SHAPE EFFECTS AND DEFINITION OF HYDRAULIC RADIUS IN MANNING'S EQUATION IN OPEN CHANNEL FLOW

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Abstract In the Manning equation the hydraulic radius can be defined as the cross-section dimension of the shape. In pipe flow the bed shear stress is assumed to be uniformly distributed along the wetted perimeter which cannot be true in open channel flow. Hence, three approximations of the true boundary shear-stress distribution are examined and more practical conveyance depth or resistance radius formulae are developed in three case to subtitute for the hydraulic radius. In this study, special emphasis is placed on a particular channel cross-section including rectangular and triangular sections. Based on the logarithmic velocity profile a formula for a normal depth of this particular channel section is also developed it is shown that the shear stress distribution may be calculated with sufficient accuracy by simpler approximation methods. Finally, a presentation is made of a numerical example comparing the proposed formulae to the classic hydraulic radius concept.

Key Words Shape Effects, Hydraulic Radius, Boundary Shear Stress Conveyance Depth, Bisectors, Isovels, Logarithmic Velocity Profile

چکیده در رابطهٔ معروف مانینگ پارامتر شعاع هیدرولیکی با استفاده از ابعاد شکل مقطع کانالها قابل تعریف است. در مقاطع لوله ای فرض می شود توزیع تنش برشی در طول محیط خیس شده بصورت یکنواخت است که چنین فرضی در سایر مقاطع کانالها بسیار نادرست است. بدین منظور سه روش تقریبی جهت توزیع تنش برشی مورد مطالعه قرار گرفته و روابطی برای عمق عبوردهی جریان (Conveyance depth) یا شعاع مقاومت جریان (Resistance radius) توسعه داده شده است، بطوریکه در هر سه حالت قید شده بجای شعاع هیدرولیکی در رابطهٔ مانینگ جایگزین گشته اند. دراین مطالعه مقطع کانالی مشتمل براشکال مستطیلی و مثلثی مورد تأکید قرار می گیرد. برمبنای پروفیل لگاریتمی سرعت جریان، رابطه ای نیز برای عمق نرمال برای مقطع کانال ذکر شده بدست آمده است. نشان داده می شود برپایهٔ این روشها توزیع تنش برشی با دقت قابل قبولی مورد محاسبه قرار می گیرد. درخاتمه جهت مقایسهٔ فرمول های داده شده مطابق روشهای جدید، با روش کلاسیک شعاع هیدرولیکی که نسبت مساحت مقطع جریان به محیط خیس شده تعریف می گردد، یک نمونه مثال عددی ارائه می شود.

INTRODUCTION

Chow [1] is a good source for the Manning formula. Up-to-date papers and discussions may be found in Yen [2] who collected papers for the centennial of Manning's with a number of the papers concentrating on resistance studies. Chow in his book reported variations in the exponent for the original experiments, and explained the choice of 2/3 as the average, by means of tables and photographs that are excellent aids for selecting an appropriate ***n*** for a wide variety of open channels.

The Darcy-Weisbach Equation, the Nikuradse experiments, and the Moody diagram contain the classic literature for circular pipes. A good representation for Moody and Nikuradse diagrams are Colebrook-White types of dimensionless equations.

It has long been acknowledged that the "hydraulic radius" concept is a poor means for determining the velocity and shear stress distributions in a channel, since it is based upon a uniform distribution of shear along the boundary. Manning's formula is an empirical formula which may be derived from Nikuradse's semi-analytical formula for the Darcy-Weisbach fricition factor for circular pipes flowing full in the rough turbulent flow range as demonstrated by Henderson [3], and later discussed by Christensen [4] based on Nikuradse [5]. The range proposed here is

$$5<R_h/k_s<340 \quad (1)$$

and approximately

$$u_* k_s/\nu>70 \quad (2)$$

in which R_h is the hydraulic radius defined as cross-sectional area divided by wetted perimeter (R_h= A/P); k_s is Nikuradse's equivalent sand roughness; u_* is the friction/shear velocity; and ν is kinematic viscosity. The discrepancy between Manning's and Nikuradse's equations is just a few percents. The turbulent transition range is

$$5 < u_* k_s / \nu < 70 \quad (3)$$

It is often permissible to extend the application range of Manning's formula. Strickler [6] and Meyer-Peter [7] came to this important point that Manning's roughness coefficient n can be related to roughness element size, and proposed the following 1/6 power formula

$$n = \frac{k_s^{1/6}}{25.6} \text{ (SI system)} \quad (4a)$$

Kamphius [8] in this literature survey on sediment transport showed that can normally be assumed the Nikuradse's sand grain roughness as follows using experimental data

$$k_s \cong 2d_{90} \quad (4b)$$

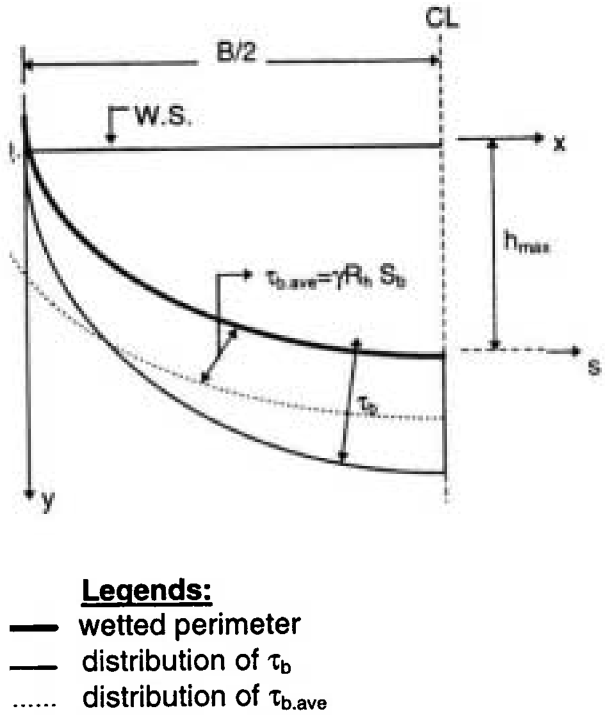
which is used by the majority of researchers although such authors used d_{50} instead of d_{90}. An up-to-date discussion of Equation 4a for n is given by French [9] Ranga Raju [10] and Yen [2]. It should be remembered that Nikuradse's friction factor formula based upon the Prandtl mixing length theory, was developed for rough turbulent fluid flow range in full flow circular pipes. We shall argue on Nikoradse's sand equivalent concept in this study. The wall shear stress τ_b, a time-mean value, is constant along the wetted perimeter in such pipes which is expressed by the bed shear stress formula

