results by the developed fuzzy expert system show good agreement with the results obtained from accurate analysis program. The fuzzy expert system can be used as a very fast performance prediction tool in optimal design processes or on-line control schemes.

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