RESEARCH NOTE

AN OPTIMUM RESOURCE ALLOCATION MODEL FOR AIRPORT PASSENGER TERMINALS

M. Saffarzadeh

Department of Civil Engineering
Tarbiat Modaress University
Tehran, Iran

Abstract According to the current Airport planning and design practices, for a given demand level there is a corresponding space requirement. While in practice, there are always trade-offs between cost and levels of service, labor and automation, equipment and fixed facilities, and expansion of existing facilities and the addition of new ones. In this research, the airport passenger Terminal Building (PTB) was divided into several segments (components of the PTB system). Each segment has its own set of characteristics such as; unit operational and maintenance cost, level of service standards, and variable traffic demand pattern. The optimization model which is discussed in this paper determines the optimum required resources for each segment of the PTB to perform its activity taking into account the operational and maintenance cost involved and the level of service provided to the users.

Key Words Airport, Passenger Terminal, Resource Allocation, Optimization, Over and Under Supply Cost

چکیده روشهای کنونی برنامه ریزی و طراحی فرودگاهها، میزان فضای موردنیاز را مستقیماً بر اساس مقدار تقاضای وارده بر سیستم محاسبه می کنند. این در حالی است که در عمل بایستی همیشه حالت تعادل و تناسبی بین متغیرهایی مانند: سطح سرویس و هزینه تمام شده، هزینه پرسنل و اتوماسیون، تجهیزات پیشرفته و تاسیسات ثابت زیربنایی، توسعه امکانات موجود و ساخت زیربناهای جدید باشد. در این مقاله سیستم ترمینال مسافری فرودگاه به چندین بخش عملیاتی و سرویس دهنده تقسیم شده است. هر بخش از ترمینال دارای ویژگیهای فنی و عملیاتی مخصوص به خود مانند هزینه ساخت، هزینه بهره برداری و نگهداری، سطح سرویس و تقاضای ترافیک متغیر می باشد. مدل توضیح داده شده در این مقاله میزان بهینه منابع موردنیاز هر بخش برای ارائه خدمات ضروری به استفاده کنندگان را در سطوح مختلف سرویس تعیین می کند.

INTRODUCTION

There has been little research to investigate optimization theory in the planning, design, and operation of airport PTBs. The only exception is the development of a design methodology, based on the heuristic modelling technique, to produce an optimum terminal design [1]. The methodology is composed of three major algorithms; facility sizing algorithm, the load assignment algorithm, and the facility layout algorithm. This methodology first determines the minimum amount of areal spaces; second the loads are assigned to the facilities in such a way that transport cost, expressed as the sum of the products of passenger flow times distance, is at minimum, and third the facilities are located relative to each other in such a manner that the transport cost is also at a minimum. The second and third steps are iterated until and optimum design has been obtained. The methodology is very useful in planning and design in terms of optimum concept selection. It does not deal with the PTB components in detail in terms of operating characteristics and stochastic demand.

In this research, the whole PTB is considered as a system in which labor, capital, and services are deployed to produce certain services to passengers. The function of this complex system may be seen as taking a passenger and providing some services to that passenger. This provision of services is associated with some cost to operators as well as passengers. For example, operating and maintenance costs which constitute a major portion of the total cost, has been almost always neglected in the current planning and design procedures. Operating and maintenance costs can be reduced by a reduction in the level of service, especially at peak periods, but at some cost to the passenger. The least cost solution may not be always the best solution for the passenger. On the other hand, terminal configurations that supposedly offer high levels of service may be expensive to operate. Those costs will be ultimately paid by the traveller either through higher fares, or other user charges. Optimizing the associated costs with the PTB operation is the subject of the optimiztion model discussed in this paper.

OPTIMIZATION THEORY

The research problem addressed in this paper is that given a fixed amount of resources, e.g., PTB space, the process should determine the allocation of all or part of the resources to a series of activities, with variable demand, in such a way that the objective function under consideration is optimized. In other words, the problem addressed here is a resource allocation problem with a series of constraints.