$$\tau_b = \gamma R_h S_b \quad (5)$$

where γ is the unit weight of fluid; and S_b is the energy grade line (egl) or energy line slope. As Figure 1 indicates, however, in open channel flow, the shear stress, β, is equal to zero at the water surface and increases along the wetted perimeter up to vertical center line of the channel cross-section. The distribution of boundary shear stress around the wetted perimeter of a channel is influenced by many factors, notably the shape of the cross-section, the longitudinal variation in planform geometry, the sediment concentration and the lateral and longitudinal distribution of boundary roughness [11]. The spatial average of the boundary shear stress, $\tau_{0.ave}$ distribution can also be seen in Figure 1.

In this figure, the horizontal x-axis is assumed to be located on the water surface beginning at the left bank. The distance from the left bank measured along the wetted perimeter is denoted s. The maximum values of s and x are P and B, wetted perimeter and top width of free surface, respectively. The maximum depth of the channel at the centre of the symmetrical cross-section is h_{max}.

The isovels (curves of constant velocity, dashed in Figure 2) and orthogonals to the isovels in a uniform flow assumption are shown in Figure 2. It



'igure 1. Boundary shear stress distributions comparison.

:an be proved that the local bed shear stress, τ_b, may ›e given by

$$\tau_b = \frac{\gamma d A S_b}{ds} \tag{6}$$

in which S_b is the bed slope ($S_b = S_0$ for uniform flow), and dA is the area between the adjacent orthogonals meeting the wetted perimeter at s=s and s=s+ds [12]. The local velocity is constant along the isovels. Because of the smaller value of the bed inclination θ, in such sections may be replaced by dA/ds approximately in Equation 6, hence the local vertical depth, **h**, giving

$$h \cong \tau_b / \gamma S_b \tag{7}$$

In flat and fairly flat sections the isovel curves assume that they are parallel to the wetted perimeter which is a reasonable approximation. It leads to Equation 6 in the following form

$$\tau_b = \gamma Z S_b \tag{8}$$

where Z is the distance, normal to the s direction from W.S. as shown in Figure 2. It can be seen that the local radius of curvature of the wetted perimeter is very long compared to the depth. According to Equations 7 and 8, the two depths intended to substitute the hydraulic radius in the Manning equation are developed as follows for a particular cross-sectional shape including rectangular and triangular. They are re-

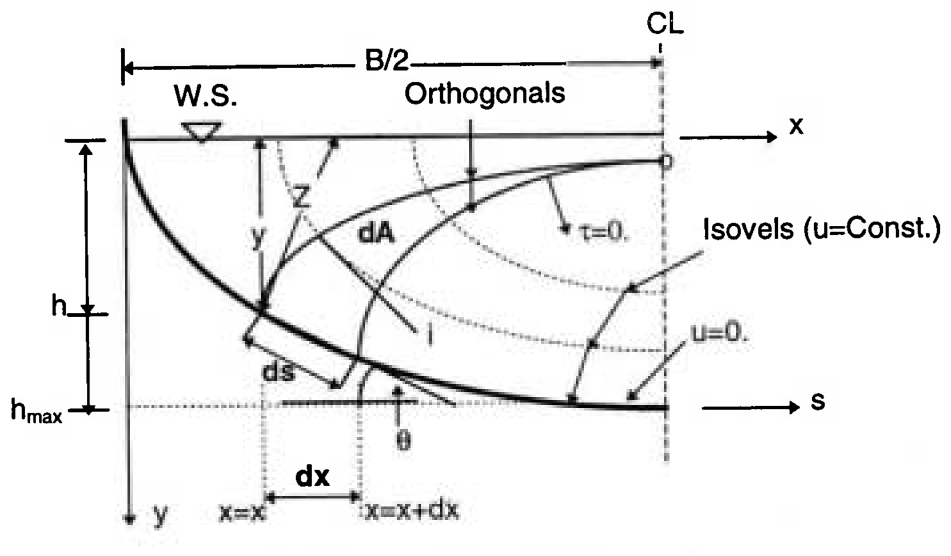


Figure 2. Definition of isovels and orthogonals.

ferred to as the conveyance depths of the R_v, and R_r and the second order R_n.

