RELIABILITYOF REPAIRABLEMULTICOMPONENT REDUNDANT SYSTEM

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Abstract This investigation deals with transient analysis of cold standby system with n units. Chapman-Kolmogorov equations are developed for repair facility with two repairmen and solved by using matrix technique. Availability and reliability factors have been obtained with probability of the system. A particular case has also been discussed.

Key Words Transient, Standby, Availability, Reliability

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INTRODUCTION

In the modern age of technology, a system is of no use if it is not reliable. However, the importance and effect of reliability could not be realized without mathematical modelling. The mathematical theory of reliability has grown out of the demands of modern technology. In general, the reliability can be defined as the freedom from failure of a component or system while maintaining a specific problem such as inventory, industrial or machine repair problem. Reliability technique is of immense importance for performance evaluation and accurate measurement of computer, communication, manufacturing systems, planning and designing of the components in electronic equipments etc. Stochastic models are becoming increasingly important for understanding or making a performance evaluation of queueing problems of real life situations. The redundant system inreliability has been developed by several researchers. Some of them are Harvill and Pipes [1] and Gopalan and Natesan [2]. These researchers have developed the techniques for stochastic behavior of one server with n units in the system.

In this paper, we consider cost analysis for such system with one additional repairman. The arrival (failure) rate and service rate are independently negative exponentially distributed with FCFS discipline. We find the expressions for steady state characteristics including stationary availability and interval reliability.

THE MATHEMATICALMODEL

In this paper, we consider the system with n dissimilar units; in which one channel is in operating condition and others are as standby Multi server facility always available with the system if more than one failed machine enter in the service system. The life times and repair times of all the units are independently distributed negative exponential variates.

The main notations used in the investigation are as follows:

λ_i	Life time rate of units, i=1,2..., n
μ_i	Repair time rate of units, i=1,2..., n
$X(t)$	Number of the operating units at time t.
$p_i(t)$	$P[X(t) = i/X(0) = n]$
$q_i(t)$	$P[X(t) = i/X(0) = n] \quad X(u) \neq 0,\ u > 0$
$p_{ij}(t)$	$P[X(t) = i/X(0) = j]$; probability that i^{th} unit is operating, given that initially unit j was operative.
P_i	$= \lim_{t \to \infty} p_i(t)$
$f^*(s)$	Laplace transform of the function f(t)
N	M + S
μ_i	$\begin{cases} \mu_r & 1 \leq i \leq r \\ 2\mu & r+1 \leq i \leq N \end{cases}$

THE BALANCE EQUATIONS AND THE SOLUTION METHOD

We consider $[X(t),\ t \geq 0]$ as a birth and death Markov process with state space. The balance equations in set of Chapman-Kalmogorov processes for $p_i(t)$ and $q_i(t)$ are as follows

$$\frac{d}{dt} p_0(t) = -\mu_1 p_0(t) + \lambda_1 p_1(t) \tag{1.1}$$

$$\frac{d}{dt} p_i(t) = -(\lambda_i + \mu_{r+1})\, p_i(t) + \mu_r p_{i-1}(t) + \lambda_{i+1} p_{i+1}(t) \qquad 1 \leq i \leq r \tag{1.2}$$

$$\frac{d}{dt} p_i(t) = -(\lambda_i + 2\mu) p_i(t) + 2\mu p_{i-1}(t) + \lambda_i{+}1 p_{i+1}(t) \qquad r < i < n \tag{1.3}$$

$$\frac{d}{dt} p_n(t) = -\lambda_n p_n(t) + 2\mu p_{n-1}(t) \tag{1.4}$$

and

$$\frac{d}{dt} q_i(t) = -(\lambda_1 + \mu_2) q_1(t) + \lambda_2 q_2(r) \tag{2.1}$$

$$\frac{d}{dt} q_i(t) = -(\lambda_i + \mu_{r+1}) q_i(t) + \mu_r q_{i-1}(t) + \lambda_{i+1} q_{i+1}(t) \qquad 2 \leq i \leq r \tag{2.2}$$

$$\frac{d}{dt} q_i(t) = -(\lambda_i + 2\mu) q_i(t) + 2\mu q_{i-1}(t) + \lambda_{i+1} q_{i+1}(t) \qquad r \leq i \leq n \tag{2.3}$$

$$\frac{d}{dt} q_n(t) = -\lambda_n q_n(t) + 2\mu q_{n-1}(t) \tag{2.4}$$

with

$$p_i(0) = q_i(0) = \delta_{in} = \begin{matrix} 1 & i = n \\ 0 & i \neq n \end{matrix} \tag{3}$$