The resource allocation problem is generally formulated as follows:

$$\begin{aligned} &\text{Minimize } f(x_1, x_2, \ldots, x_n) \\ &\text{subject to: } \textstyle\sum_{j=1}^{n} (x_j) = N,\ x_j > 0 \qquad (1) \\ &\qquad j = 1, 2, 3, \ldots, n \end{aligned}$$

That is, given one type of resource whose total amount is equal to N, it is desired to allocate it to n segments which serve an uncertain number of customers so that the objective value $f(x)$ becomes as small as possible. The variable x_j in Equation 1 respresents the amount of resource allocated to segment j. If the resource is not divisible, e.g., persons, processors, then the variable x_j is a discrete variable that takes nonnegative integer values. In this case, the constraint x_j = integer, $j = 1, 2, 3, \ldots, n$ is added to Equation 1.

The objective function in general form, i.e., Equation 1, cannot be used for airport PTBs due to the fact that one may have more resources than what is required. A special objective function for this problem should be developed in such a way that the allocated resources may be smaller or equal to the total resource available. The objective function for this research problem is as follows:

$$\text{Minimize: } \textstyle\sum_{j=1}^{n} c_j x_j$$

$$\text{subject to: } \textstyle\sum_{j=1}^{n} (x_j) \le N,\ x_j > 0 \qquad (2)$$

where,

$c_j(x_j)$ = expected over-and under-supply cost of allocating x_j to segment j,

x_j= resource allocated to segment j, e.g., space,

n = total number of PTB segments,

N = total amount of available resource, e.g., PTB passenger processing area.

There are two types of costs associated with the allocation of resources, i.e., over-and under-supply cost. Over-supply cost is the cost of providing resources more than what is required and under-supply cost is the cost of not providing enough resources to meet the demand. Moreover, allocation of resources depends on the demand placed upon the facility. The demand at each segment is also uncertain and depends mainly on the flight schedule. Taking all the variables into consideration, the expected over- and under-suply cost function for the PTB is found as follows:

Assume that y is the demand variable at each segment and $p_j(y)$ is the Probability Mass Fuction (PMF) for variable y at segment j. This means that the probability of having y units of demand at segment j is $p_j(y)$. It is also assumed that each unit of demand needs θ_j units of resource at segment j, e.g., the amount of space that each passenger occupies. If x_j is the amount of resource supplied to segment j, then the expected amount of over-supply resources would be:

$$\sum_0^{\delta_j} (x_j - \theta_j y)\, p_j(y) \tag{3}$$

where,

x_j = the amount of resource supplied,

y = demand variable, i.e., number of passengers,

$p_j(y)$ = probability of having y units of demand at segment j,

θ_j = the amount of resource needed by each demand unit, LOS,

δ_j = integer (x_j/θ_j).

To calculate the cost associated with the amount of over-supply resources, the unit cost of over-supply at segment j should be found. If α_j is assumed to be the unit cost of over-supply at segment j, then the over-supply cost at this segment is as follows:

$$\alpha_j \sum_0^{\delta_j} (x_j - \theta_j y) p_j(y) \tag{4}$$

As was mentioned, if the resources supplied to segment j were less than required then there would be an under-supply cost. Following the same process and assuming β_j to be the unit cost of under-supply, the expected under-supply cost would be:

$$\beta_j \sum_{\delta_j}^{Y} (\theta_j y - x_j) p_j(y) \tag{5}$$

where

Y= maximum expected demand for segment j,

β_j= the unit cost of under-supply.

Therefore, the total cost associated with the allocation of x_j resources to segment j is the sum of the two preceding cost elements,

$$C_j X_j = \alpha_j \sum_0^{\delta_j} (x_j - \theta_j y)\, p_j(y) + \beta_j \sum_{\delta_j}^{Y} (\theta_j y - x_j) p_j(y) \tag{6}$$

By solving Equation 6 for different values of x_j, the optimum resource value associated with the minimum total cost, for one specific segment, can be found. Since the PTB system consists of several segments for which resources should be allocated, the total expected over-and under-supply cost for the whole system would be as follows:

$$C_T = \sum_{j=1}^{n} C_j X_j = \sum_{j=1}^{n} [\alpha_j \sum_0^{\delta_j} (x_j - \theta_j y)\, p_j(y) + \beta_j \sum_{\delta_j}^{Y} (\theta_j y - x_j) p_j(y)] \tag{7}$$

where,

C_T= total expected over-and under-supply cost of the PTB system,

n = maximum number of PTB segments.