CONVEYANCE DEPTH OF THE FIRST ORDER

With regard to the bed shear stress distribution formula, based on Equation 7, the mean of the time-mean velocity, u_m, may be given by Manning's formula as follows

$$u_m = \frac{1}{n} R_v^{2/3} S_0^{1/2} \text{ (SI system)} \tag{9}$$

where R_v is the unknown conveyance depth of the first order. Keeping in mind that the almost horizontal shear stress acting in the verticals in this case must be nearly zero, the spatial mean velocity in the vertical of the depth, may be written

$$u_{m,x} = \frac{1}{n} h^{2/3} S_0^{1/2} \text{ (SI system)} \tag{10}$$

Integrating $u_{m,x}$ over the total width of the cross-section area and substituting the result into Equation 9 yields

$$u_m = \frac{1}{A}\int_A u_{m,x}\, dA = \frac{1}{A}\int_0^B \frac{1}{n} y^{2/3} S_0^{1/2} \cdot y dx = \frac{1}{n} R_v^{2/3} S_0^{1/2} \tag{11}$$

or,

$$R_v = \left(\frac{1}{A}\int_0^B y^{2/3} dx\right)^{3/2} \tag{12}$$

Equation 12 is the general formula which is known as the conveyance depth of the first order. This formula is now examined for the cross-section and then R_h and R_v are compared for this particular section. Consider the symmetrical cross-section shown in Figure 3. Comparison of the three prediction methods for the shear stress distribution is also indicated in this case.

For the section shown in Figure 3, the corresponding formula for the cross-sectional area, wetted perimeter, and hydraulic radius are

$$A = \frac{B}{2}(h_{max} + h) \tag{13}$$

$$P = 2h + \frac{B}{\cos\theta} \tag{14}$$

$$R_h = \frac{A}{P} = \frac{B(h_{max} + h)}{2\left(2h + \frac{B}{\cos\theta}\right)} \tag{15}$$

Using Equation 12 for the cross-section considered earlier yields

$$R_v = \left[\frac{1}{A}\int_0^B y^{5/3} dx\right]^{3/2} = \left\{\frac{4}{B(h_{max} + h)}\int_0^{B/2}\left[h + \frac{2x(h_{max} + h)}{B}\right]^{5/3} dx\right\}^{3/2} \tag{16}$$

in which

$$y = h + \frac{2x}{B}(h_{max} - h) \tag{17}$$

is the local depth of the cross-section at an arbitrary point along the wetted perimeter, and x is the horizontal distance of the element from left bank. After a simple integration, the result is

$$R_v = \left\{\frac{3}{4(h_{max}^2 - h^2)}\left[h_{max}^{8/3} - h^{8/3}\right]\right\}^{3/2} \tag{18}$$

For given values of θ, Q, B, n, and longitudinal bed slope, S_0, the normal depth, h_{max}, may easily be computed by introduction of Equations 15 and 18, into the following Manning's equation for R_h

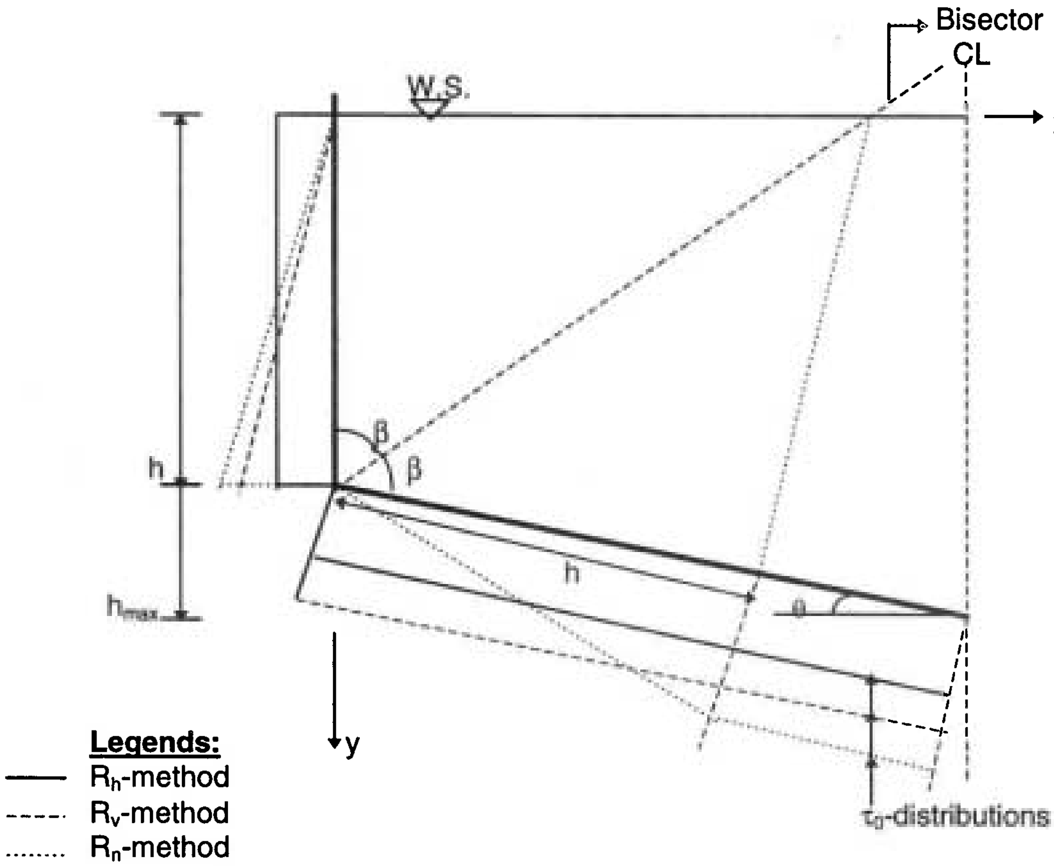


Figure 3. Channel cross-section, boundary shear stress distribution.

$$Q = \frac{1}{n} A R_h^{2/3} S_0^{1/2} \tag{19}$$

and for R_n

$$Q = \frac{1}{n} A R_v^{2/3} S_0^{1/2} \tag{20}$$

Using Equations 15 and 19, the normal depth of the channel will be as follows

$$h_{max} = \frac{\left(h + \dfrac{B}{\cos\theta}\right)^{2/5}}{B/2} \left(\frac{nQ}{\sqrt{S_0}}\right)^{3/5} - h \tag{21}$$

and similarly, from Equations 18 and 20, the normal depth of the channel will be as follows,

$$h_{max} = \left[\frac{8\,nQ}{3\,BS_0^{1/2}}\,(h_{max} - h) + h^{8/3}\right]^{3/8} \tag{22}$$

Equations 21 and 22 give normal depths in uniform flow related to $\mathbf{R_h}$ and $\mathbf{R_v}$, respectively. Those equations also cover the triangular (h = 0) section.

