SINGLE SERVER BULK QUEUE WITH SERVICE INTERRUPTION, TWO PHASE REPAIRS AND STATE DEPENDENT RATES

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Abstract This paper reports a study on a single server queue with bulk arrival and bulk service patterns wherein the incoming traffic depends on the state of the server which may be in operating or breakdown state. The repair of the breakdown server is performed in two phases. The operating duration of server, repair duration of both phases of repairing as well as job's inter-arrival times and service times are exponentially distributed. By using generating function approach, the transient analysis for the queue size distribution has been carried out. For steady state, the stability condition and average queue length for various states have been derived. By taking a numerical illustration, the effect of various parameters on the average queue length has been depicted graphically.

Key Words Bulk Queue, Breakdown, Phase Repairs, State Dependent Rates, Generating Functions, Queue Length

چکیده این مقاله، مطالعه ای را در یک صف با خادم تنها (در صفی با یک خادم) با ورود درهم (فلهّ ای) و الگوهای خدمت درهم گزارش می کند که در آن ترافیک ورودی به حالت خادمی که در حالت انجام کار یا خارج از مدار باشد مربوط می شود. مرمّت (تعمیر) خادم از مدار خارج شده در دو فاز انجام می گیرد. مدّت زمان فعّال خادم، مدت زمان مرمّت فازهای تعمیر همچنین زمانهای ورود داخلی شغلها و زمانهای خدمت، به طور نمایی توزیع می یابند. با بکار بردن روش (راه حل) تابع مولد، تحلیل گذرا برای اندازه توزیع صف انجام داده شده است. در حالت پایا، شرط پایداری (اصطلاحاً معیار پایداری می گویند) و طول میانگین صف برای حالتهای مختلف به دست آورده شده اند. با در نظر گرفتن تصویری از مقادیر عددی، اثر پارامترهای مختلف بر روی طول میانگین صف به طور ترسیمی مجسّم (رسم) می شود.

INTRODUCTION

The phenomena of service interruption due to failure of service channel are common in many real life queueing situations ranging from computer and communication systems to manufacturing and production processes. During the server-breakdown period when the service of the jobs is interrupted, the arriving units may be discouraged to join the queue. Thus, server's breakdown also influences the input traffic so that it is essentially important to study the effect of server's status on the behaviour of customers as well as on the performance of the system. One of the earlier important works done in this direction was performed by Yechiali and Naor [1]. Shagon [2] studied a single server queueing model wherein the arrival rate depends on server-breakdown. By assuming Exponential and Erlange k distribution for operational and failed periods respectively, he obtained steady state waiting time and average queue length. The service breakdown and repair processes for the system where an incoming stream of jobs is split into N substreams each of which is directed to a separate server and its associated queues served by N stations subject to breakdown and repair was addressed by Wartenhorst [3]. Jain [4] studied optimal N-policy for state dependent markovian queue with single removable and unreliable server.

Madan [5] included batch sevice in single server queueing model with service interruption. Madan [6] extended his work by incorporating two phase repairs. However, the arrival rate was not considered dependent on server's state which might being either in operationaı or breakdown mode. The state dependent markovian queue with bulk service was investigated by Ganesan [7]. Jayaraman [8] analyzed a general bulk service queueing model with arrival rate depending upon the server's breakdowns. He suggested a numerical method based on matrix-geometric approach to obtain steady state probability vector of the number of customers in the system. The state dependent markovian queue with bulk service was also investigated by Ganesan [7].

Several modes of transportation such as buses, trains, ships, airplanes, elevators, conveyor belts etc. are suitable examples of bulk arrival and bulk service system where incoming customers may be affected by the status of the server which is subject to breakdown and repair. We develop a bulk service queueing model wherein customers arrive in batches of varying size at the service facility which is subject to breakdown and repair in two phases. The customer's arrival rate and batch size distribution depend upon the server's state which may be in operating or breakdown mode. The repair rates for both phases of repairs also depend on the system state (empty or non-empty). By assuming exponential distributions for duration of different states of the system, the differential difference equations are constructed in terms of transient transition probabilities of the number of jobs present in various states. For steady state, the equilibrium condition and average queue size distribution have been derived in explicit form.