For stochastic process $\{X(t),\ t \geq 0\}$, the stationary distribution $\{p_i\}$ is the limiting solution of (1) as $t \to \infty$ with derivatives being replaced by zero.

$$\mu_1 P_0 + \lambda_1 P_1 = 0 \tag{4.1}$$

$$(\lambda_1 + \mu_{r+1})\, P_1 + \mu_r P_{i-1} + \lambda_{i+1} P_{i+1} = 0 \qquad 1 \leq i \leq n \tag{4.2}$$

$$(\lambda_1 + 2\mu)\, P_i + 2\mu P_{i-1} + \lambda_{i+1} P_{i+1} = 0 \qquad 1 \leq i \leq n \tag{4.3}$$

$$\lambda_n P_n + 2\mu P_{n-1} = 0 \tag{4.4}$$

$$\lambda_i P_i = \mu_{i-1} \qquad i = 0, 1, 2, \ldots n$$

Thus
$$P_i = P_0 \prod_{j=1}^{i} (\mu_j/\lambda_j) \tag{5}$$

where

$$P_0 = [1 + \sum_{i=1}^{n} (\prod_{j=1}^{1} \mu_j / \lambda_j)]^{-1}$$

We consider pointwise system availability to be

$$A(t) = 1 - p_0(t) \text{ with } A(\infty) = 1 - P_0 \tag{6}$$

and Reliability of the system would be

$$R(t) = \sum_{i=1}^{n} q_i(t) \tag{7}$$

Mean time to system failure is

$$E(T) = \int_0^{\infty} R(x)dx = \sum_{i=1}^{n} \int_0^{\infty} q_i(x)dx = \sum_{i=1}^{n} q_i^*(0) \tag{8}$$

The probability of busy repairman at t, is

$$B(t) = 1 - p_n(t) \tag{9}$$

The expected fraction of time for under repair system is

$$\lim_{t \to \infty} B(t) = 1 - P_n \tag{10}$$

After taking Laplace transformation of Equations 1 and 2 with matrix notation, we get $A(s)\,P = I_{n+1}$ with $P' = \{P_0'(s)\,.\,P_1'(s)\ ...,\ P_n'(s)\}$ (11)

and

$B(s)\,Q = I_n$ with $Q' = \{Q_1'(s)\,.\,Q_2'(s)\}\ ...,\ Q_n'(s)\}$ (12)

and

$I'_{n+1} = (0, 0, ..., 1)$ (13)

where

$$A(s)=\begin{pmatrix} s+\mu_1 & -\lambda_1 & 0 & 0\ldots \\ -\mu_1 & (s+\lambda_1+\mu_2) & -\lambda_2 & 0\ldots \\ \ldots & & & \\ \ldots\mu_{r-1} & (s+\lambda_{r-1}+\mu_r) & -\lambda_r\ldots & \\ \ldots\mu_r & (s+\lambda_r+2\mu) & -\lambda_{r+1}\ldots & \\ \ldots 2\mu & (s+\lambda_{r+1}+2\mu) & -\lambda_{r+2} & \\ \ldots & & & \\ \ldots -2\mu & (s+\lambda_n) & & \end{pmatrix}$$

$$B(s)=\begin{pmatrix} (s+\lambda_1+\mu_2) & -\lambda_2 & 0\ldots \\ -\mu_2 & (s+\lambda_2+\mu_3) & -\lambda_3\ldots \\ \ldots & & \\ \ldots\mu_{r-1} & (s+\lambda_r+2\mu) & -\lambda_{r+1}\ldots \\ \ldots\mu_r & (s+\lambda_{r+1}+2\mu) & -\lambda_{r+2}\ldots \\ \ldots & & \\ \ldots -2\mu & s+\lambda_n & \end{pmatrix}$$

Using Cramer's rule, equations (11) and (12) give

$p_i'(s) = |A_i(s)|/|A(s)|\quad 0 \le i \le n$ (14.1)

and

$q_i'(s) = |B_i(s)|/|B(s)|\quad 1 \le i \le n$ (14.2)

We can find $A_i(s)$ and $B_i(s)$ from $A(s)$ and $B(s)$ by replacing ith column by unit vector in RIIS of 11 and 12.