ENGELUND METHOD FOR A CONVEYANCE DEPTH

Assume the bed shear stress, τ_0, along the wetted perimeter is a constant value. In the case of open channel flow, this assumption is incorrect, however Engelund [13] intended to investigate the effect of nonuniform shear stress on the applicability by power

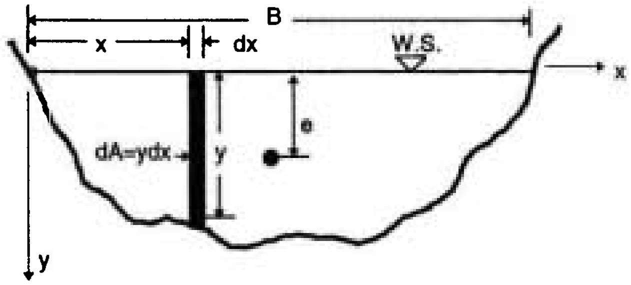


Figure 4. Cross-sectional area and element for integration scheme

formulae. Figure 4 indicates the cross-sectional area of water surface.

The area of the cross-section is given by

$$A = \int_A dA = \int_0^B y\, dx \tag{23}$$

in which **A** is the area; **B** is the width of water surface; and **y** is the local depth. Because of the usually higher value of **B** compared to the depth in open channel flow, the length **P** of the perimeter is approximately equal to **B**, hence

$$R = \frac{A}{P} \cong \frac{A}{B} = H_m \tag{24}$$

where H_m is the mean depth of the flow in channel. It would therefore be a reasonable approximation to propose that the local boundary shear stress, τ_0, proportional to the depth can be calculated from

$$\tau_0 = \gamma R_h S_0 = \gamma y S_0 \tag{25}$$

in which γ is the water specific gravity; S_0 is the channel bed slope and R_h is the hydraulic radius. It has been shown that the power formula for the flow in open channel will be very inaccurate [13]. Alternatively, we shall now investigate the possibility of replacing the hydraulic radius with a value such as R_r, which is known as the resistance radius or conveyance depth as given by Engelund method. He proposed the following formula for $\mathbf{R_r}$

$$\sqrt{R_r} = \frac{1}{A} \int_0^B y^{3/2}\, dx \tag{26}$$

For wide rectangular channels $\mathbf{y = H_m = h}$, and $\mathbf{A = Bh}$, from which

$$R_r = h = R_h \tag{27}$$

and for triangular sections, it can be seen

$$R_r = 0.64(h_{max} - h) = 1.28 R_h \tag{28}$$

Equation 26 may be used for a particular cross-section shown in Figure 6. Using Equations 13 and 17, Equation 26 gives

$$R_r = \left[\frac{4}{5\,(h^2_{max} - h^2)} (h^{5/2}_{max} - h^{5/2}) \right]^2 \tag{29}$$

and from the Manning Equation as follows

$$Q = \frac{1}{n} A R_r^{2/3} S_0^{1/2} \tag{30}$$

and also using Equations 29 and 30, the normal depth may be obtained

$$h_{max} = \frac{2nQ \,/\, (B \sqrt{S_0})}{\left[\frac{4}{5\left(h^2_{max} - h^{5/2}\right)} (h^{5/2}_{max} - h^{5/2}) \right]^{4/3}} - h \tag{31}$$

It should be noted that there is no significant difference between Engelund method and first order conveyance depth method. For an arbitrary cross-section (see Figure 4) the following formula may be given which is dependent upon the central gravity of cross-sectional area and the first moment of area around x-axis [13].

$$R_r = R_h\left[1 + \frac{3}{4}\left(\frac{e}{H_m} - \frac{1}{2}\right)\right]^2 \tag{32}$$

where **e** is centre gravity or cross-sectional centre from water surface, $\mathbf{H_m} = \mathbf{h_{max}}$ is the mean depth or normal depth, and $\mathbf{R_h}$ is the hydraulic radius. In this case for a shape considered $\mathbf{R_h}$ comes from Equation 15, and **e** may be found as

$$A.e = \frac{B}{2}\left[h^2 + \frac{1}{3}\left(h_{max}^2 + hh_{max} - 2h^2\right)\right] \tag{33}$$

and then dividing extremes of this equation by A which is obtained from Equation 13, yields

$$e = \frac{h^2 + \frac{1}{3}\left(h_{max}^2 + hh_{max} - 2h^2\right)}{h_{max} + h} \tag{34}$$

As an example, for given **h**= 50mm, $\mathbf{h_{max}}$= 70mm, and **B** = 165mm, it can be seen that **e**= 30.278mm, $\mathbf{R_h}$= 36.7mm, and $\mathbf{R_r}$= 0.9014 $\mathbf{R_h}$ and hence $\mathbf{R_r}$= 33.08mm which is lower than $\mathbf{R_h}$ by about 10%. It can be suggested that for calculation of the mean velocity and also discharge passing a cross-section, $\mathbf{R_r}$ may be replaced by $\mathbf{R_h}$ in Manning's formula. The advantage of using the resistance radius instead of hydraulic radius is that we get a logical coherence between the normal hydraulic power formulae and the theoretical basis, and that of the cross-section is taken into account [13].