THE MODEL AND GOVERNING EQUATIONS

The assumptions underlying the model are as follows:

- The System may be in one of the following disjoint states:

(B) the server is busy in providing service to customers in batches of fixed size **b** (≥ 1).

$(R^{(1)})$ the server has broken down and phase 1 repairing is being done.

$(R^{(2)})$ the phase 2 repairing is being performed on the server.

(I) the server is operational but idle.

- The jobs arrive at the system in poisson fashion with mean rate $\lambda(\lambda_0)$ in batch with P.G.F. $C(z) = \sum_{i=1}^{k} c_i z^i$ when the server is operating (idle). When the server breaks down, the customers are assumed to arrive in poisson fashion but with a different mean rate λ_j in batches whereas the system state is $R^{(j)}$, $j = 1,2$.

- The service times and durations of operating times are exponentially distributed with the means $1/\mu$ and $1/\alpha$, respectively.

- The repair times of j phase repair, $j = 1,2$ are exponentially distributed with mean rate β, (β_j) when the system is non-empty (empty).

- As soon as the server breaks down, it stops providing service to jobs and enters into the first phase of repair. After completing the first phase of repair, it immediately undergoes for the second phase of repair.

- After completing both phases of repairing, the server restarts the service of the jobs in batch of constant size b $(b \geq 1)$. When the server fails while providing service to a batch of jobs, the service done is lost.

- The repair is perfect and the switch over time from operating to repair state and vice versa are instantaneous.

Let $W_n(t)$ and $R_n^j(t)$, $j = 1,2$ be the probability that there are n customers present in the system at time t when the server is in states (B) and $(R^{(j)})$, respectively. We exclude that batch of jobs which is in service or on suspended service. We denote $Q(t)$ as the probability that the system is in state I at time t. The laplace transform of f(t) is denoted by f(s). W_n, $R_n^{(j)}$ and Q denote the corresponding steady state probabilites.

The differential difference equations governing the model are:

$$\frac{d}{dt} W_n(t) = -(\lambda + \mu + \alpha) W_n(t) + \sum_{i=1}^{n} \lambda c_i W_{n-1}(t) + \mu W_{n+b}(t), \quad (n \geq 1) \tag{1}$$

$$\frac{d}{dt} W_0(t) = -(\lambda + \mu + \alpha) W_0(t) + \lambda_0 Q(t) + \mu \sum_{k=1}^{b} W_k(t) + \beta_2 R^{(2)}{}_0(t) \tag{2}$$

$$\frac{d}{dt} R_n^{(1)}(t) = -(\lambda + \beta_1) R_n^{(1)}(t) + \sum_{i=1}^{n} \lambda_1 c_i^{(1)} R^{(1)}{}_{n-i}(t) + \alpha W_n(t), \quad (n \geq 1) \tag{3}$$

$$\frac{d}{dt} R_0^{(1)}(t) = -(\lambda_1 + \beta_1') R_0^{(1)}(t) + \alpha W_0(t) \tag{4}$$

$$\frac{d}{dt} R_n^{(2)}(t) = -(\lambda_2 + \beta_2) R_n^{(2)}(t) + \sum_{i=1}^{n} \lambda_2 c_i^{(2)} R^{(2)}{}_{n-i}(t) + \beta_1 R_n^{(1)}(t), \quad (n \geq 1) \tag{5}$$

$$\frac{d}{dt} R_n^{(2)}(t) = -(\lambda_2 + \beta_2') R_0^{(2)}(t) + \beta'_1 R_0^{(1)}(t) \tag{6}$$

$$\frac{d}{dt} Q(t) = -\lambda_0 Q(t) + \mu W_0(t) \tag{7}$$

The initial condition is

$$W_0(0) = \delta_{n,m} \tag{8}$$

where $\delta_{n,m}$ is kronecker delta.