Now we apply elementary row and column transformations on $|A(s)|$ and $|B(s)|$ we get

$|A(s)| = s|\phi(s)|$ (15.1)

and

$|B(s)| = |\psi(s)|$ (15.2)

with

$$\phi(s)=\begin{pmatrix} (s+\lambda_1+\mu_2) & -\sqrt{\lambda_1\mu_2} & 0\ldots \\ & (s+\lambda_2+\mu_2) & -\sqrt{\lambda_2\mu_3}\ldots \\ \ldots & & \\ \ldots -\sqrt{\lambda_{r-1}\mu_r} & (s+\lambda_r+\mu_r) & -\sqrt{2\lambda_r\mu}\ldots \\ \ldots -\sqrt{2\lambda_r\mu} & (s+\lambda_{r+1}+2\mu) & -\sqrt{2\lambda_{r+1}\mu}\ldots \\ \ldots & & \\ \ldots -\sqrt{\lambda_{n-1}2\mu} & (s+\lambda_n+2\mu) & \end{pmatrix}$$

$$\psi(s)=\begin{pmatrix} s+\lambda_1 & -\sqrt{\lambda_1\mu_2} & 0\ldots \\ -\sqrt{\lambda_1\mu_2} & (s+\lambda_2+\mu_2)\ldots & \\ \ldots & & \\ \ldots -\sqrt{\lambda_{r-1}\mu_r} & (s+\lambda_r+\mu_r)\ldots & \\ \ldots -\sqrt{\lambda_r 2\mu} & (s+\lambda_{r+1}+2\mu)\ldots & \\ \ldots & & \\ \ldots -\sqrt{\lambda_{n-1}2\mu} & (s+\lambda_n+2\mu) & \end{pmatrix}$$

By using the eigen values of tridiagonal positive definite matrices from the method developed by Gregory and Young [5] where the zeros of polynomials $|\phi(s)|$ and $|\psi(s)|$ are the negatives of eigen values of matrices $a(0)$ and $b(0)$.

$|\phi(n)|$ and $|\psi(n)|$ at $\phi_1, \phi_2, ..., \phi_n$ and $\psi_1, \psi_2, ...\ \psi_n$ are the zeros of $|\phi(n)|$ and $|\psi(n)|$.

$|A(s)| = s\prod_{k=1}^{n}(s-\phi_k)$ (16.1)

and

$|B(s)| = s\prod_{k=1}^{n}(s-\psi_k)$ (16.2)

with

$P_i'(s) = |A_i(s)|s\prod_{k=1}^{n}(s-\phi_k)\quad 0\le i\le n$ (17.1)

$q_i'(s) = |B_i(s)|/\prod_{k=1}^{n}(s-\psi_k)\quad 1\le i\le n$ (17.2)

The partial fractions of 17.1 and 17.2 give

$p_i'(s)=s^{-1}a_{0i} + \sum_{k=1}^{n} a_{ki}/(s-\phi_k)\quad 0\le i\le n$ (18.1)

$q_i'(s) = \sum_{k=1}^{n} b_{ki}/(s-\psi_k)\quad 1\le i\le n$ (18.2)

$a_{0i} = |A_i(0)|/\prod_{j=1}^{n}\phi_k$ (19.1)

$$a_{ki} = ¤A_i(f_k)¤/ f_k \underset{j=1\ Úk}{\overset{n}{P}} (f_k - f_j) \qquad (19.2)$$

$$b_{ki} = ¤B_i(y_k)¤/ \underset{j=1\ Úk}{\overset{n}{P}} (y_k - y_j) \qquad (19.3)$$

Inverse Laplace transforms of 18.1 and 18.2 give

$$P_i(t) = a_{0i} + \overset{n}{\underset{k=1}{Ś}} a_{ki}\, e^{f_k t} \quad 0 À i À n \qquad (20.1)$$

and

$$q_i(t) = \overset{n}{\underset{k=1}{Ś}} b_{ki}\, e^{y} \quad 1 À i À n \qquad (20.2)$$

With the help of these terms we can find steady state characteristics and other desired transient indices of the system.