CONVEYANCE DEPTH OF THE SECOND ORDER

Based on Equation 8 and assuming isovels parallel to the wetted perimeter and a substantial radius of curvature of the wetted perimeter when compared with local depths, the conveyance depth of the second order may be defined. Because of the influence of the curvature radius, deriving a simple formula for a general cross-section is much more difficult in this case [12]. In current study, the development of a formula for $\mathbf{R_n}$ in therefore limited to a particular channel cross-section including rectangular and triangular sections considered earlier. To simplify the problem, it is assumed that the local shear stress in the flow direction is equal to zero along the bisectors of the angles between walls and angular slopes. This simplification is reasonable in open channel flow; see for example, Knight, Yuen and Alhamid [11]; Knight and Lai [14]; Patel [15]. The cross-sectional geometry considered is shown in Figure 5.

There is a limitation in this case that a bisector intersects on the water surface before intersecting the bisector from the other half of the cross-section. In other words, it can be implied that

$$B > 2h \tag{35}$$

For more detail readers may refer to Keulegan [16] and Christensen [17].

The area elements dA considered in Figure 5 are

$$dA_1 = z_1 ds_1 = s_1 \tan\beta \, ds_1 \tag{36}$$

$$dA_2 = z_2 ds_2 = s_2 \tan\beta \, ds_2 \tag{37}$$

in which

$$\beta = \frac{\pi}{4} + \frac{\theta}{2} \tag{38}$$

for this particular shape, The Manning formula with $\mathbf{R_n}$ instead of hydraulic radius, $\mathbf{R_h}$, may now be given by

$$u_m = \frac{1}{n} R_n^{2/3} S_0^{1/2} = \frac{1}{A} \int_P u_{m,s} \, dA \quad (39)$$

in which

$$u_{m,s_2} = \frac{1}{n} (s_2 \tan\beta)^{2/3} S_0^{1/2} \qquad (0. < s_2 < h) \quad (40)$$

and

$$u_{m,s_2} = \frac{1}{n} (s_2 \tan\beta)^{2/3} S_0^{1/2} \qquad (0. < s_2 < h) \quad (41)$$

Equations 40 and 41 are approximations subject to the assumption of isovels being parallel to the wetted perimeter made in this cross-section. Equation 26 may now be extended as

$$u_{m,s} = \frac{1}{n} R_n^{2/3} S_0^{1/2} = \frac{2}{A} \int_0^h \frac{1}{n} (s_1 \tan\beta)^{5/3} S_0^{1/2} ds_1 + \frac{2}{A} \int_0^h \frac{1}{n} (s_2 \tan\beta)^{5/3} S_0^{1/2} ds_2 + \frac{2}{A} \left\{ \left(\frac{1}{n} h_{max}^{2/3} S_0^{1/2} \right) \left[\frac{B h_{max}}{4} + \frac{h}{2} \tan\beta \left(\frac{B}{2 \cos\theta} - h_{max} - h \right) \right] \right\} \quad (42)$$

Hence the following Equation can easily be derived

$$A R_n^{2/3} = \frac{3}{2} h^{8/3} (\tan\beta)^{5/3} + h_{max}^{2/3} \left[\frac{B h_{max}}{2} + h \tan\beta \left(\frac{B}{2 \cos\theta} - h_{max} - h \right) \right] \quad (43)$$

and eventually using Equation 13, R_n may be obtained as follows

$$R_n = \left\{ \frac{3}{B(h_{max} + h)} h^{8/3} (\tan\beta)^{5/3} + \frac{2 h_{max}^{2/3}}{B(H_{max} + h)} \left[\frac{B h_{max}}{2} + h \tan\beta \left(\frac{B}{2 \cos\theta} - h_{max} - h \right) \right] \right\}^{3/2} \quad (44)$$

Equation. 44 is the R_n-equivalent of Equations 15 and 18. The corresponding formula for h_{max} can be derived by using Manning's Equation and Equation 44

$$h_{max} = \frac{2nQ / (B \sqrt{S_0})}{\frac{3}{2} h^{8/3} (\tan\beta)^{5/3} + h_{max}^{2/3} \left[\frac{B h_{max}}{2} + h \tan\beta \left(\frac{B}{2 \cos\theta} - h_{max} - h \right) \right]} - h \quad (45)$$

Equation 45 is the normal depth equivalent using R_n as in Equation 21 and 22. A similar equation can be derived for the narrow channel case; i.e. when

$$B < 2h \quad (46)$$

All equations mentioned can be solved by a simple numerical iteration method using the following first estimate for the indeterminate h_{max}

$$h_{max} = \left(\frac{nQ / \sqrt{S_0}}{B} \right)^{3/5} \quad (47)$$

To support the above mentioned method to define a simple formula for the hydraulic radius, the following example may be demonstrated. Bisectors and secondary flows in a trapezoidal open channel may be seen (Figure6).

In the case of trapezoidal cross-section, bisectors are drawn at the meeting point of the bed and wall. It can be seen that these bisectors are as orthogonals and there is no interaction among the secondary currents.