Taking Laplace transform of set of Equations (1-7) and using Equation 8,we get

$$(s + \lambda + \mu + \alpha) W_n(s) = \delta_{n,m} + \sum_{i=1}^{m} \lambda c_i W_{n-i}(s) + \mu W_{n+b}(s) + \beta_2 R_n^{2}(s), \quad (n \geq 1) \tag{9}$$

$$(s + \lambda + \mu + \alpha) W_0(s) = \delta_{0,m} + \lambda_0 Q(s) + \mu \sum_{k=1}^{b} W_k(s) + \beta'_2 R_0^{(2)}(s) \tag{10}$$

$$(s + \lambda_1 + \beta_1) R_n^{(1)}(s) = \sum_{i=1}^{n} \lambda_1 c_i^{(1)} R^{(1)}{}_{n-i}(s) + \alpha W_n(s), \quad (n \geq 1) \tag{11}$$

$$(s + \lambda_1 + \beta'_1) R_0^{(1)}(s) = \alpha W_0(s) \tag{12}$$

$$(s + \lambda_2 + \beta_2) R_n^{(2)}(s) = \sum_{i=1}^{n} \lambda_2 c_i^{(2)} R_{n-i}^{(2)}(s) + \beta_1 R_n^{(1)}(s), \quad (n \geq 1) \tag{13}$$

$$(s + \lambda_2 + \beta'_2) R_0^{(2)}(s) = \beta'_1 R_0^{(1)}(s) \tag{14}$$

$$(s + \lambda_0) Q(s) = \mu W_0(s) \tag{15}$$

THE GENERATING FUNCTION METHOD

We define the following generating functions:

$$W(s,z) = \sum_{n=0}^{\infty} W_n(s) z^n, \quad R^{(j)}(s,z) = \sum_{n=0}^{\infty} R_n^{(j)}(s) z^n, \; j = 1,2. \tag{16}$$

$$C(z) = \sum_{i=1}^{\infty} c_i z^i, \qquad C^{(j)}(z) = \sum_{i=1}^{\infty} c_i^{(j)} z^i, \; j = 1,2. \tag{17}$$

$$P(s,z) = \sum_{n=0}^{\infty} p_n(s) z^n \tag{18}$$

Performing, z^b times (10) $+\sum_{n=1}^{\infty} z^{n+b}$ times (9); (12)+ $\sum_{n=1}^{\infty} z^n$ times (11); (13)+ $\sum_{n=0}^{\infty} z^n$ times (14) and using (16) - (18), and on rearranging various terms, we have

$$[\{s + \lambda - \lambda C(z) + \mu + \alpha\} z^b - \mu] W(s,z) = z^{m+b} + \lambda_0 z^b Q(s) - \mu W_0(s) + \mu \sum_{r=1}^{b-1} (z^b - z^r) W_r(s) + \beta_2 z^b R^{(2)}(s,\alpha) + (\beta'_2 - \beta_2) z^b R_0^{(2)}(s) \tag{19}$$

$$\{s+\lambda_1-\lambda_1 C^{(1)}(z)+\beta_1\}R^{(1)}(s,z)=\alpha W(s,z)+(\beta_1-\beta'_1)R_0^{(1)}(s) \quad (20)$$

$$\{s+\lambda_2-\lambda_2 C^{(2)}(z)+\beta_2\}R^{(2)}(s,z)=\beta_1 R^{(1)}(s,z)(\beta_2-\beta'_2)R_0^{(2)}(s)-(\beta_1-\beta'_1)R_0^{(1)}(s) \quad (21)$$

From Equations 20 and 21, we get

$$R^{(1)}(s,z)=\frac{\alpha W(s,z)+(\beta_1-\beta_1')R_0^{(1)}(s)}{s+\lambda_1-\lambda_1 C^{(1)}(z)+\beta_1} \quad (22)$$

$$R^{(2)}(s,z)=\frac{\beta_1 R^{(1)}(s,z)+(\beta_2-\beta_2')R_0^{(2)}(s)-(\beta_1-\beta_1')R_0^{(1)}(s)}{s+\lambda_2-\lambda_2 C^{(2)}(z)+\beta_2}$$

$$=\frac{\beta_1\,\alpha W(s,z)+F(s)}{(\delta_1+\beta_1)(\delta_2+\beta_2)} \quad (23)$$

where

$$F(s)=\delta_1\{(\beta_2-\beta'_2)R_0^{(2)}(s)-(\beta_1-\beta'_1R_0^{(1)}(s)\}+\beta_1(\beta_2-\beta'_2)R_0^{(2)}(s)$$

