BUS ASSIGNMENT TO IRANIAN PILGRIMS DURING PILGRIMAGE

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Abstract Among the multitude of activities of Hajj Organization which require careful planning bus assignment to the Iranian pilgrim caravans during pilgrimage is the most important one. The objective of this assignment is to have a minimum number of standing passengers/empty seats, while satisfying a large number of operational and religious constraints. This problem was formulated as a large scale zero-one programming model which had to be solved heuristically. In this paper, the special nature of the problem was exploited which allowed it to be reformulated as a Linear Programming model. Computational experience with the actual data shows the success of this new approach.

Key Words Bus Scheduling, Zero-One Integer Programming, Linear Relaxation

چکیده دربین فعالیتهای بیشماری که توسط سازمان حج و زیارت در رابطه با زائران خانه خدا صورت می گیرد و نیاز به برنامه ریزی دقیق دارند، مساله تخصیص اتوبوس به کاروانها درایام مناسک حج است که خود از اهمیت ویژه ای برخوردار است. هدف از این تخصیص داشتن حداقل تعداد زایر سرپا و یا صندلی خالی در اتوبوسها است، ضمن اینکه محدودیتهای عدیده اجرایی و مذهبی هم باید مدنظر قرار گیرد. این مساله ابتدا به شکل یک مدل برنامه ریزی صفر و یک فرموله گردید که به علت بزرگی مدل مستقیماً قابل حل نبود. لذا دراین تحقیق نشان داده شده است که چگونه این مدل بزرگ به یک مدل برنامه ریزی خطی تبدیل می شود. درضمن محاسبات کامپیوتری انجام شده با داده های واقعی دال برموفقیت این روش جدید است.

INTRODUCTION

The Iranian annual pilgrimage to Mecca starts by registering with predefined caravans which constitute the basis for organizing the entire pilgrimage process. Among the multitude of activities which require careful planning is the bus transportation planning for the pilgrims caravans inside Saudi Arabia during the pilgrimage. For this purpose, buses are rented by the Hajj Organization. To reduce the rental cost, ordinarily the number of rented seats are 5 to 10 percent less than the number of pilgrims, thus, the caravans end up with a number of standing passengers. The buses are rented from different companies which differ in two ways:

1) Capacity, (i.e. the number of seats of the bus), and

2) Type of the bus (roofed and unroofed buses).

The following constraints should also be considered:

1. Each caravan transports its members independently (Mixing of caravan members is not allowed).

2. Caravans consist of men and women, the number in each one of these two groups varies from one caravan to another and in total.

3. The men must be transported on buses with no tops because of Shiite tenet, but it does not make any difference for women.

4. The number of standing passengers and/or empty seats should not exceed a prescribed limit.

5. There must be a sufficient number of buses to

transport all members of all caravans.

The object of this bus scheduling is to allocate the buses so that the sum of standing passengers/empty seats for the whole Iranian pilgrims is minimized, and the optimal use of the rental buses is made. The bus scheduling problem which has been seen as a multiple depot bus scheduling problem, is the problem of chaining the trips together to construct days of work for buses so that:

1) all operational constraints are satisfied

2) each bus returns to the depot it started from

3) each depot operates no more than a specified number of buses, and;

4) each trip is covered by exactly one bus.

These must be done so as to minimize the sum of the fixed cost of renting buses and the variable cost of operating buses. This problem has been studied by many researchers [1-11] with a range of suggested heuristic and exact algorithms.

It is well known that when restricted to one depot, this problem can be modeled as a network flow problem and solved very quickly [4]. But when it is extended to more than one depot, the problem becomes NP-hard [2]. Forbes et al. [1], have proposed an exact algorithm for multiple depot bus scheduling, based on a linear programming relaxation of a new formulation, with a subsequent branch and bound stage.

The bus assignement problem which has been considered here, is basically different from the previous vehicle scheduling problems because of its additional (religious) constraints. This problem has thus far been solved by trial and error, in the Hajj Organization [12]. In this paper, however, it is formulated as a zero-one model. A great number of integer variables had to be included to this large scale model which made it highly time consuming even by a main frame computer. The special nature of the model was, therefore, exploited to restructure and reformulate it by the help of non-integer variables. The main contribution of this paper is the remodeling of a large scale integer programming model into a linear programming model such that it could be solved even by a personal computer.

In the next sections, the required data processing in executing the model, computer programming in different environments, and the software utilized in solving the linear programming are presented. Computational experience with real data of Iranian pilgrims in several years and also with many simulated data shows the success of this approach.