LOGARITHMIC VELOCITY PROFILE FOR THE CROSS-SECTION CONSIDERED

The adoption of a logarithmic velocity distribution along a normal to the boundary in an open channel was introduced by Keulegan [16]. However, instead of the local friction velocity he applied the mean friction velocity over the solid boundary as a "reference velocity", [12], as used in Equation 53 related to the velocity distribution. Limitations of the methods

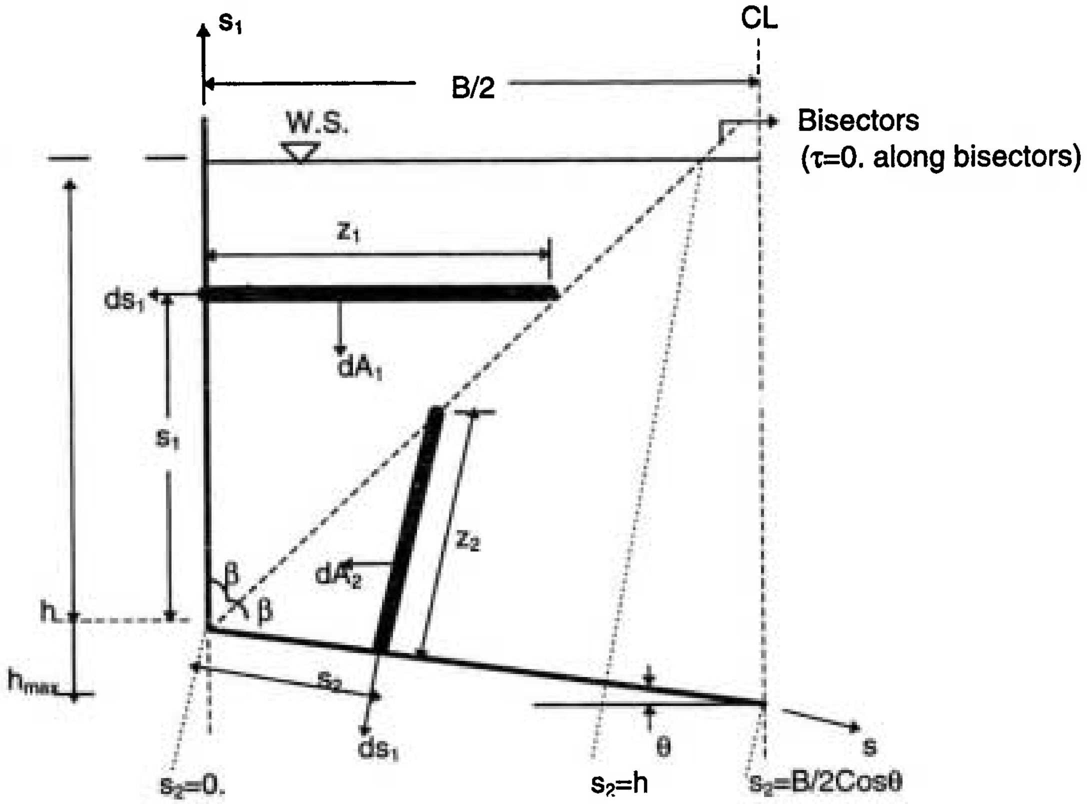


Figure 5. Integration scheme for derivation R_n

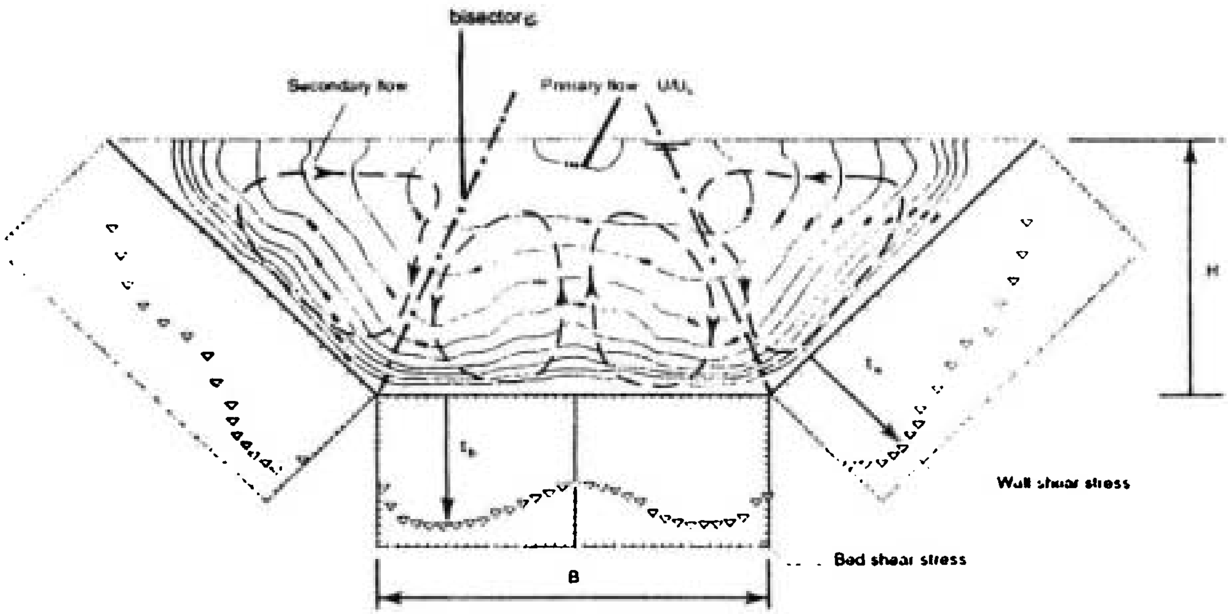


Figure 6. Typical relationship between boundary shear stress distribution, secondary flows, primary flow Fr= 3.24, Asp= B/H=1.52 [11], and bisectors at the joining point of walls and bed in a trapezoidal channel.

discussed in the preceding sections were the rough flow range and the range of roughness showed by

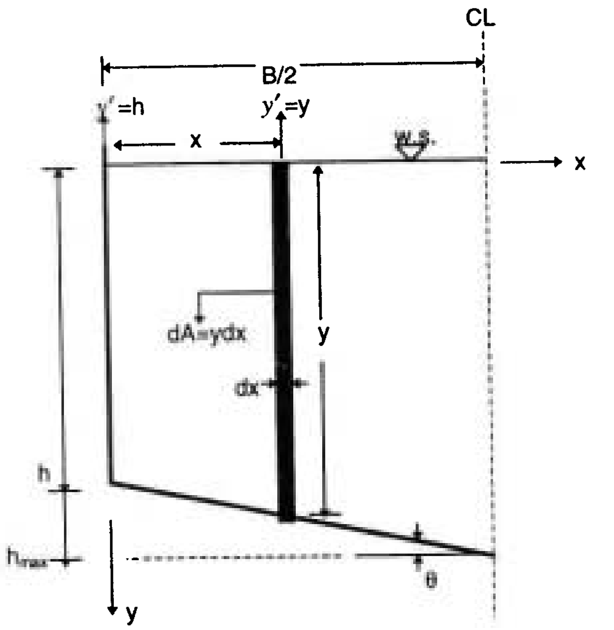


Figure 7. Element scheme for Integration over cross-sectional area.