A SPECIAL CASE

For n = 3, the system becomes

$$¤A(s)¤ = \begin{vmatrix} s+m_1 & -l_1 & 0 & 0 \\ -m_1 & s+l_1+m_2 & -l_2 & 0 \\ 0 & -m_2 & l_2+2m & -l_3 \\ 0 & 0 & -2m & l_3 \end{vmatrix} \qquad (21.2)$$

and

$$¤B(s)¤ = \begin{vmatrix} s+l_1+m_2 & -l_2 & 0 \\ -m_2 & s+l_2+2m & -l_3 \\ 0 & -2m & s+l_3 \end{vmatrix} \qquad (21.2)$$

with

$$¤f(s)¤ = \begin{vmatrix} s+l_1+m_1 & -¡\overline{l_1m_2} & 0 \\ -¡\overline{l_1m_2} & s+l_2+m_2 & ¡\overline{l_2m_3} \\ 0 & -¡\overline{l_3 2m} & s+l_3+2m \end{vmatrix} \qquad (22.1)$$

and

$$¤y(s)¤ = \begin{vmatrix} s+l_1 & -¡\overline{l_1m_2} & 0 \\ -¡\overline{l_1m_2} & s+l_2+m_2 & -¡\overline{l_2m_3} \\ 0 & -¡\overline{l_3 2m} & s+l_3+2m \end{vmatrix} \qquad (22.2)$$

We can calculate the factors of 22.1 and 22.2 as f_m and y_m (m= 1,2,3) with suitable techniques. These factors will be helpful to find

$$p_0(t) = \frac{l_1l_2l_2}{f_1f_2f_3} + \frac{l_1l_2l_3}{f_1(f_1-f_2)(f_1-f_3)} e^{f_1t} + \frac{l_1l_2l_3}{f_2(f_2-f_3)(f_2-f_1)} e^{f_2t} + \frac{l_1l_2l_3}{f_3(f_3-f_1)(f_3-f_2)} e^{f_3t} \qquad (23)$$

$$p_1(t) = \frac{m_1l_2l_3}{f_1f_2f_3} + \frac{(f_1+m_1)l_2l_3}{f_1(f_1-f_2)(f_1-f_3)} e^{f_1t} + \frac{(f_2+m_1)(l_2l_3)}{f_2(f_2-f_3)(f_2-f_1)} e^{f_2t} + \frac{(f_3+m_1)l_2l_3}{f_3(f_3-f_1)(f_3-F_2)} e^{f_3t} \qquad (24)$$

$$p_2(t) = \frac{m_1m_2l_3}{f_1f_2f_3} - \frac{[(f_1+m_1)(f_1+m_2)+f_2l_1]l_3}{f_1(f_1-f_2)(f_1-f_3)} e^{f_1t} - \frac{[(f_2+m_1)(f_2+m_2)+f_2l_1]l_3}{f_2(f_2-f_3)(f_2-f_1)} e^{f_2t} - \frac{[(f_3+m_1)(f_3+m_2)+f_3l_1]}{f_3(f_3-f_1)(f_3-f_2)} e^{f_3t} \qquad (25)$$

$$p_3(t) = \frac{2m_1m_2m}{f_1f_1f_3} - \frac{(f_1+m_1)[(f_1+m_2)(f_1+2m)+f_1l_2]}{f_1(f_1-f_2)(f_1-f_3)} e^{f_1t} - \frac{(f_2+m_1)[(f_2+m_2)(f_2+2m)+f_2l_2]}{f_2(f_2-f_3)(f_2-f_1)} e^{f_2t} - \frac{(f_3+m_1)[(f_3+m_2)(f_3+2m)+f_3l_2]}{f_3(f_3-f_1)(f_3-f_2)} e^{f_3t} \qquad (26)$$

$$q_1(t) = \frac{l_2l_3}{(y_1-y_2)(y_1-y_3)} e^{y_1t} + \frac{l_2l_3}{(y_2-y_1)(y_2-y_3)} e^{y_2t} + \frac{l_2l_3}{(y_3-y_1)(y_3-y_2)} e^{y_3t} \qquad (27)$$

$$q_2(t) = \frac{l_3(y_1+l_1+m_2)}{(y_1-y_2)(y_1-y_3)} e^{y_1t} + \frac{l_3(y_2+l_1+m_2)}{(y_2-y_1)(y_2-y_3)} e^{y_2t} + \frac{l_3(y_3+l_1+m_2)}{(y_3-y_1)(y_3-y_2)} e^{y_3t} \qquad (28)$$