$$\delta_1=s+\lambda_1-\lambda_1 C^{(1)}(z)$$

$$\delta_2=s+\lambda_2-\lambda_2 C^{(2)}(z)$$

Using Equations19 and 23, we obtain **W(s,z)** as

$$W(s,z)=\frac{N(s,z)}{D(s,z)} \quad (24)$$

where

$$N(s,z)=z^{m+b}+\lambda_0 z^b Q(s)-\mu W_0(s)+\mu\sum_{r=1}^{b-1}(\alpha^b-\alpha^r)W_r(s)+(\beta'_2-\beta_2)z^b R_0^{(2)}(s)+\frac{\beta_2 z^b F(s)}{(\delta_1+\beta_1)(\delta_2+\beta_2)} \quad (25)$$

$$D(s,z)=(\delta+\mu+\alpha)z^b-\mu-\frac{\alpha\beta_1\beta_2 z^b}{(\delta_1+\beta_1)(\delta_2+\beta_2)} \quad (26)$$

$$\delta=s+\lambda-\lambda C(z) \quad (27)$$

Equation 24 has (b +1) unknown $Q(s)$ and $W_r(s)$, $r=0,2,\ldots,b-1$. These unknowns can be determined by using Rouche's theorem according to which the denominator $D(s,z)$ has b zeros lying within circle $|z|=1$. At zeros of $D(s,z)$, the $N(s,z)$ will vanish (since $W(s,z)$ is analytic) so that we find b linear equations. By using Equation 15 and b linear equations, we can compute all unknowns. Once these unknowns are obtained, by using Equation 2, 23 and 24, we obtain $R^{(1)}(x,z)$, $R^{(2)}(x,z)$ and $W(s,z)$.

THE STEADY STATE SOLUTION

We attempt the steady state solution for a particular case when there are single arrivals and single departures so as $c_1=c_1^{(1)}=c_1^{(2)}=1$, $b=1$ and $c_i=c_i^{(1)}=c_i^{(2)}=0$ for $i\neq 1$. Also $C(z)=C^{(1)}(z)=C^{(2)}(z)=z$. Using Tauberian property $\lim_{s\to 0}sg(s)=\lim_{t\to\infty}g(t)$, Equations 22, 23 and 24 reduce to

$$R^{(1)}(z)=\frac{\alpha W(z)+r_1}{\lambda_1(1-z)+\beta_1} \quad (28)$$

$$R^{(2)}(z)=\frac{\beta_1\alpha W(z)+F(0)}{(\lambda_1-\lambda_1 z+\beta_1)+(\lambda_2-\lambda_2 z+\beta_2)} \quad (29)$$

$$W(z)=\frac{N(z)}{D(z)} \quad (30)$$

Where

$$N(z)=\lambda_0 zQ-\mu W_0-zr_2+\frac{\beta_2 zF(0)}{(\lambda_1-\lambda_1 z+\beta_1)(\lambda_2-\lambda_2 z+\beta_2)} \quad (31)$$

$$D(z)=\{\lambda(1-z)+\mu+a\}z-\mu-\frac{\mu\beta_1\beta_2 z}{(\lambda_1-\lambda_1 z+\beta_1)(\lambda_2-\lambda_2 z+\beta_2)} \quad (32)$$

$$r_j=(\beta_j-\beta'_j)R_0^{(j)},\quad j=1,2 \quad (33)$$

$$F(0)=\lambda_1(1-z)(r_2-r_1)+\beta_1 r_2 \quad (34)$$

Applying Tauberian property, Equations 12, 14 and 15 yield

$$R_0^{(1)} = \alpha W_0/(\lambda_1 + \beta') \quad (35)$$

$$R_0^{(2)} = \alpha\beta'_1/(\lambda_1 + \beta'_1)(\lambda_2 + \beta'_2) \quad (36)$$

$$\lambda_0 Q = \mu W_0 \quad (37)$$

We can determine the only unknown Q by using normalizing condition

$$W(1) + R^{(1)}(1) + R^{(2)}(1) + Q = 1 \quad (38)$$

Since the first three terms of L.H.S. of Equation 38 are of the 0/0 form, by applying L-Hospital rule, we compute