Equations 1 and 2. This range is the ususal range for the majority of open channel flows. "The constraints imposed by Equation 1 and the limited accuracy of the Manning formula as an approximation to the generally accepted logarithmic formula for the Darcy-Weisbach friction factor, may be avoided by neglecting the Manning formula, or any other power formula for that matter, and basing the flow formula directly on the logarithmic formula as practised in the UK" [18]. Consider the symmetrical cross-section as shown in Figure 7.

A simple formula for the above-mentioned cross-section developed from Nikuradse's logarithmic velocity distribution in the rough flow range.

The shear velocity may be written

$$u_{*,x} = \sqrt{g y S_0 \cos\theta} \qquad (48)$$

where $u_{*,x}$ is the shear velocity and y is the local depth of the element shown in Figure 7. The value of y can be given by Equation 17. Combining Equations 17 and 48, yields

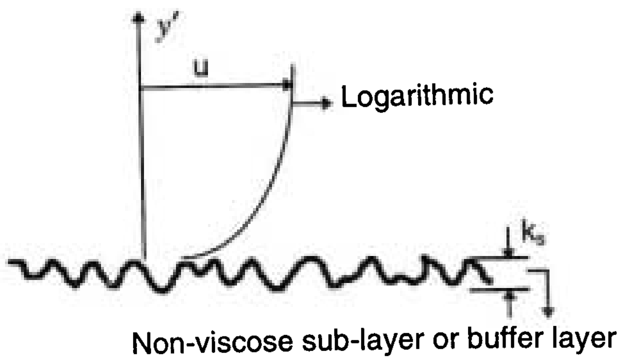


Figure 8. Logarithmic velocity profile for rough boundary.

$$u_{*,x} = \left\{ g \left[h + \frac{2x\,(h_{max} - h)}{B} \right] S_0 \cos\theta \right\}^{1/2} \qquad (49)$$

Considering Figure 8, Prandtl's logarithmic velocity distribution law is applicable to rough boundaries which is valid for Equation 2, as follows

$$\frac{u}{u_{*,x}} = \frac{1}{\kappa} \ln \frac{y'}{y_0} \qquad (50)$$

in which κ is the von-Karman constant, and y′ is the vertical distance from bed. Assume

$$y_0 = \eta \mathbf{k}_s \qquad (51)$$

where η is a constant (i.e. y'_0 directly proportional to the size of roughness excrescence's k_s). Inserting Equation 51 into Equation 50 gives

$$\frac{u}{u_{*,x}} = \frac{1}{\kappa} \ln \frac{y}{k_s} - \frac{1}{\kappa} \ln \eta \qquad (52)$$

Using this approach, investigators have experimentally obtained values η of 1/30 and for κ of 0.4. Inserting these values, the velocity profile in the vertical located at **x = x** may be given by

$$\frac{u}{u_{*,x}} = 8.5 + 2.5 \ln \frac{y}{k_s} \qquad (53)$$

TABLE 1. The Result of a Numerical Example Obtained for Comparison of the Proposed Methods

Method	Normal Depth, h_{max}(m)	Area, A(m^2)	Hydraulic Radius, R(m)	Mean Vel., $\bar{u}$(m/s)	Reynolds R_e*10^{-6}	Froude F_r	Friction Factor, f	Shear Velocity, $u_* = \sqrt{gRS_0}$	Shear Stress, $\tau_0 = \rho u_*^2$
R_h	1.520	4.623	0.791	1.082	3.003	0.280	0.0529	0.0881	7.762
R_v	1.343	3.916	1.017	1.277	4.557	0.352	0.0491	0.0999	9.980
R_r	1.342	3.912	1.012	1.278	4.538	0.352	0.0490	0.0996	9.920
R_n	1.300	3.744	1.085	1.335	4.530	0.374	0.0476	0.1074	11.535
Log	1.366	4.008	0.986	1.248	4.318	0.341	0.0497	0.0983	9.670

Method	Ratio, $\frac{h_{max}}{h_{maxh}}$	Ratio, A/A_h	Ratio, R/R_h	Radio, $\bar{u}/\bar{u}_h$	Radio, R_e/R_{eh}	Radio, F_r/F_{rh}	Radio, f/f_h	Radio, u_*/u_{*h}	Radio, t_0/t_{0h}
R_h	1	1	1	1	1	1	1	1	1
R_v	0.884	0.8469	1.286	1.218	1.517	1.257	0.9282	1.134	1.286
R_r	0.883	0.8462	1.379	1.177	1.511	1.257	0.9263	1.131	1.278
R_n	0.855	0.8099	1.372	1.234	1.508	1.336	0.8998	1.219	1.486
Log	0.899	0.8668	1.247	1.153	1.438	1.218	0.9395	1.116	1.246

R_r-Method; Equation 31

$$h_{max} = \frac{1.976}{\left[\frac{0.8}{\left(h_{max}^2 - h^2\right)}\left(h_{max}^{5/2} - h^{5/2}\right)\right]^{4/3}} - h \qquad (63)$$

R_n-Method; Equation 45

$$h_{max} = \frac{1.976}{2.717h^{8/3} + h_{max}^{2/3}\left[2h_{max} + 1.428h\,(2.128 - h_{max} - h)\right]} - h \qquad (64)$$

Log-Method; Equation 59

$$h_{max} = h + .0768\left[h_{max}^{5/2}(l\,n\,h_{max} + 4.659) - h^{5/2}(l\,n\,h + 4.659)\right] \qquad (65)$$

The h_{max} values, the corresponding cross-sectional areas, **R** related to methods, and also the corresponding mean velocity, Reynolds and Frouds numbers, friction facor and velocity, shear stress, and finally the ratios of these parameters are shown in Table 1, together with the Equations from which those values have been obtained.

