REDUCING THE LIMITATION ON APPLICATION OF SYNCHRONOUS DECORRELATING DETECTOR CDMA SYSTEMS

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Abstract In CDMA (Code - Division Multiple - Access) systems multi - user accessing of a channel is possible. Under the assumptions of Optimum multi - user and decorrelating detector in CDMA systems. By using signals with zero and / or identical cross correlations, a simple and expandable decorrelating detector with optimum efficiency which can be easily implemented are proposed. Constructing these signals is accomplished by encoding of the data stream by a member of orthogonal code which is then multiplied by a special signature sequences

Key Words Decorrelating Detector, Conventional Detector, Signature Sequence, Orthogonal Signals

چکیده در سیستمهای CDMA دسترسی چند کاربر بصورت همزمان به یک کانال امکان‌پذیر است. در این مقاله با فرض انتقال همزمان سیگنالها در سیستم CDMA، گیرندهٔ قراردادی تک کاربره، با گیرندهٔ بهینهٔ چندکاربره و آشکارساز غیر همبسته‌کننده مورد مقایسه قرار می‌گیرند. با بکاربردن سیگنالهای با همبستگی متقابل یکسان یک آشکارساز غیر همبسته‌کننده با راندمان بهینه، تعداد کاربران قابل افزایش بوده و پیاده‌سازی بسادگی امکان‌پذیر می‌شود. سیگنالهای مذکور با انکود نمودن داده‌ها توسط کدهای متعامد و سپس ضرب آنها در دنباله‌های امضایی مخصوص ساخته می‌شوند.

INTRODUCTION

CDMA is a technique where several users access simultaneously a common channel by modulating preassigned linearly independent signature sequences. If the sequences were orthogonal, then a bank of match filters would achieve optimum demodulation. Let {S(n), n=O, ... , M-1} be kth user signature sequence. If the users maintain symbol synchronization, the receiver observes:

$$r(n) = \sum_{k=1}^{k} b_k(j)s_k(n - jM) + \sigma N(n); \quad jM \leq n \leq jM+M-1 \tag{1-1}$$

where N is white Gaussian noise and $b_k(j) \in \{-1,1\}$ is equally likely information sequence of kth user.

The outputs of matched filters for a symbol interval (j=0) is:

$$y_k <r(n) , s(n) > , k=1 ,..., k \tag{1-2}$$

where < , > denotes inner product $\mathbf{y} = (y_1, ..., y_k)$ depends on $\mathbf{b} = (b_1, ..., b_k)$ in the following way:

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{N} \tag{1-3}$$

H is the matrix of cross correlations between the assigned sequences, where:

$$H_{ij} = \sum_{n=0}^{M-1} s_i(n)s_j(n); \; i,j = 1, ..., k; Hi\ i = W_i > 0 \tag{1-4}$$

where W_i is energy-per bit of the ith user, and N is a zero-mean Gaussian K-vector with covariance matrix equal to O^2H, conventional detector yielding the decisions, $\hat{b}_k = \text{sgn}\, y_k$ for the kth user. Asymptotic efficiency [1] of this detector is equal to one if orthogonal signature sequences is used.

A large number of users require a large set of orthogonal signals which are not practically feasible. On the other hand, even with low cross correlations, the powers of the received signals cannot be very dissimilar (near-far problem).

Optimum multi-user detector [2] achieves important performance gains over conventional detector by selecting the most likely hypothesis:

$$\hat{b}_k \in \arg\min \sum_{n=0}^{M-1} \left| r(n) - \sum_{k=1}^{K} b_k s_k(n) \right|^2 \tag{1-5}$$

This detector have the best asymptotic efficiency and is near-far resistant at the expense of computational complexity that grows exponentially with the number of users. On the other hand computational complexity of decorrelating detector [1] is linear and yields the following decisions:

$$\hat{b} = \text{sgn}\,(\mathbf{H}^{-1}.\mathbf{y}) = \text{sgn}\,(\mathbf{b} + \mathbf{H}^{-1}\mathbf{N}) \tag{1-6}$$

Since $\mathbf{H}^{-1}.\mathbf{N}$ is correlated noise $\mathbf{H}^{-1}.\mathbf{Y}$ optimum decisions.