$$W(1) = \lim_{z\to 1} W(z) = \frac{\lambda_0 \beta_1 \beta_2 Q + \lambda_2 \beta_1 r_2 + \lambda_1 \beta_2 r_1}{(\mu-\lambda)\beta_1\beta_2 - \alpha(\lambda_1\beta_2 + \lambda_2\beta_1)} \quad (39)$$

$$R^{(1)}(1) = \lim_{z\to 1} R^{(1)}(z) = \frac{\alpha W(1) + r_1}{\beta_1} \quad (40)$$

$$R^{(2)}(1) = \lim_{z\to 1} R^{(2)}(z) = \frac{\beta_1 \alpha W(1) + \beta_1 r_2}{\beta_1 \beta_2} \quad (41)$$

Using Equations 39 to 41 in Equation 38 and simplifying, we have

$$Q = \frac{\sigma\zeta}{(\sigma + \lambda_0 \beta_1 \beta_2)\zeta + \lambda_0 \alpha\theta} \quad (42)$$

where

$$\sigma = (\mu - \lambda)\beta_1\beta_2 - \alpha(\lambda_1\beta_2 + \lambda_2\beta_1)$$

$$\zeta = \mu(\lambda_1 + \beta_1')(\lambda_2 + \beta_2')$$

$$\theta = \mu\{\lambda_2 + \beta_2')(\beta_1\beta_2 + \beta_2\lambda_1 + \beta_1\lambda_1) + \beta_1\beta_1'(\beta_2 + \lambda_2)\} + (\beta_1 - \beta_1')\beta_2(\lambda_2 + \beta_2')(\lambda_1 - \lambda) + \beta_1'\beta_1(\beta_2 - \beta_2')(\lambda_2 - \lambda) + \alpha(\lambda_1 - \lambda_2)\{(\beta_1 - \beta_1')\lambda_2 + \beta_1\beta_2' - \beta_1'\beta_2\}$$

Equation 42 can be rewritten as

$$Q = 1 - \frac{\lambda_0(\alpha\theta + \beta_1\beta_2\zeta)}{(\sigma + \lambda_0\beta_1\beta_2)\zeta + \lambda_0\alpha\theta} \quad (43)$$

which implies that traffic utilization ρ is

$$\rho = \frac{\lambda_0(\alpha\theta + \beta_1\beta_2\zeta)}{(\sigma + \lambda_0\beta_1\beta_2)\zeta + \lambda_0\alpha\theta} \quad (44)$$

so that the condition for the steady state equilibrium is given by

$$\frac{\lambda\beta_1\beta_2 + \alpha(\lambda_1\beta_2 + \lambda_2\beta_1)}{\mu\beta_1\beta_2} < 1 \quad (45)$$

THE AVERAGE QUEUE LENGTH

After a little algebra, the generating function $W(z)$ can be written as

$$W(z) = \frac{N_1(z)}{D_1(z)} =$$

$$\frac{\lambda_0 Q(\lambda_1 - \lambda_1 z + \beta_1)(\lambda_2 - \lambda_2 z + \beta_2) + z\{\lambda_1\lambda_2 r_2(1-z) + \beta_1\lambda_2 r_2 + \beta_2\lambda_1 r_1\}}{(\mu - \lambda z)(\lambda_1 - \lambda_1 z + \beta_1)(\lambda_2 - \lambda_2 z + \beta_2) - az\{\lambda_1\lambda_2(1-z) + \beta_1\lambda_2 + \beta_2\lambda_1\}}$$