Note 1: g= 9.807 (m/s²), ρ_w= 1000 (Kg_m/m^3), T_w= 20°C, and ν= 0.00000114 (m²/s).

Note 2: $h_{max} / h_{max\,h}$, A/A_h, R/R_h, $\bar{u}/\bar{u}_h$, R_e/R_{eh}, F_r/F_{rh}, f/f_h, u_*/u_{*h} *and* t_0/t_{0h} are ratios of normal depth, area, hydraulic radius, mean velocity, Reynolds number, Froude number, friction factor, shear velocity, and boundary shear stress of each method over traditional hydraulic radius method, respectively.

CONCLUDING REMARKS

The use of the traditional hydraulic radius which was derived from pipe flow analysis is very inaccurate in open channels. From the values obtained using the five methods compared in the preceding section, there is a clear difference between the hydraulic radius method and the others. It can also be seen that taking the boundary shear stress distributions when using the Manning equation does not have a insignificant influence on the results. This is because the boundary shear stresses are small.

Considering normal depths, areas and hydraulic radius ratios presented in Table 1 show the differences. For example areas derived from the local normal depth method show that the vertical depth method R_v, Engelund method R_r and the local normal depth method R_n give the percentages such as 11.6%, 11.7%, and 14.5% smaller than those found by conventional hydraulic radius method. The logarithmic-method yields results in the same range as the R_n-method. This method however is of a more general nature, because it is not limited to the roughness range proposed by Equation 1, that is restricting the use of the Manning Equation. Computing velocity and shear stress distributions and comparing them with those obtained from the work done by other people by the way of 2-D methods will be very useful to evaluate the results of the methods presented in this study [11]. By means of the logarithmic-method which seems to be the simplest and most accurate method on the basis of Prandtl's mixing length theory, it can be seen that the R_n-method is sufficient for most practical purposes. By this method, the boundary shear is computed directly from the area between bisectors and normal to the boundary. It should be emphasized that this method gives quite good results if we assume that the channel profile is smooth. The other useful oarameters for comparison purposes is also shown in Table 1. This work needs laboratory experiments before the Equations can be recommended for general use.

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NOTATIONS

The notations used in this study are defined where they first appear and in the following list:

A	$[L^2]$	: Cross-sectional area
B	[L]	: Top width of the channel
e	[L]	: Central-gravity of cross-sectional area from the water surface
g	$[LT^2]$	: Gravity acceleration
H_m	[L]	: Mean depth of the cross-section (=A/B)
h	[L]	: Water depth of the wall part only
h_1	[L]	: Water depth of inclined part only
h_{max}	[L]	: Water depth of inclined part only
k_s	[L]	: Nikuradse's equivalent sand roughness
n	[--]	: Manning's roughness coefficient
P	[L]	: Wetted perimeter
P_b	[L]	: Bed wetted perimeter
P_w	[L]	: Wall wetted perimeter
Q	$[L^3T^{-1}]$	: Discharge
R_h	[L]	: Hydraulic radius or conveyance depth in hydraulic radius method
R_n	[L]	: Hydraulic radius or conveyance depth in normal depth method
R_v	[L]	: Hydraulic radius or conveyance depth in vertical depth method
S_0	[L/L]	: Longitudinal bed slope
S_1	[--]	: Distance measured along the wall part of wetted perimeter of the channel
S_2	[--]	: Distance measured along the sloping part of wetted perimeter of the channel
S_b	[L/L]	: Longitudinal energy grade line or bed slope
u	$[LT^1]$	: Velocity
u_*	$[LT^1]$	: Friction/Shear velocity
u_{*x}	$[LT^1]$	: Friction/Shear veloicty in the boundary
u_m	$[LT^1]$	: Mean of the time-mean velocity
$u_{m,x}$	$[LT^1]$	: Mean velocity in the boundary
y	[L]	: Local vertical depth at x = x

y' [L] : Vertical distance from the bed

y_0' [L] : A parameter which is proportional to the roughness excrescence ($y_0' = \eta k_s$)

Z [L] : Distance normal to s direction from W.S.

z_1 [L] : distance between wetted perimeter and bisector along to the wall part

z_2 [L] : Distance between wetted perimeter and bisector along to the sloping part

η [--] : An experimental coefficient for k_s in mixing length theory

θ [--] : Angle between bed parts and horizontal

β [--] : Angle between bisectors at the point of wall and bed junctions

κ [--] : Universal constant charcterising the turbulence or von-Karman constant

ν [L^2T^{-1}] : Kinematic viscosity of water

ρ [ML^{-3}] : Water density

γ [ML^2T^{-2}] : Unit weight of water

τ [ML^3T^{-2}] : Shear stress in direction of flow

τ_b [ML^3T^{-2}] : Boundary shear stress

τ_0 [ML^3T^{-2}] : Shear stress in direction of flow at boundary

$\tau_{0.ave}$ [ML^3T^{-2}] : Spatial average of the boundary shear stress along wetted perimeter

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