It is hardly possible to find an absolute mathematical solution for the preceding equation in which the resources and demand are assumed indivisible. It is possible to solve this equation numerically or by computer programs and provide the values of x_j for all predefined segments of the PTB.

However, if the resources and demand were assumed to be divisible, then x_j and y are continuous variables that can take any nonnegative real values. In this case following the same procedure of indivisibility, the total over-and under-supply cost function for segment j would be as follows:

$$C_j X_j = \alpha_j \int_0^{\delta_j} (x_j - \theta_j y)\, dF_j(y) + \beta_j \int_{\delta_j}^{\infty} (\theta_j y - x_j)\, dF_j(y) \tag{8}$$

where,

$F_j(y)$ = cumulative distribution function of demand at segment j which is continuous and increasing,

θ_j = constant representing the amount of required resource for each unit of demand function,

α_j = unit cost of over supply at segment j,

β_j = unit cost of under supply at segment j.

The cost function for the continous case, Equation 8, can be rewritten as follows:

$$C_j X_j = \alpha_j x_j \int_0^{\delta_j} dF_j(y) - \alpha_j \theta_j \int_0^{\delta_j} y dF_j(y) + \beta_j \theta_j \int_0^{\infty} y dF_j(y) - \beta_j \theta_j \int_0^{\delta_j} y dF_j(y) - \beta_j x_j \int_0^{\infty} dF_j(y) + \beta_j x_j \int_0^{\delta_j} dF_j(y) \tag{9}$$

If μ_j is defined as the mean of $F_j(y)$ then by using the principles of probability theory such as:

$$\int_0^{\infty} dF_j(y) = 1.0 \quad ; \quad \int_0^{\infty} y dF_j(y) = \mu_j \tag{10}$$

the preceding equation would simplify to the next equation, i.e:

$$C_j X_j = \beta_j \theta_j \mu_j - (\alpha_j + \beta_j)\, \theta_j \int_0^{\delta_j} y dF_j(y) + (\alpha_j + \beta_j) x_j \int_0^{\delta_j} dF_j(y) - \beta_j x_j \tag{11}$$

Equation 11 would be further simplified to,

$$C_j X_j = \beta_j(\theta_j \mu_j - x_j) + (\alpha_j + \beta_j)(x_j F_j \delta_j) - (\alpha_j + \beta_j)\theta_j \int_0^{\delta_j} y dF_j(y) \tag{12}$$

Therefore, the only integral left in Equation 12 is analyzed as:

$$\int_0^{\delta_j} y dF_j(y) = \int_0^{\infty} y dF_j(y) - \int_0^{\infty} y dF_j(y) \tag{13}$$

The first part of Equation 13 is equal to μ_j and the second part can be solved by using the following expected value theory [2]:

$$\int (x > \alpha)\, x dF(x) = \alpha(1 - F(\alpha)) + \int_{\delta_j}^{\infty} (1 - F(x))\, dx \tag{14}$$

Replacing X with y, α_j with δ_j and $F(x)$ with $F_j(y)$ leads to,

$$\int_{\delta_j}^{\infty} y dF(y) = \delta_j (1 - F(\delta_j)) + \int_{\delta}^{\infty} (1 - F(y))\, dy \tag{15}$$

The integral in Equation 15 is analyzed to,

$$\int_{\delta_j}^{\infty} (1 - F(y)) dy = \int_0^{\infty} (1 - F(y)) dy - \int_0^{\delta_j} (1 - F(y)) dy \tag{16}$$

The second integral can be broken into two parts, Equation 16 would simplify as;

$$\int_{\delta_j}^{\infty} (1 - F(y))\, dy = \mu_j - \delta_j + \int_0^{\delta_j} F(y)\, dy \tag{17}$$

Finally by substituting Equation 17 into 13 and substituting Equation 13 into 12, the total cost function would be simplified to a determinate function in which all of its elements can be calculated,

$$C_j X_j = \beta_j \theta_j (\mu_j + \delta_j) + (\alpha_j + \beta_j)\theta_j \int_0^{\delta_j} F(y)\, dy \tag{18}$$