Let $L_q^{(B)}$, $L_q^{(1)}$ and $L_q^{(2)}$ denote the average number of the waiting jobs when server is busy, under first phase of repair and second phase of repair respectively. Now

$$L_q^{(B)} = W'(1) = \frac{N_1'(1)D_1(1) - N_1(1)D_1'(1)}{\{D_1(1)\}^2} \quad (47)$$

where

$$N(1) = \lambda_0 Q\,\beta_1\beta_2 + (\beta_1\lambda_2 r_2 + \beta_2\lambda_1 r_1)$$

$$D(1) = (\mu - \lambda)\beta_1\beta_2 - \alpha(\lambda_1\beta_2 + \lambda_2\beta_1)$$

$$N'(1) = -\lambda_0 Q(\lambda_1\beta_2 + \lambda_2\beta_1) - \lambda_1\lambda_2 r_2 + (\beta_1\lambda_2 r_2 + \beta_2\lambda_1 r_1)$$

$$D'(1) = (\lambda - \mu - \alpha)(\lambda_1\beta_2 + \lambda_2\beta_1) + a\lambda_1\lambda_2 - \lambda\beta_1\beta_2$$

After some algebraic manipulation, Equation 47 yields.

$$L_q^B = [\lambda Q\{\alpha\beta_2(\lambda_1 + \beta_1)(\lambda_1\beta_2 + \lambda_2\beta_1) + \beta_1^2(\alpha\lambda_2^2 + \lambda\beta_2^2)\} + (\mu-\lambda)\{\beta_1^2\lambda_2^2 r_2 + r_1\lambda_1\beta_2(\lambda_1\beta_2 + \lambda_2\beta_1) + \mu\beta_1\beta_2(\beta_1\lambda_2 r_2 + \beta_2\lambda_1 r_1)\} + \alpha\lambda_1^2\lambda_2\beta_2(r_2 - r_1)]/\{D(1)\}^2 \quad (48)$$

$$L_q^{(1)} = \left[\frac{d}{dz}R^{(1)}(z)\right]_{z=1} = \frac{\alpha}{\beta_1}W'(1) + \frac{\lambda_1\{\alpha W(1) + r_1\}}{\beta_1^2} \quad (49)$$

$$L_q^{(2)} = \left[\frac{d}{dz}R^{(2)}(z)\right]_{z=1} = \frac{\alpha}{\beta_2}W'(1) + \frac{\alpha(\lambda_1\beta_2 + \lambda_2\beta_1)W(1) + (\beta_1\lambda_2 r_2 + \beta_2\lambda_1 r_1)}{\beta_1\beta_2^2} \quad (50)$$

A SPECIAL CASE

When repair rates of two phases are equal irrespective of the server's state (i.e., $\beta_1 = \beta_2 = \beta'_1 = \beta'_2 = \beta$) and the arrival rate is constant (i.e., $\lambda_0 = \lambda_1 = \lambda_2 = \lambda$), Equations 48 to 50 reduce to

$$L_q^{(B)} = \frac{\lambda^2\{\alpha(2\lambda + \beta) + \beta^2\}}{\mu\beta\{(\mu - \lambda)\beta - 2\alpha\lambda\}} \quad (51)$$

$$L_q^{(1)} = \frac{\alpha\lambda^2\{\alpha(\lambda + 2\beta) + \beta(\mu - \lambda + \beta)\}}{\mu\beta^2\{(\mu - \lambda)\beta - 2\alpha\lambda\}} \quad (52)$$

$$L_q^{(2)} = \frac{\alpha\lambda^2\{\alpha(2\beta - \lambda) + \beta(2\mu - 2\lambda + \beta)\}}{\mu\beta^2\{(\mu - \lambda)\beta - 2\alpha\lambda\}} \quad (53)$$

We also get

$$L_q = \frac{\lambda^2\{\alpha(4\alpha + 4\beta + 3\mu) + \beta^2\}}{\mu\beta\{(\mu - \lambda)\beta - 2\alpha\lambda\}} \quad (54)$$

Equation 54 coincides with the result obtained by Madan [6].

NUMERICAL RESULTS

To analyze the effect of various parameters on the average queue length L_q, we consider the special case having a constant arrival rate λ and a constant repair rate β. By fixing $\lambda = 1.5$, we plot L_q against different parameters in Figures 1-4.