$$q_3(t) = \frac{(y_1+l_1)(y_1+2m+l_2)+m_2(y_1+2m)}{(y_1-y_2)(y_1-y_3)} e^{y_1t} \frac{(y_2+l_1)(y_2+2m+l_2)+m_2(y_2+2m)}{(y_2-y_1)(y_2-y_3)} e^{y_2t} + \frac{(y_3+l_1)(y_3+2m+l_2)+m_2(y_3+2m)}{(y_3-y_1)(y_3-y_2)} e^{y_3t} \qquad (29)$$

This method can be applied for all multiple unit redundant system with n Â 3. We can find various characteristics given in Equations 6-10 by using Equations 23-29.

ILLUSTRATION

For numerical illustration purpose, we consider n=2. In this case, we have

$$f_1 = -1/2\,[(l_1 + l_2 + m_1 + m_2) - \{(l_1 + l_2 + m_1 + m_2)^2 - 4(l_1 l_2 + m_1 (l_2 + m_2))\}^{1/2}] \quad (30.1)$$

$$f_2 = -1/2[(l_1 + l_2 + m_1 + m_2) + \{(l_1 + l_2 + m_1 + m_2)^2 - 4(l_1 l_2 + m_1(l_2 + m_2))\}^{1/2}] \quad (30.2)$$

$$y_1 = -1/2[(l_1 + l_2 + m_2) - \{(l_1 + l_2 + m_2)^2 - 4 l_1 l_2\}^{1/2}] \quad (30.3)$$

$$y_2 = -1/2[(l_1 + l_2 + m_2) - \{(l_1 + l_2 + m_2)^2 - 4 l_1 l_2\}^{1/2}] \quad (30.4)$$

which gives

$$p_0(t) = \frac{l_1 l_2}{f_1 f_2} + \frac{l_1 l_2}{f_1(f_1 - f_2)} e^{f_1 t} + \frac{l_1 l_2}{f_2(f_2 - f_1)} e^{f_2 t} \quad (31)$$

$$p_1(t) = \frac{m_1 l_2}{f_1 f_2} + \frac{l_1 l_2}{f_1(f_1 - f_2)} e^{f_1 t} + \frac{l_2(m_1 + f_2)}{f_2(f_2 - f_1)} e^{f_2 t} \quad (32)$$

$$p_2(t) = \frac{m_1 m_2}{f_1 f_2} + \frac{\{(m + f_1)(m_2 + f_1) + l_1 f_1\}}{f_1(f_1 - f_2)} e^{f_1 t} + \frac{\{(m_1 + f_2)(m_2 + f_2) + l_1 f_2\}}{f_2(f_2 - f_1)} e^{f_2 t} \quad (33)$$

$$q_1(t) = l_2\left[\frac{\{(e^{y1})(e^{y2})\}}{(y1 - y2) + y2 - y1}\right] \quad (34)$$

$$q_2(t) = \frac{(l_1 + m_2 + y_1)}{y_1 - y_2} e^{y_1 t} + \left[\frac{(l_1 + m_2 + y_2)}{y_2 - y_1}\right] e^{y_2 t} \quad (35)$$