Considering that $\delta_j = x_j / \theta_j$ then the preceding equation can be written with respect to x_j,

$$C_j X_j = \beta_j \theta_j (\mu_j - \frac{x_j}{\theta_j}) + (\alpha_j + \beta_j)\theta_j \int_0^{x_j/\theta_j} F(y)\, dy \qquad (19)$$

If the demand function is known, the absolute value for x_j can be found by solving Equation 19 mathematically. To find the optimum value for x_j, the derivative of the final equation with respect to x_j should be taken, i.e:

$$C_j X_j = -\beta_j + (\alpha_j + \beta_j)\theta_j(\mu_j - \frac{x_j}{\theta_j}) \qquad (20)$$

If the value of derivative is substituted by zero, and then by solving the derivative with respect to x_j, the absolute value of x_j can be found. From Equation 19 and its derivative (F is increasing) it is also clear that the cost function is convex with respect to variable x_j which means that there is a minimum point in the cost function.

So far, the equation for finding the optimum resource value for one typical segment of the PTB based on the minimum over-and under-supply cost was found. The objective function was to minimize the expected total cost of operating the whole PTB system consisting of several segments. Therefore, the problem would be a resource allocation problem with a separable convex objective function. There are few approaches to solve the allocation problems of the type in Equation 1 in which the total amount of resources, N, would be allocated to the segments. If the demand function and the values for α_j and β_j are known, then Equation 1 can be written as a series of nonlinear separable convex functions which have to be optimized. In other words, the values of x_j should be found in such a way that they minimize the expected cost function. Then knowing all the variables and constants, algorithms can be developed, e.g., RANK or RELAX, to find the optimal values for x_j [3]. It should be noted that several assumptions are inherent in these algorithms. For example, these algorithms minimize the sum of convex objective functions of one variable under a simple constraint that all variables sum to a given constant, i.e., maximum resource available,. They also assume that each objective function is strictly convex, i.e., has a defined mathematical function whose derivation is increasing in x_j. These assumptions are not supported for airport PTBs in which hardly all resources are fully utilized and the demand function cannot be defined mathematically.

The objective function of this research problem is more complex due to the fact that the sum of allocated resources could be less than or equal to the maximum resource available. Except for some approximation procedures, no formal, computationally mathematical solution exist for optimally solving Equation 2. In addition, more complexities exist within the equation such as the exact demand function, and over-and under-supply costs for the unit. Due to the stochastic nature of passenger arrival and departure at the PTB, no specific mathematical function can represent the actual demand on the system at each instant of time.

Another difficulty associated with the mathematical approach is finding values for α_j and β_j. The value of α_j depends on the unit cost of over-supply of a facility or the activity which is going to be performed, e.g., design, construction, operation and maintenance. The value of α_j can be obtained by going through a cost allocation process. First, all the cost items associated with the PTB's operation should be estimated. Second, the sum of these costs should be divided by the total amount of resources available. Then the cost of providing one unit of extra resource can be obtained. For example, the procedure for the estimation of the operational cost of over-supply, α_j, is summarized as follows:

$$\alpha_j = OPS_{total} / RES_{total} \qquad (21)$$

where,

OPS_{total} = total PTB operational and maintenance cost
RES_{total}= total resource available such as space, labor.

The value of β_j is even more difficult to estimate. The question to be answered is how much the cost to the operator would be if resources are provided one unit less than what is required. In the case of multiple airports, the operator at one airport may lose customers due to the availability of better service at another location. One approach is to put monetary values on the amount of discomfort such as: congestion, delay, walking distance, etc. experienced by passengers. This approach is interpreted as a social cost estimation which would give an impression of the under-supply cost from a users' point of view.

OPTIMIZATION MODEL DEVELOPMENT

Existing optimization algorithms, e.g., RANK, RELAX, in combinatorial optimization were examined to find out if they could be used to solve the developed optimization problem. The results were negative due to the fact that in an airport PTB, not all resources need to be allocated and no mathematical function can represent the variable demand distribution on the PTB segments. Since the assumptions of these algorithms are not supported by the real life PTB operation, an algorithm was developed from first principles. A simplified flow chart of optimization program is shown in Figure 1.

