RESEARCH NOTE

ON FINITE QUEUE WITH TWO TYPES OF FAILURES AND PREEMPTIVE PRIORITY

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Abstract We study the single server queueing system with two types of failure to service channels including the preemptive priority to the repair of major failure. The units arrive at the system in a poisson fashion and are served exponenitally. The steady - state probabilities of various states by using generating function have been obtained.

Key Words Queue, Preemptive Priority, Probability Generating Function (PGF), Major and Minor Failures, Finite Waiting Space

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INTRODUCTION

The queueing system subject to failure in two modes and having preemptive priority to repair major failure was stuided by Madan [1]. Jain and Sharma [2,3] discussed machine interference problems with two types of failure and preemptive priority. Recently, Reddy [4] studied the optimization of k-out-of-N queueing system subject to failures in two modes and repair provision. There exist many practical situations where only finite waiting spaces are available for customers. Taking such real life problems into consideration, we extend Madan's [1] model by including finite waiting space. The steady-state probability generating functions for the number of customers present in the system for various states have been obtained explicitly.

THE MODEL AND ANALYSIS

We consider a single repair facility system where units arrive to service station in poisson fashion and are served exponentially by single server. The service channel is subject to failure in two modes major and minor failures with rate a_1 and a_2 respectively. The units are served in FCFS discipline with mean service rate m in normal conditions and slower rate n ($n < m$) in case of minor failure of service channel. The repair to major failure is given preemptive priority over the minor one, i.e., in case repairman is busy in minor failure with repair rate b_1 and a major failure occurs, the repairman will switch over to the major failure and will repair it with repair rate β_2. Our main purpose is to obtain the steady state probability generating function (GF) for the number of customers present in the system for different states. We introduce the following notations for our model:

$W_n^{''}$ ($W_n^{'}$) : Probability that there are $n(\geq 0)$ units in the system and service channel is in the normal working state (in working state with only major component, and minor component is under repair).

$F_n^{''}$ ($F_n^{'}$) : Probability that there are n units in the system and service channel is in the failed (completely failed) state due to the major failure (major as well as minor failures) and major component is under repair.

The steady state difference equations satisfying our model are:

$$(\lambda + \mu + a_1 + a_2) W_n^{''} = (\lambda W_{n-1}^{''} + \mu W_{n+1}^{''} + \beta_1 F_n^{'} + \beta_2 W_n^{'}) \qquad (1 \leq n \leq N-1) \quad (1.1)$$

$$(\lambda + a_1 + a_2) W_0^{''} = \mu W_1^{''} + \beta_1 F_0^{'} + \beta_2 W_0^{''} \quad (1.2)$$

$$(\lambda + a_1 + a_2) W_0^{''} = \mu W_1^{''} + \beta_1 F_0^{'} + \beta_2 W_0^{''} \quad (1.3)$$

$$(\lambda + \nu + a_1 + \beta_2) W_n^{'} = \lambda W_{n-1}^{'} + a_2 W_n^{''} + \beta_1 F_n^{''} + \nu W_{n+1} \qquad (0 \leq n \leq N-1) \quad (1.4)$$

$$(\nu + a_1 + \beta_2) W_N^{'} = \lambda W_{N-1}^{'} + a_2 W_N^{''} + \beta_1 F_N^{'} \quad (1.5)$$

$$(\lambda + a_1 + \beta_2) W_0^{''} = \nu W_1^{''} + a_2 W_0^{''} + \beta_1 F_0^{''} \quad (1.6)$$

$$(\lambda + a_2 + \beta_1) F_n^{'} = \lambda F_{n-1}^{'} + a_1 W_n^{''} \qquad (1 \leq n \leq N-1) \quad (1.7)$$

$$(a_2 + \beta_1) F_N^{'} = \lambda F_{N-1}^{'} + a_1 W_N^{''} \quad (1.8)$$

$$(\lambda + a_2 + \beta_1) F_0^{'} = a_1 W_0^{''} \quad (1.9)$$

$$(\lambda + \beta_1) F_n^{''} = \lambda F_{n-1}^{''} + a_1 W_n^{'} + a_2 F_n^{'} \qquad (0 \leq n \leq N-1) \quad (1.10)$$

$$\beta_1 F_N^{''} = \lambda F_{N-1}^{''} + a_1 W_N^{'} + a_2 F_N^{'} \quad (1.11)$$

$$(\lambda + \beta_1) F_0^{''} = a_1 W_0^{'} + a_2 F_0^{'} \quad (1.12)$$

By applying the following probability generating functions

$$W^{'}(z) = \sum_{n=0}^{N} W_n^{'} z^n \;;\; W^{''}(z) = \sum_{n=0}^{N} W_n^{''} z^n$$

$$F^{'}(z) = \sum_{n=0}^{N} F_n^{'} z^n \;;\; F^{''}(z) = \sum_{n=0}^{N} F_n^{''} z^n$$

in the above set of Equations 1, we get

$$F^{'}(z) = \frac{a_1}{d^{'}} W^{''}(z) + \frac{\lambda z^N}{d^{'}}(1-z) F_N^{'} \quad (2)$$