However if we let $\tilde{y}_k = \frac{y_k}{\sqrt{w_k}}$ then:

$$\text{sgn}\,(H^{-1}.y) = \text{sgn}\,(W^{-1/2} R^{-1}.\, W^{-1/2}.\, y)$$

$$= \text{sgn}\,(W^{-1/2} R^{-1}.\, \tilde{y}) = \text{sgn}\,(R^{-1}.\, \tilde{y}) \tag{1-7}$$

where $\mathbf{R}$ is the matrix of normalized cross correlations, i.e. $\mathbf{R} = \mathbf{W}^{-1/2}.\mathbf{H}.\mathbf{W}^{-1/2}$ and $\mathbf{W} = \text{diag}\,[W_1, \ldots, W_k]$.

Equation 1-7 indicates that this detector dose not require knowledge of the energies of users. Therefore, its near-far resistance is equal to the optimum decision. Its asymptotic efficiency which is independent of the energy and interference of other users is equal to:

$$\eta_k = \frac{1}{R_{kk}^{-1}} \tag{1-8}$$

The efficiency of this detector is maximum if energies of all users are fixed and:

$$|H^{-1}_{jk}| \leq H^{-1}_{kk} \text{ for all } j \neq k \tag{1-9}$$

The organization of the rest of the paper is as follows: limitations of the application of decorrelating detector are explained in the comming section. An application of this detector which uses signals with identical cross correlations, without the limitations discussed in the comming section, is proposed next. Coding methods for obtaining the same correlations are then described. Finally, increasing the number of users and transmitter / receiver configuration of proposed detector is discussed

LIMITATIONS ON APPLICATION OF DECORRELATING DETECTOR

1. The detector must know all signature sequences in order to specify $\mathbf{R}^{-1}$. This results in low reliability and security.
2. Although the detector is linear, the complexity of computation of $\mathbf{R}$ do not increase linearly when one user is added.
3. Because the $(k+1) \times (k+1)$ matrix $\mathbf{R}^{-1}$, is totally different from the $k \times k$ matrix $\mathbf{R}^{-1}$, the detector implementation must be modified when one user is added.
4. The invertibility condition of $\mathbf{R}$ is linear independent of signature sequences [1], but the implementation of $\mathbf{R}^{-1}$ is not always possible.

AN IMPLEMENTABLE METHOD

In order to specify $\mathbf{R}$, the detector needs not know signature sequences if we obtain a set of linear independent signals with the same cross correlations. With such a signal $\mathbf{R}^{-1}$ can be easily specified and implemented. As shown in the coming section, when one user is added, the

$k \times k$ matrix $\mathbf{R}^{-1}$, can be easily expanded to the $(k+1) \times (k+1)$ matrix $\mathbf{R}^{-1}$; therefore, this method does not have the limitation discussed in the previous section.

SIGNALS WITH THE SAME CROSS CORRELATIONS

In this paper we used the orthogonal coding as a mean of spreading [3]. Information bits of any user encoded by a member of orthogonal code of length M. A M bit signature sequence , is then multiplied by the code word before transmission. An orthogonal code reciving recent interest is the Walsh Hadamard (WH) code [3] with the property:

$$WH_i(L).WH_j(L) = WH_{i\oplus j}(L) \quad (4\text{-}1)$$

where L, denotes chip position within a code sequence and $i \oplus j$ denotes the bit by bit modulo-2 addition. An information bit of a user encoded by this code and then multiplied by a signature sequence denoted by X_1 is given by [4]:

$$bWH_p(L)\ X_1\ (L) = \{bWH_p\ (0)\ X_1\ (0)\ , \dots ,\ bWH_p\ (M-1)X_1(M-1)\} \quad (4\text{-}2)$$

where WH_p is one of the M,M bit orthogonal code words. The synchronous receiver correlates this signal with all M possible scrambled code words. This is done by forming detection statistics:

$Z_1\ (J = 0, \dots , M-1)$ *given by*

$$Z_j = < bWH_p\ X_1, WH_j X_1 > = b< WH_p, WH_j > = b\theta_{p,j}(0) \quad (4\text{-}3)$$

where $\theta_{p,j}(0)$ is the correlation between WH_p *and* WH_j. $\theta_{p,j}(0)$ equals M if p=j and zero otherwise. Thus the decision as to which code word was sent (j) is given by the index of the Z_j with the largest magnitude.

The decision as to which information bit was sent is determined from sign of Z_j.

Consider the presence of synchronous interfering signal, b $WH_q X_2$.

$$r(L) = bWH_p(L)X_1(L) + b\ WH_q\ (L)X_2(L),\ L = 0, 1, \dots, M-1 \quad (4\text{-}4)$$

The receiver again correlates the received signal with the M scrambled code words during this code period, which results:

$$Z_j = b\theta_{p,j}(0) + b\sum_{L=0}^{M-1} WH_q(L)WH_j(L)X_1(L)X_2(L) \quad (4\text{-}5)$$

Using (4-1), (4-5) becomes:

$$Z_j = b\theta_{p,j}(0) + b\sum_{L=0}^{M-1} WH_{i\oplus j}\ (L)\ X_1\ (L)\ X_2\ (L) \quad (4\text{-}6)$$

For reasons discussed before, $X_1(L)X_2(L)$ must have the same correlation with all possible code words. This correlation normalized by 1/M gives rise to the WH transform. Thus, the WH transform of the product of any two signature sequence must have a constant magnitude. Such sequences are a type of ben't sequences of length $M=2^n$ (n-even).

There are two types of bent sequences: linearly-based bent and bent-based bent sequences [5]. The first approach generates √M and the second M/2 signature sequences. The correlation (1-6) for all q and j and all different signature sequences is equal to | √M | [4] Figure 1(a) shows k user transmitter which used M orthogonal code word (WH_j) and M/2 bent based bent signature sequences (X_i).

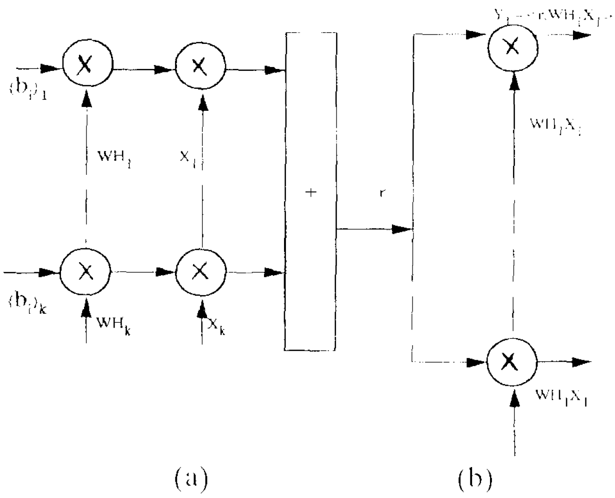

Figure 1. k user transmitter (a) and k user receiver (b).