The algorithm of the program consists of mainly two parts, i.e., optimization and sub-optimization. In the optimization part no constraint has been set for the maximum amounts of available resources, while in the sub-optimization part the maximum amounts of available resources are limited. The mechanism of the optimization program is summarized as follows:

The only inputs to the program are the Probability Mass Functions (PMFs), obtained from a Terminal simulation model and variables of α_j, β_j and θ_j. The PMFs for different segments of the PTB are saved in separate data files. Each data file contains the population distribution function for 24 hour time periods of a typical day for a specific segment of the PTB. The relative values of α_j and β_j are assumed based on engineering judgment or historical data. The variable θ_j will represent the level of service concept within the optimization. In other words, θ_j represents the rquired resource value for each demand unit at each LOS, e.g., square meters per person.

The most common quantitative factors that influence level of service in PTBs are congestion which is measured in terms of number of passengers per unit area, queue length, and waiting time [4,5,6]. Transport Canada [4] proposed a comprehensive level of servie assessment method based on providing "space" at different PTB components. The method which was subsequently adopted by International Air Transport Association [5], established six different levels of service based on space provision, i.e., square meters per person. The boundaries for the various PTB facilities are shown in Table 1.

Data files containing the PMFs are opened and scanned into the program. The time of the day is divided into equal time periods, e.g., one hour long. Within one time period, the procedure will find the optimum required resources for different segments of the PTB. The program calculates the over-and under-supply costs associated with each resource value. Then the resource value associated with the minimum total over-and under-supply cost is called optimum required resource.

The sum of optimum resource values for all PTB segments during first time period is compared with the maximum existing resources, e.g., total PTB processing area. If the sum of resources is smaller than the maximum value, then the time is incremented and the same process is repeated for all other time periods. If the sum of resources for all segments at a specific time period is greater than the maximum existing resource, the process will start to sub-optimize