$$F^{''}(z) = \frac{a_1 W^{'}(z)}{d^{''}} + \frac{a_1 a_2}{d^{'} d^{''}} W^{''}(z) + \frac{a_2 \lambda z^N (1-z)}{d^{'} d^{''}} F_N^{'} + \frac{\lambda z^N (1-z)}{d^{''}} F_N^{''} \quad (3)$$

$$W^{'}(z) = \frac{1}{c^{'} - a_1 \beta_1 z / d^{''}} [a_2 z (1 + a_1 \beta_1 / d^{'} d^{''}) W^{''}(z) + \lambda z^{N+1} (1-z) \{W_N^{'} + (a_2 F_N^{'}/d^{'} + F_N^{''}) \beta_1 / d^{''}\} + (z-1) \nu W_0^{'}] \quad (4)$$

and

$$W^{''}(z) = \frac{(1-z)}{c^{''} - a_1 \beta_1 z / d^{'} - \dfrac{(a_2 z + a_1 a_2 \beta_1 z / d^{'} d^{''})}{c^{'} - a_1 \beta_1 z / d^{''}} \beta_2 z} \times$$

$$\left[\lambda z^{N+1}\frac{\beta_2 z}{c'-a_1\beta_1 z/d''}(W_N'+a_2\beta_1F_N'/d'd''+\beta_1F_N''/d'')\right.$$

$$\left.+\beta_1F_N'/d'+W_N''+\mu W_N''+\frac{\lambda W_0'\beta_2 z}{c'-a_1\beta_1 z/d''}\right] \qquad (5)$$

where

$d' = \lambda(1\text{-}z) + a_2 + \beta_1$

$d'' = \lambda(1 - z) + \beta_1$

$c' = \{\lambda(1-z) + \nu + a_1 + \beta_2\} z - \nu$

$c'' = \{\lambda(1\text{-}z) + \mu + a_1 + a_2\} z - \mu$

In limiting case when $z \to 1$, we get

$$F'(1) = \frac{a_1}{a_2+\beta_1}W''(1) \qquad (6)$$

$$W'(1) = \frac{a_2(a_1+a_2+\beta_1)}{\beta_2(a_2+\beta_1)}W''(1) \qquad (7)$$

and

$$W''(1) + W'(1) + F'(1) + F''(1) = 1 \qquad (8)$$

$$F''(1) = \frac{a_1a_2(a_1 + a_2 + \beta_1 + \beta_2)}{\beta_1\beta_2(a_2 + \beta_1)}W''(1) \qquad (9)$$

and simplifing, we get

$$W''(1) = \frac{\beta_1\beta_2(a_2+\beta_1)}{(a_1+\beta_1)[\beta_1\beta_2+a_2(a_1+a_2+\beta_1+\beta_2)]} \qquad (10)$$

which is the steady-state probability that the system is in the normal working state.

To determine W_0' and W_0'' involved in Equation 5, we use L' Hospital's rule and get

$$W_1''(1) = \frac{\left[\mu W_0'' + \nu W_0' - \lambda(W_N' + F_N' + W_N'' + F_N'')\right]\beta_1\beta_2(a_1+\beta_1)}{a_2\beta_1(a_1+a_2+\beta_1) + \mu\beta_1\beta_2(a_2+\beta_1) - \lambda(a_1+\beta_1)\left[\beta_1\beta_2+a_2(a_1+a_2+\beta_1+\beta_2)\right]} \qquad (11)$$

On comparing the coefficient of R. H. S. of Equations 10 and 11, we obtain

$$W_0'' = \frac{\beta_1\beta_2(a_2 + \beta_1)}{(a_1+\beta_1)[\beta_1\beta_2+a_2(a_1+a_2+\beta_1+\beta_2)]} \qquad (12)$$

$$W_0' = \frac{a_2\beta_1(a_1 + a_2 + \beta_1)}{(a_1+\beta_1)[\beta_1\beta_2+a_2(a_1+a_2+\beta_1+\beta_2)]} \qquad (13)$$

$$W_N' + F_N' + W_N'' + F_N'' = \frac{a_2(a_1 + a_2 + \beta_1 + \beta_2)}{[\beta_1\beta_2 + a_2(a_1 + a_2 + \beta_1 + \beta_2)]} \qquad (14)$$

In Particular (i) when there are identical failure and repair rates, $a_1 = a_2 = a$ and $\beta_1 = \beta_2 = \beta$ and (ii) when there is no major failure, $a_1 = 0$, similarly in case of only major failure (no minor failure), we put $a_2 = 0$.

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REFERENCES

1. Madan, K. C., "A Priority Queueing System with Service Interruptions", ***Statistica Neerlandica***, Vol. 27, No. NR 3, (1972), 115-123.
2. Jain, M. and Sharma, G. C., "A Finite Difference Approach to Machine Interference Problem with Preemptive Priority", Ph.D. Thesis, Agra University, Agra, (1986).
3. Jain, M. and Sharama, G. C., "An Iterative Approach for Solving Machine Interference Problem with Interrupted Service Including Priority", ***Journal of M.A.C.T.***, Vol. 17, (1984), 76-84.
4. Reddy, C. R., "Optimization of k-out-of N Systems Subject to Common Cause Failure with Repair Provision", ***Journal of Micro-electron. Reliab.***, Vol. 33, No. 2, (1993), 175-183.