1. MODEL DEVELOPMENT

In this section the system parameters are presented followed by the model.

1.1 System Parameters

Suppose there are N caravans. Let us call them 1,2,...,N. There are K types of buses which may differe in capacity and type (roofed or unroofed). There are U_k number of buses of kind k(k= 1 to K).

The immediate question here is how the K types of buses will be assigned to the caravan i, i= 1 to N. This is answered by considering all possible combinations of these assignments with respect to the constraints 1-4 in section 1. For example, consider a caravan with 160 pilgrims, consisting of 110 men and 50 women, and a prescribed upper limit of 12 standing passengers/empty seats. One possible combination is to assign two unroofed buses with 52 seats capacity to men, and one roofed bus with 47 seats capacity to women. The resulting standing passengers for this assignment is 6 men plus 3 women. Let S_{ij} show the number of passengers/empty seats of assigning the combination j to caravan i, when this value should be less than or equal to some prescribed upper limit. Hence, for this allowable assignment S_{ij} is equal to 9. However, in this case, assigning two unroofed buses with 47 and 50 seats capacity to men and one roofed bus wiht 52 seats capacity to women is not allowed.

Here, the empty seats and the difficulty it creates in this model, need explanation. Since the number of rented seats is already fewer than the number of pilgrims, it seems unreasonable to have empty seats. But in some cases the combinations with empty seats are the only possible combinations while maintaining the feasibility of the problem and giving the optimal solution. The major difficulty with this case is when S_{ij} is less than zero. Since the objective is to minimize, then the optimal solution would tend toward these negative coefficients in the objective function, which is not favorable. Therefore, these coefficients have been supplanted by zero. This would imply the combinations with empty seats will not be present in the final optimal solution, unless it is compulsory.

There are many possible combinations of assigning buses to caravan i, but all of them are not necessarily allowed. Let us call the number of combinations which is allowed to caravan i by m_i.

Suppose, a_{ijk} is the number of buses in type k,(k= 1 to K) assigned to caravan i, (i= 1 to N) when the allowed combination is j, (j= 1 to m_i). For example, if the unroofed buses with 47, 50, 52 and 54 seat capacity and roofed buses with 47, and 48 seat capacity are referred to by number 1,2,3,4,5, and 6 respectively, then a prescribed limit of 12 standing passengers will be the pattern shown in Figure 1, (i.e. a_{i12}= 2, a_{i15}= 1 for the first allowable combination).

The above illustration is just an example of some allowable combinations, which is very similar to the possible cutting pattern in the cutting-stock problem [7, page 207]. Therefore, the problem is to choose one and only one allowable combination for each caravan, such that the total standing passengers/ empty seats are minimized.

Figure 2, is a generalization of Figure 1 for this whole problem. It encompasses the range of indices and the whole parameters of the proceeding model.

1.2 Model Constraints

There are basically six types of constraints:

(a) Mixing of caravans members are not allowed.

(b) The men in each caravan should be transported in unroofed buses.

(c) Caravans consist of men and women, the number of each group varies from one caravan to another, and in total.

(d) The number of standing passengers/empty seats are not to exceed a predefined upper limit.

(e) There must be sufficient buses to transport all members of all caravans.

(f) Only one allowable combination should be assigned to each caravan.

Constraints (a) to (d) are already considered in

Caravan No. i	Allowable Combination j	Type	Unroofed				Roofed		No. of Standing Passengers/ Empty seats
		Capacity	47	50	52	54	47	48	
		k / a_{ijk}	1	2	3	4	5	6	
1	1	a_{i1k}		2			1		9
	2	a_{i2k}			1	1		1	2

Figure 1. Numerical example of allowable combinations.

Caravan No.	Allowable Combination	Type of Buses 1	2	...	K	Standing passengers /Empty Seats
1	1	a_{111}	a_{112}	...	a_{11k}	S_{11}
	2	a_{121}	a_{122}	...	a_{12k}	S_{12}
	.	.	.		.	.
	.	.	.	...	.	.
	.	.	.		.	.
	m_1	a_{1m_11}	a_{1m_12}	...	a_{1m_1k}	S_{1m_1}
i	1	a_{i11}	a_{i12}	...	a_{i1k}	S_{i1}
	2	a_{i21}	a_{i22}	...	a_{i2k}	S_{i2}
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	m_i	a_{im_i1}	a_{im_i2}	...	a_{im_ik}	S_{im_i}
N	1	a_{N11}	a_{N12}	...	a_{N1k}	S_{N1}
	2	a_{N21}	a_{N22}	...	a_{N2k}	S_{N2}
	.	.	.	.	.	.
	.	.	.	.	.	.
	.	.	.	.	.	.
	m_N	a_{Nm_N1}	a_{Nm_N2}	...	a_{Nm_Nk}	S_{Nm_N}
No. of Available Buses		u_1	u_2	...	u_k	

Figure 2. Parameters of model.