In k user reciver - Figure 1(b) - the first match filler output in a bit time interval equals to

$WH_1\ (L)\ WH_k\ (L)\ X_1\ (L)\ X_k\ (L) + N_1$

$$y_1 = b_1 \sum_{L=0}^{M-1} WH_1\ (L)\ WH_1\ (L)\ X_1\ (L)\ X_1\ (L) + \ldots$$

$$+\ b_k \sum_{L=0}^{M-1} \ldots$$

$$Y_1 = b_1 M + b_2 r + \ldots b_k r + N'$$

where $r = +\sqrt{M}$ or $-\sqrt{M}$ and in matrix form L:
$\mathbf{Y = HB + N}$

where $H_{ii} = M$, $H_{ii} = \sqrt{M}$ or $-\sqrt{M}$ for all $i=J=1, \ldots K$

The use of positive correlations having appropriately chosen orthogonal codes (discussed at the end of this section) results in, $H_{ij} = \sqrt{M}$ and $\mathbf{H} = \sqrt{M}\ \mathbf{R}$ where, $R_{ii}\text{-}\sqrt{M}$ and $R_{ij} = 1$
for all $i \neq j = 1, \ldots, k$.
Therefore, for any number of users $\mathbf{R}^{-1}$ is equal to:

$$\frac{1}{c} R^{-1}_{ii} = a, \frac{1}{c} R^{-1}_{ij} = -1 \text{ For all } i=j=1,\ldots,k. \quad (4\text{-}8)$$

where $a = \sqrt{M} + k - 2$ and c is a nonzero positive constant. K user decorrelating detector decision by using matrix $\mathbf{R}$ with the same entries, is equal to:

$$\hat{\mathbf{b}} = \mathrm{sgn}(\mathbf{H}^{-1}\mathbf{Y}) = \mathrm{sgn}\left(\frac{\sqrt{M}}{c}\mathbf{R}^{-1}.y\right) = \mathrm{sgn}(\mathbf{R}^{-1}.y) \quad (4\text{-}9)$$

Figure 2, which is the special case of a two user detector, can obtain all features discussed before. When the number of users increases, the process of modification of this detector consists of changing the parameter "a" and the number of adders and their inputs only (one adder per user). As mentioned in section 1 if (1-9) is satisfied, then asymptotic efficiency of this detector is optimum for all users. In this method (1-9) is equivalent to "$a \geq 1$" or $k \geq 3-\sqrt{M}$

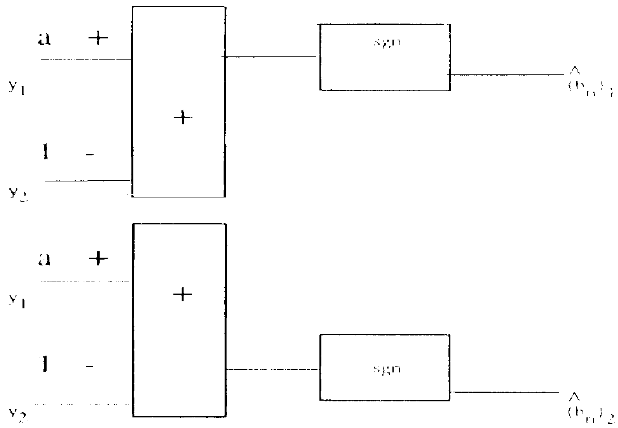

Figure 2. Special case of two users detector.

which is always the case.

As mentioned in this section, the cross correlations between signals must be positive. This results in reducing the limitation on application of decorrelating detector. Furthermore, as will be seen in the next section, the number of users can also be increased in a simple way. It is possible to obtain code words with positive correlations by simulation. For example in case of $M=4$, WH orthogonal codes and bent sequences are as follows:

$WH_1 = [+1+1+1+1]$, $WH_2 = [+1-1+1-1]$,
$WH_3 = [+1-1+1-1]$, $WH_4 = [+1-1-1+1]$

$X_1 = [+1+1+1+1]$,
$X_2 = [+1+1+1-1]$ (4-10)

If all possible cross correlation,

$$\sum_{L=0}^{3} WH_i(L) WH_j(L) X_k(L) X_m(L),$$

For all i, j, k, m are computed, it will be seen that only the following two correlations become negative.

$$\sum_{L=0}^{3} WH_2 WH_3 X_1 X_2 = \sum_{L=0}^{3} WH_1 WH_4 X_1 X_2 = -2 \quad (4\text{-}11)$$