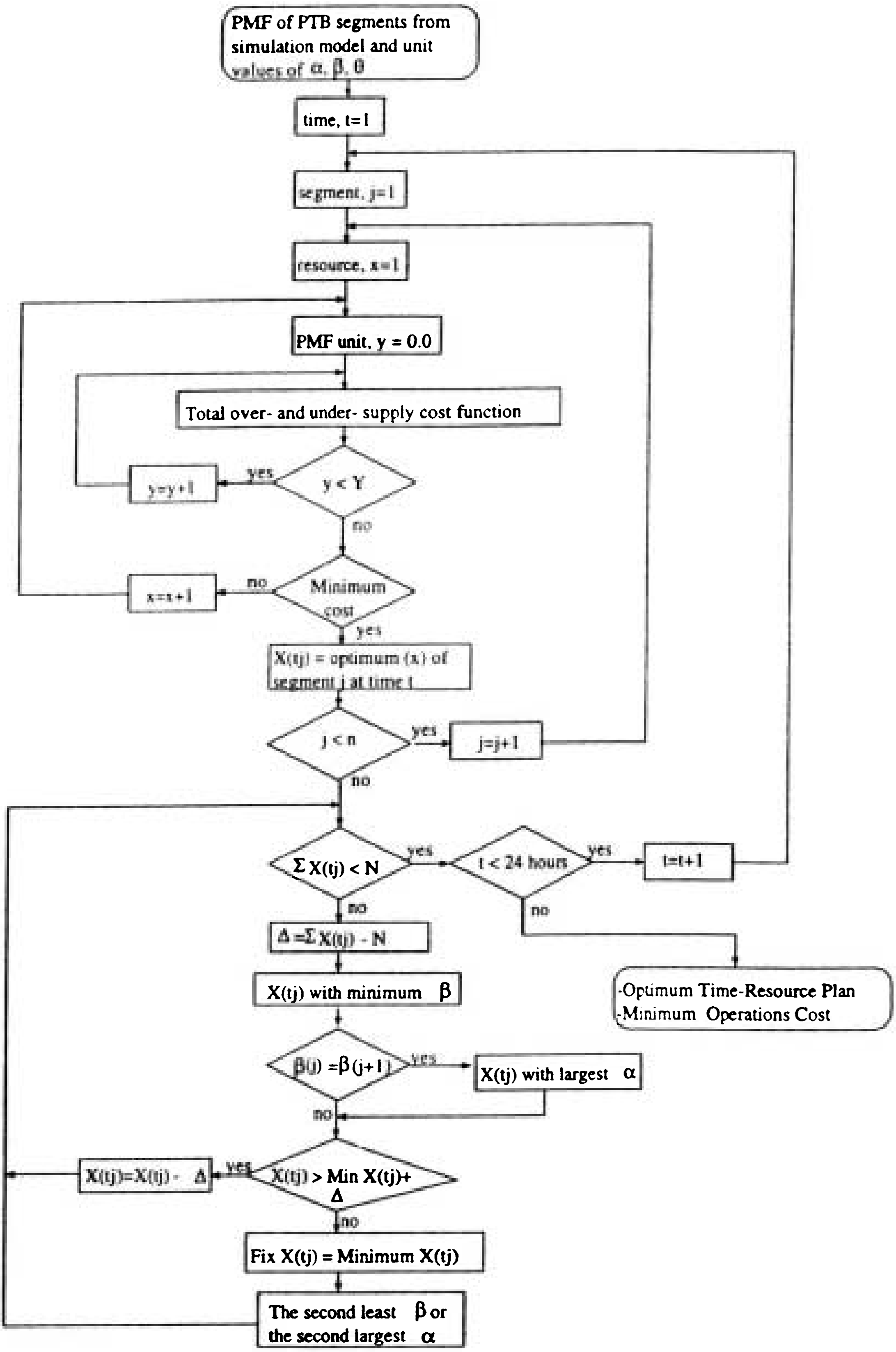

Figure 1. Flowchart of the optimization program.

TABLE 1. LOS Targets for PTB Components, [4].

Terminal Component	A to B m^2/ person	B to C m^2/person	C to D m^2/person	D to E m^2/person	E to F m^2/person
Check-in	1.6	1.4	1.2	1.0	0.8
Waiting areas	2.7	2.3	1.9	1.5	1.0
Holdroom	1.4	1.2	1.0	0.8	0.6
Baggage claim	1.6	1.4	1.2	1.0	0.8
PIL area[1]	1.4	1.2	1.0	0.8	0.6

1: Preliminary Inspection Lines for Passport Control.

the system. The optimization process will find the optimum resources without any constraint at a global minimum total cost. Having the constraint of limited total resources, the optimum values will be adjusted at a price of increased total cost.

The sub-optimization process will be done in such a way that it would minimize the increased cost to the total expected over-and under-supply cost. Therefore, the process will find the optimum resource value of the segment associated with the minimum value of under-supply unit cost, β_j. The optimum resource value of the segment will be decremented until the sum of resources for all segments is equal to the maximum existing resource. It should be noted that the resource value of any segment cannot be decremented lower than its minimum value. The minimum resource value for each segment could be the required resorces at the lowest operating service level. Transport Canada [4] recommends the level of service C as a design standard, as it provides good level of service at a reasonable cost. However, the minimum level of service can be defined by the user.

The sum of the new optimum resources of various segments will be compared with the maximum existing resources and if it is still higher than the maximum, another segment with the second lowest β_j will be chosen for sub-optimization. If the values of β_j for two segments are equal, then the segment with the higher unit cost of over-supply, α_j, will be chosen. The rationale is to choose the segments for sub-optimization which have the least impact on the total cost increase. As mentioned earlier, the cost of operating different segments of the PTB may be different due to the type and the cost of facilities involved in their operation. From the analysis, it was found that the lowest β_j and the highest α_j have the minimum impact on the total over-and under-supply cost. This process is repeated until the sum of the optimum required resources are equal to the maximum availabe resources.

The output of the optimization program would be the optimum resource values in a variable time-space plan format. The program will also provide the associated supply costs of resources for a 24-hour period. How close one can bring the practical plan to the theoretical plan depends on the flexibility of the physical layout and other constraints, e.g., traffic demand pattern.

The sum of optimum resource values from vairous segments multiplied by the unit cost of providing resources is the total cost of operating the PTB at each instant of time. If all the conditions are met, the operational and maintenance cost would be a function of demand distribution. Therefore, one of the objectives of this research, which was to produce a variable time-cost plan as opposed to a fixed cost plan, is achieved. This optimization procedure, if applied properly, will result in signification savings on the operation and maintenance costs of PTBs over long time periods.

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