Figure 2, but constraints (e) and (f) will be considered in the following model.

1.3 The Objective Function

The main objective of the model is to find the total minimum standing passengers in all caravans.

1.4 Zero-One Model

Based on the parameters in Figure 2, let us define the following descision variable:

$$X_{ij} = \begin{cases} 1 & \text{If the allowed combination } j \text{ is assigned to caravan } i \\ 0 & \text{If the allowed combination } j \text{ is not assigned to caravan } i \end{cases}$$

Therefore, the following zero-one model will be obtained:

$$Min\ Z = \sum_{i=1}^{N} \sum_{j=1}^{m_i} S_{ij}\, X_{ij} \qquad (1)$$

S. t:

$$\sum_{j=1}^{m_i} X_{ij} = 1 \qquad i = 1,\ldots,N \qquad (2)$$

$$\sum_{i=1}^{N} \sum_{j=1}^{m_i} a_{ijk} X_{ij} \le U_k \qquad k = 1,\ldots,K \qquad (3)$$

$$X_{ij} = 0,1 \qquad i = 1,\ldots,N \text{ and } j = 1,\ldots,m_i$$

Where, (1) is the main objective function of the model, (2) indicates the constraint of type (f), and (3) is the constraint of type (e).

1.5 Conversion to Linear Programming

In the previous zero-one model, if the number of caravans is 800, and the average number of admissible combinations per caravan are 60, then the number of (0-1) variable will be 48000. It is highly time consuming and non economical if this problem would have to be solved even by a main frame computer such as IBM/4341. We tried to solve a simple problem even with smaller dimension, using MPSX/370 (Mathematical Programming System Extended). We had to abort running after one hour and half, because there was no result and the computer was still running. In this research, however we were successful in converting the previous large scale zero-one model into a linear programming model such that it can be solved by the personal computer. In order to overcome this difficulty, the following modifications have been made in the whole zero-one model:

1. Let us partition the caravans into several smaller groups calling each caravan type i (i = 1 to n) where n is suggested to be a small number such as one digit number. Therefore in the new linear programming model the index i, shows the new defined type of the caravan.

2. Instead of considering the actual number of men and women, let us consider the distribution of men and women in each caravan of type i. For instance, if we consider the entire caravan with 120 pilgrims, then the distribution of men and women in these caravans would be different. But if we define the caravans of 120 pilgrims with an average of 10,20,30,...120 men, then we come up with a new stratification such as 1,2,...,p,...,12. Then the stratum p, would change from 1 to n_i(here 1 to 12).

3. Let f_{ip} be the frequency of stratum p in caravan type i. For example, if 32 caravans of the caravans with 120 pilgrims (i.e. i = 2) have on the stratum average 70 men, (i.e. p = 7), then f_{27} would be equal to 32. Therefore, the dimensions in Figure 2, are extended by one more index p. In other words, the parameters a_{ijk} and S_{ij} are changed to a_{ijpk} and S_{ipj} respectively. But the other parameters remain the same.

Now let us define the new non integer variable and the rest of new parameters as follows:

X_{ipj}= The fraction of stratum p in caravan of type i to have the allowed combination j.

a_{ipjk}= Number of buses of type k, which is the jth allowable combination of j in stratum p are assigned to the caravan of type i.

S_{ipj}= Number of standing passengers, related to the jth allowable combination of stratum p in caravan type i.

f_{ip}= Number of caravan of type i in stratum p.

n = The type of caravan.

n_i= Number of stratum in carvan type *i*.

n_{ip}= Number of allowable combination for stratum p in caravan type i.

U_k= The available buses of type k.

K= Total number of buses.