Therefore code words (WH_2, WH_3) and (WH_1, WH_4) can not be used simultaneously. Hence the maximum number of users is $k=2$

and simultaneously usable pair of codes are as follows:

$(WH_1, WH_2), (WH_2, WH_3), (WH_1, WH_3), (WH_1, WH_4)$, (4-12)

By using simulation negative correlations can be detected for all $M=2^n$ (n-even)

INCREASING THE NUMBER OF USERS

Considering that each user must have either a code word or a signature sequence that is different from those of others, the number of users can be increased. Among, M orthogonal M bit code, L number of which having positive correlations are used. An appropriate implementation of transmitter is shown is Figure 3.

This figure indicates that the maximum number of users is equal to LM/2. Observing that L is proportional to M (e.g.M=4 results in, L=M/2=2) we can conclude that the number of users is proportional to M^2. as Figure 4 indicates, the cross correlation matrix of transmitted signals is equal to:

$$H=\sqrt{M}\begin{bmatrix} \underbrace{\begin{vmatrix}\sqrt{M}&1&1\\1&\sqrt{M}&1\\1&1&\sqrt{M}\end{vmatrix}}_{\frac{M}{2}\times\frac{M}{2}} & & \ddots \\ \underbrace{\begin{vmatrix}0&1\ldots1\\1&0\ldots1\\1&1\ldots0\end{vmatrix}}_{\frac{M}{2}\times\frac{M}{2}} & \ddots & \\ \begin{matrix}\text{all off diaconal}\\\text{Submatrics}\\\text{are in this form}\end{matrix} & \ddots & \underbrace{\begin{vmatrix}\sqrt{M}&1\ldots1\\1&\sqrt{M}\ldots1\\1\ldots1&\sqrt{M}\end{vmatrix}}_{\frac{M}{2}\times\frac{M}{2}}\end{bmatrix}_{\frac{LM}{2}\times\frac{LM}{2}}=\sqrt{M}R$$

This matrix is obtained because

$$\sum_{l=0}^{M-1} WH_i(L)WH_i(L)X_j(L)X_k(L) = \begin{cases}\sqrt{M} & k=j\\ \sqrt{M} & k\neq j\end{cases}$$

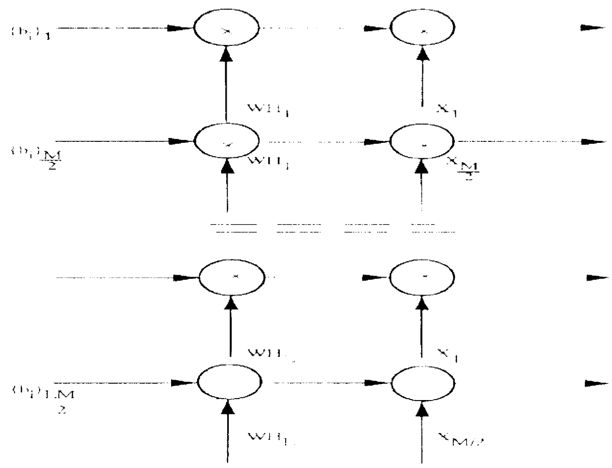

Figure 3. Transmitter in which the number of users is proportional to M^2.

$$\sum_{l=0}^{M-1} WH_i(L)WH_m(L)X_j(L)X_k(L) = \begin{cases}0 & k=j\\ \sqrt{M} & k\neq j\end{cases} \quad i\neq m \tag{5-2}$$

Taking into consideration that some of the entries of **R** are zero, $\mathbf{R}^{-1}$ can not be obtained in the previous section. However in this case the inverse of this matrix can be obtained using its toplites property.