In Figure 1, we set $\theta = 1.0$ and vary α for different values of μ. It is observed that L_q increases nearly linearly with the failure rate α. The increase is more prominent for the lower service rate of μ. Figure 2 exhibits the effect of repair rate β on L_q for $\alpha = 0.1$ and different value of μ. With increasing β and μ, L_q decreases, but the effect diminshes for higher values of β. Figures 3 and 4 display the graphs for L_q vs. μ for $\beta = 1.0$, $\alpha = 0.1$ (0.1) 0.4 and $\alpha = 0.1$, $\beta = 1.0$ (0.2) 1.8, respectively. We note that L_q decreases sharply for smaller values of μ, but for larger values of μ there is no significant effect. Consequently, the average queue length can be reduced by some extent by improving the service rate μ and the repair rate β upto some threshold value, after which its effect is negligible.

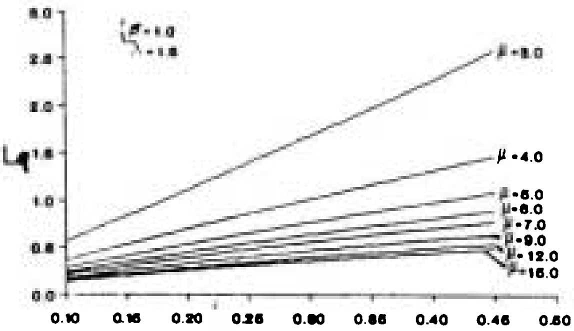

Figure 1. Average queue length vs. α.

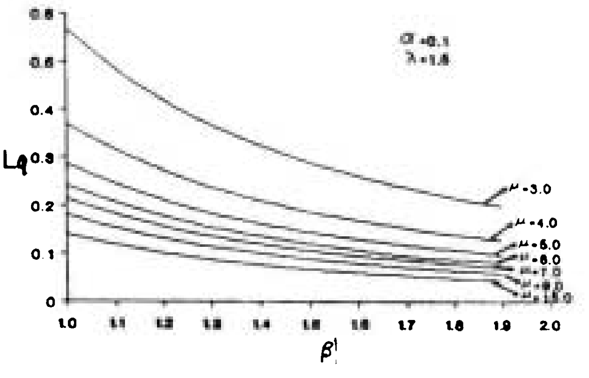

Figure 2. Average queue length vs. β.

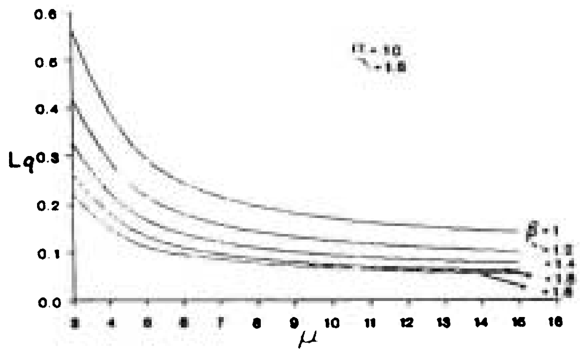

Figure 4. Average queue length vs. μ.

DISCUSSION

In the present paper, we have investigated a model of bulk queueing situation where a server is subject to breakdown. The incorporation of state dependent input rate and phase repair make the considered model more feasible to realistic situation as sometimes a failed server needs more than one kind of attention of the repairman before returning to the operating state. An exact solution of the queue length distribution is obtained by using generating functions of Laplace transform of state probabilities.

Bulk arrival and bulk service are common patterns in transportation processes wherein servers are subject to breakdown and repair. Similar phenomena may occur in the area of batch processing manufacturing, telecommunication switching and computer systems. In such systems failure and repair have a major impact on the flow of jobs that have to be handled by the server.

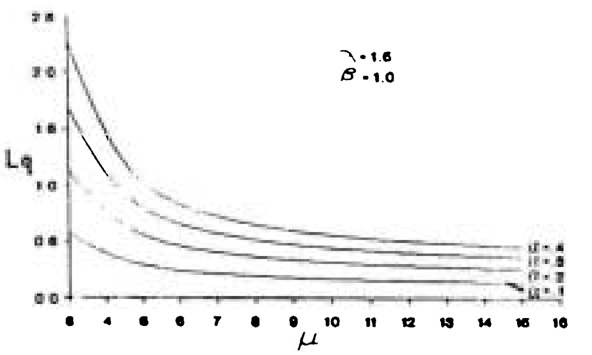

Figure 3. Average queue length vs. μ.

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