The new linear programming with respect to the above definition would be:

$$\text{Min } Z = \sum_{i=1}^{n} \sum_{p=1}^{n_i} \sum_{j=1}^{n_{ip}} f_{ip} S_{ipj} X_{ipj} \tag{4}$$

S.t.

$$\sum_{j=1}^{n_{ip}} X_{ipj} = 1 \qquad p = 1,\ldots,n_i\,,\; i = 1,\ldots,n \tag{5}$$

$$\sum_{i=1}^{n} \sum_{p=1}^{n_i} \sum_{j=1}^{n_{ip}} f_{ip}\, a_{ipjk}\, X_{ipj} \le U_k \qquad k = 1,\ldots,K \tag{6}$$

$$X_{ipj} \ge 0 \qquad i = 1,\ldots,n, \quad p = 1,\ldots,n_i, \; j = 1,\ldots,n_{ip}$$

Where (4) shows the total standing passengers to be minimized, (5) indicates that only one allowable combination should be assigned to one type of caravan; and (6) means there must be sufficient number of buses to transport all Iranian pilgrims. Here X_{ipj} is no longer integer.

Based on the regulation of the Hajj Organization, the caravans may only have groups of 100, 120, 140 or 160 pilgrims. Therefore, n is equal to four. If we suppose that the average number of classes for each n is 10 (n_i= 10) and as before the average number of admissible combinations are 60 (n_{ip}= 60), then the number of variables in this linear programming will be $4\times10\times60 = 2400$, which is solvable by a personal computer.

To clarify this LP model, a numerical example may be helpful. Suppose in the optimal solution of this linear programming we have X_{243}= 0.25. It means, to 25% of stratum 4 of caravan type 2, third allowable combination is assigned. Suppose the number of caravans in this stratum is f_{24}= 29. Then, the buses should be assigned to $f_{24}\times X_{243}$= 7.257 caravans based on the third allowable combination. Then this value can be rounded for example to 7. Fortunately, the final result is not sensitive to this number whether it is 7 or 8. However, the sensitivity analysis of linear programming could rigorously overcome any doubts.

2. DATA PROCESSING

The initial model in this research was a large scale integer programming, which could not be solved through personal computers. Therefore a main frame computer such as IBM/4351 was chosen. The MPSX (Mathematical Programming System eXtended) was the only software available for this IBM computer. MPSX could accept the input data, such as objective function coefficients, the constraints coefficients and the vector of right hand side through a predefined file called Mathematical Programming System (MPS). The conversion of the original data base files into the MPS file, was not an easy task. The procedure will be described here very briefly.

The original database files (DBF) for caravans and buses were converted to two new DBFs. The information in the new Caravan. DBF file contained:

1. Caravan no.
2. Type of caravan,
3. Total number of pilgrims in this caravan,
4. Number of men, and
5. Number of caravans which have this number of men.

The new BUS.DBF file contained the following information:

1. Bus no.
2. Type of bus (i.e. roofed, unroofed),
3. Bus type code (code 1 for roofed bus and code 0 for unroofed bus).
4. Number of seats, and
5. Number of available buses with these specifications.

Then these files were converted to text files, in order to be programmable for all environments, such as Turbo Pascal. In Turbo Pascal environment two identical but independent routines were used in order to convert bus text file and caravan text file, into a new file as a data file with predefined fields. In the text file which has the role of intermediate file between DBF and data file, the number of combinations, number of caravans, the allowable combinations and the number of standing passengers/empty seats (slack/surplus) was held the same as illustrated in Figure 3. These pieces information were held in a data file too, in Turbo Pascal.

Therefore, the intermediate text file could work both in database and Turbo pascal environments. In the next step a new file called MPS was made by the

comb. No.	Caravan No.	Type of bus	Type of bus	Type of bus	Type of bus	Slack/ surplus
001,1	001	roofed 55	roofed 42	_____	_____	3
001,2	001	roofed 55	roofed 47	_____	_____	-2
001,3	001	roofed 42	roofed 47	_____	_____	11
002,1	002	unroofed 52	roofed 42	roofed 42	_____	6
002,2	002	unroffed 50	roofed 50	roofed 42	_____	0

Figure 3. Sample data in text file.