$$\mathbf{R}^{-1} = A_p\,\mathbf{P}^{-1}_p\,.\,\mathbf{A}^T_p \tag{5-3}$$

where

$$\mathbf{A}_p = \begin{vmatrix} 1 & 0 & 0 & 0 \\ a_{p,1} & 1 & 0 & 0 \\ a_{p,2} & a_{p-1,1} & 1 & 1 \\ \vdots & & & \\ a_{p,p} & a_{p-1,p-1} & a_{1,1} & 1 \end{vmatrix}$$

$$\mathbf{P}_p = \begin{vmatrix} \rho_p & P & \ldots & P \\ 0 & \rho_{p-1} & 0 & 0 \\ \vdots & & & \\ 0 & & \rho_1 & 0 \\ 0 & & & \rho_0 \end{vmatrix} \tag{5-4}$$

and p=(LM/2) -1 is the rank of **R**. In the above matrices a and ρ are obtainable by Levinion Dorbin algorithm with low complexity.

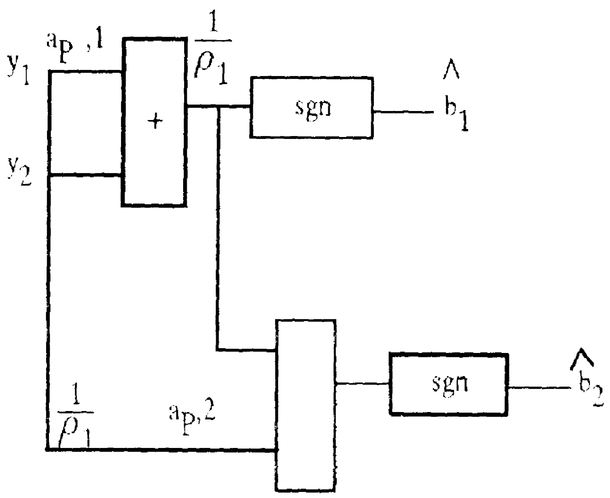

Figure 4. Two users detector described by (5-4),(5-5),(5-6).

If the elements of first row of **R** are named as R(O),..., R(LM/2)-1) we would have.

$$\begin{cases} a_{k,i} = a_{k-1,i} + a_{k,k}a_{k-i,k-i} \qquad i = 1,...k-1 \\ a_{k,k} = -[R(K) + \sum_{i=1}^{k-1} a_{k-1,1}\, R(K-L)] / \rho_{k-1} \\ \rho_k = (1-|a_{k,k}|^2 \rho_{k-1} \;;\; \rho_o = R(0) = \sqrt{M} \end{cases} \qquad (5\text{-}5)$$

By increasing the number of users, **R** becomes augmented and the parameters, a and ρ remain unchanged. Therefore, according to (5-5) only new parameters are obtained. The parameters a and ρ are fixed and can be computed once for all $M=2^n$ (n-even).

Therefore, it is not necessary to compute them for every detection. The block diagram of decorrelating detector described by the following equation is shown in Figure 5 for two users:

$$\mathbf{b} = \mathrm{sgn}\,(\mathbf{R}^{-1}.y) = \mathrm{sgn}(\Lambda_p.\Lambda_p^{-1}.\Lambda^T_p.y) \qquad (5\text{-}6)$$

In Figure 4, the number of adders and the corresponding inputs increase as the number of users increases. There are 2 (k-1) adders in this detectors, therefore, each new user adds two adders. This indicates that the implementation complexity of this detector is linear.

CONCLUSION

In this paper we have used a special linear transformation which is applied to the output of a matched filter bank. This transformation exhibits a good asymptotic efficiency and substantially higher performance than the conventional single user detector, while maintaining a comparable ease of computation. By fixing cross correlation between signals through using orthogonal coding and bent signature sequences, we managed to implement this transformation in a simple and expandable way. Its asymptotic efficiency and near-far resistance is comparable to the optimum multi-user detector. Research is needed to determine an optimal set of signature sequences designed specifically for an asynchronous environment in which orthogonal coding is used to spread information signals.

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