Choosing

$l_1 = 1/80$, $l_2 = 1/60$, $m_1 = 1/70$, $M_2 = 1/50$

we have

$f_1 = -0.01443$, $f_2 = -0.04905$, $y_1 = -0.00448$, $f_2 = 0.04472$

so that

$p_0(t) = 0.2857 - 0.4e^{-0.01443t} + 0.1176e^{-0.04905t}$

$p_1(t) = 0.3412 + 0.00435e^{-0.01443t} - 0.3418e^{-0.04905t}$

$p_2(t) = 0.40857 + 0.36072e^{-0.01443t} - 0.2343e^{-0.04905t}$

$q_1(t) = 0.415\,(e^{-0.00448t} - e^{-0.04472t})$

$q_2(t) = 0.6963\,e^{-0.00448t} + 0.3037\,e^{-0.04472t}$

Hence the system characteristics are

$A(t) = 0.7143 + 0.4\,e^{-0.01443t} - 0.1176e^{-0.04905t}$

$R(t) = 1.1113\,e^{-0.00448t} - 0.1113\,e^{-0.04472t}$

$B(t) = 0.59143 - 0.36072\,e^{-0.01443t} - 0.2343e^{-0.04905t}$

For similar units case we take

$l_1 = l_2 = 1/80$ and $m_1 = m_2 = 1/50$

Now we have

$f_1 = -0.0167$, $f_2 = -0.0483$, $y_1 = -0.00389$, $y_2 = -0.0411$

So that

$p_0(t) = 0.1975 - 0.3019\,e^{-0.0167t} + 0.1046\,e^{-0.0483t}$

$p_1(t) = 0.3086 - 0.0779\,e^{-0.0167t} + 0.2288\,e^{-0.0483t}$

$p_2(t) = 0.4938 - 0.3757\,e^{-0.0167t} + 1287\,e^{-0.0483t}$

$q_1(t) = 0.3359\,(e^{-0.00389t} - e^{-0.0411t})$

$q_2(t) = 0.7689\,e^{-0.00389t} + 0.2318\,e^{-0.0411t}$

Hence the system characteristics are

$A(t) = 0.8025 + 0.3019\,e^{-0.0167t} - 0.1046\,e^{-0.0483t}$

$R(t) = 1.1048\,e^{-0.00389t} - 0.1041\,e^{-0.0411t}$

$B(t) = 0.5062 - 0.3757\,e^{-0.0167t} - 0.1287\,e^{-0.0483t}$

The reliability and availability curves for the similar and dissimilar units system are shown in Figures 1 and 2 respectively. It can we noticed that reliability and availability derease as time increases. The transient state probabilities by varying time for the case of dissimilar and similar units are summarized in Tables 1 and 2 respectively.

CONCLUSION

In this paper, the transient analysis of cold standby system with n dissimilar units is discussed. The reliability and availability indices for redundant system are obtained by solving balance equations. We have also illustrated numerically the tactability of the approach by taking n = 2 for similar as well as dissimilar units.

6. REFERENCES

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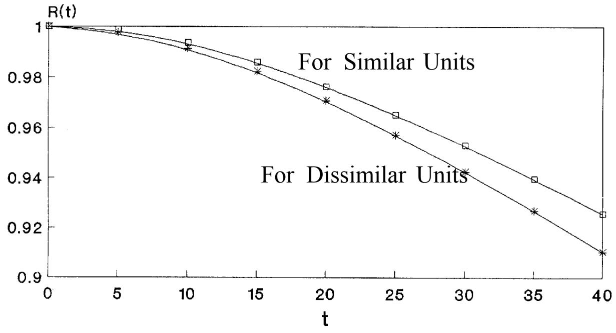

Figure 1. Reliability for two units system.

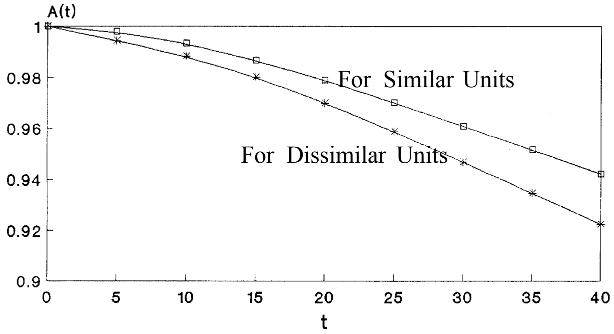

Figure 2. Availability for two units system.

TABLE 1. Transition Probabilities for the System with Similar Units.

t	$p_0(t)$	$p_1(t)$	$p_2(t)$
5	0.0055	0.0777	0.9275
10	0.0115	0.1357	0.8643
15	0.0198	0.1809	0.7814
20	0.0300	0.2164	0.7667
25	0.0413	0.2440	0.7288
30	0.0533	0.2656	0.6964
35	0.0654	0.2825	0.6684
40	0.0776	0.2956	0.6443

TABLE 2. Transistion Probabilities for the System with Dissimilar Units.

t	$p_0(t)$	$p_1(t)$	$p_2(t)$
5	0.0020	0.0582	0.9404
10	0.0065	0.1016	0.8911
15	0.0132	0.1371	0.8486
20	0.0211	0.1657	0.8118
25	0.0299	0.1889	0.7798
30	0.0392	0.2077	0.7516
35	0.0485	0.2230	0.7269
40	0.0579	0.2355	0.7050

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