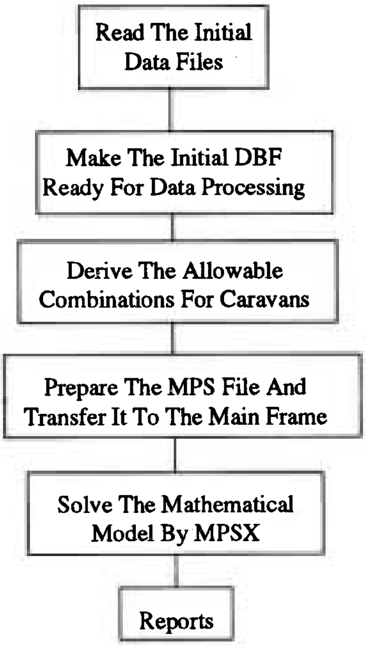

Figure 4. The entire computational procedure.

help of data files. In fact, the definition of variables and its coefficients in the objective function and in the constraints and the amount of right hand side were derived from the information in data file. Fortunately, the format of MPS is the same as Text, and its content is accessible and retrievable. All of the previous files, even MPS could be constructed for Personal Computer (PC) environment. The entire procedure is depicted in Figure 4. The next step would be to work with MPSX in the main frame using the MPS file.

3. COMPUTATIONAL EXPERIENCE

The computational experience in this research may be presented in two different categories:

(i) Comparing the initial integer programming model with the final linear programming model.

(ii) Comparing the final optimal solution of linear programming with the result of previous heuristic approach (trial and error method by Hajj organization).

The initial mathematical models were large scale zero-one models with more than 40000 variables. Therefore, we had to work with a main frame computer. An IBM/4341 and MPSX software were available. Several numerical examples with real data and also with simulated data were solved. The results were as follows:

A tentative problem with 5300 variable and 154 constraints was generated. The linear programming relaxation of this problem took only 1.1 minutes, while this problem with integer variables took 40.61 minutes to run with MPSX. A real problem with about 40000 zero-one variables and 4858 constraints was also tested. The LP relaxation took less than 10 minutes, while the zero-one variable problem took more than 100 minutes of CPU time with no result. So it was aborted because it was non-economical and discouraging. Many actual and simulated problems within this range were examined which showed the sensitivity of time to number of integer variables. Therefore, we had to do some essential modifications. The special structure of the problem was exploited. The problem was restructured and reformulated by the help of linear programming (see section 1.5). Several computational experiences with real and simulated data have been done with the final linear programming model. Since we had started to work with the main frame, we stayed with it all along the research. The MPSX could solve the LP problem with 4000 to 9000 variables in less than 2 minutes. However, the final mathematical model in one experience with real data was converted to a linear programming model with 6544 variables, and 62 constraints. The solution of this problem with MPSX took only 1.2 minutes while it took more than 24 hours through a 586 personal computer by the heuristic method developed in the Hajj Organization [12]. Additionally, the number of standing pilgrims in the mathematical approach was one fourth of the trial and error method. However, in this special example there were 800 caravans, 102485 pilgrims, 2023 buses with 22 types or 97062 seats. In other words, at least 5423 standing passengers would be inevitable both in the mathematical approach and in the heuristic method. But, 57 standing passengers out of 100000 pilgrims, while considering a large number of operational and religious constraints would be insignificant.

Other experiments with many actual and simulated data also showed the high relative efficiency and strength of the linear programming model with respect to the previous heuristic approach.

The advantage of the new approach is not only about reducing the number of standing passengers, but also about two other areas: First, a complicated decision process was organized into a standard problem, which is comprehensive, systematic, and reliable. Secondly, the time for bus assignment in Saudi Arabia, is very short because the bus information is not ready 2 or 3 days before pilgrimage. There is only one person, testing his own trial and error method continuously in order to reach a reasonable solution. But in the mathematical approach, after completing the bus information, a linear program should be run, where other pieces information such as the distribution of men and women could be obtained much earlier. The execution time of the model is short and results are rigorous, where the procedure is not monopoleable either.

CONCLUSION

The complete model of bus assignment based on Shiite tenet has been developed here, but the main contribution of this paper was to exploit the nature structure of a large scale zero-one problem and remodel it into a linear programming one.

The advantages of the mathematical approach over the previous trial and error method are as follows:

1. The new approach (NA) is systematic, rigorous, comprehensive, and reliable.

2. The NA is not time consuming, and it is compatible for the short period before the pilgrimage.

3. The NA gives the optimal solution, and it is always superior to a reasonable solution of the heuristic method.

4. The NA is not costly, because the personal computer and the software for linear programming are already available.

5. The NA is not monopoleable.

Since the final model is in a closed form of linear programming, therefore the number of caravans and the number of pilgrims would not hardly modify the model, but the number of pilgrims in each caravan (types of caravans) should be a predefined and fixed number. Otherwise, the problem gets extremely large and will have the "curse of dimensionality".

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