CONCEPTS AND APPLICATION OF THREE DIMENSIONAL INFINITE ELEMENTS TO SOIL-STRUCTURE-INTERACTION PROBLEMS

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Abstract This study is concerned with the formulation of three dimensional mapped infinite elements with 1/r and $1/\sqrt{r}$ decay types. These infinite elements are coupled with conventional finite elements and their application to some problems of soil structure interaction are discussed. The effeciency of the coupled finite-infinite elements formulation with respect to computational effort, data preparation and the far field representation of the unbounded domain is investigated and brought out.

Key Words Exponential Decay Type, Mapped Infinite Element, Ascent and Descent Formulation Computational Aspect

چکیده اصول فرمول بندی المان های نامحدود سه بعدی ایزوپارامتریک در این مقاله بررسی شده است. کوپل کردن این المان ها با المان های محدود، اقتصادی بودن این المان ها از نظر کاهش وقت کامپیوتری و همچنین مدل کردن واقعی مسائل half space با حل چند نمونه مسئله اندر کنش سازه، پی و خاک مورد مطالعه قرار گرفته است.

INTRODUCTION

In many of the unbounded domains, related to engineering and sciences, an engineer is concerned with displacements, strains and stresses at a point far away from the points of the load application, e.g., problems of deeply located tunnels, structure-foundation interaction, wave propagation, fluid-stucture foundation interaction, etc. Before the development of infinite elements, an analyst used to truncate the domain of analysis to some large, but finite distance which would then be considered as the approximation to infinity.

Unfortunately this approach (i.e., truncation approach) is often expensive, sometimes inaccurate and frequently both. These shortcomings have been overcome with the development of infinite elements. Bettes [1,2], Bettes and Bettes [3], Beer and Meek [4], Zienkiewicz, et. al. [5], Curnier [6], Marques and Owen [7], Kumar [8], Viladkar, et. al [9] and Angelov (10, 11) have used various types of infinite elements for the far field representation of 1-D, 2-D and 3-D problems. Broadly speaking, these infinite elements can be catagorised as (i) Exponentional decay type, where a finite element is stretched to an infinite element and (ii) Mapped element with $1/r^n$ decay, where an infinite element is mapped to a finite element. These two catagories are widely recognized as the displacement descent formulation and co-ordinate ascent formulation, respectively.

Considering numerical integration, Zienkiewicz, et.al. [5] found that an infinite element with displacement descent formulation requires numerical integration over semi-finite range, which by and large is inconvenient. However, the infinite element with coordinate ascent formulation can be integrated conveniently using a conventional Gauss-Legendre numerical scheme. Therefore, these elements are attractive due to their application to real engineering problems and their implementation in any finite element package.

This study is mainly concerned with the formulation of 3-D infinite element of mapped type, their shape functions with 1/r and $1/\sqrt{r}$ decay type, and their coupling with usual finite elements. Their applicability has been well established by analysing the problems of: **a)** Raft-Soil-System, and **b)** Space Frame-Raft-Soil System.

FORMULATION OF 3-D ISOPARAMETRIC INFINITE ELEMENTS

Table 1 shows an eight noded infinite element which is mapped into finite element i.e., $-1<\xi<+1$ by using expression

$$x = \sum_{i=1}^{n} N_i x_i \tag{1}$$

In general, the shape function, N_i should satisfy the following conditions:

a) It should have the value of unity at node i and zero at all the other nodes,

b) $$\sum_{i=1}^{n} N_i = 1, \tag{2}$$

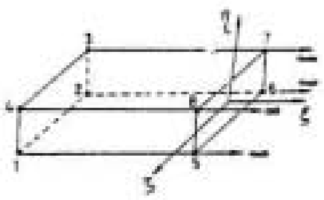
c) For $\xi = +1$ the value of N_i should tend to infinity, and

d) $$\sum_{i=1}^{n} \frac{\partial N_i}{\partial \xi} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial \eta} = \sum_{i=1}^{n} \frac{\partial N_i}{\partial \zeta} = 0 \tag{3}$$

where n = Number of nodes per element.

Table 1 shows the shape function of such an eight noded infinite element of serendipity type. The inverse mapping of this element can be expressed using Equation 1 as

TABLE I. Mapping/Shape Functions for 3-D Serendipity Type Isoparametric Infinite Elements.

Type of Element	Element Figure	Mapping/Shape Functions (l/r) type decay	Mapping/Shape Functions $(1/\sqrt{r})$ type decay
8. noded		$N_1 = \frac{-\xi(1-\eta)(1+\zeta)}{2(1-\xi)}$	$\frac{1}{3}[1-\frac{1}{(1-\xi)^2}](1-\eta)(1+\zeta)$
		$N_2 = \frac{-\xi(1-\eta)(1-\zeta)}{2(1-\xi)}$	$\frac{1}{3}[1-\frac{1}{(1-\xi)^2}](1-\eta)(1-\zeta)$
		$N_3 = \frac{-\xi(1+\eta)(1-\zeta)}{2(1-\xi)}$	$\frac{1}{3}[1-\frac{1}{(1-\xi)^2}](1+\eta)(1-\zeta)$
		$N_4 = \frac{-\xi(1+\eta)(1+\zeta)}{2(1-\xi)}$	$\frac{1}{3}[1-\frac{1}{(1-\xi)^2}](1+\eta)(1+\zeta)$
		$N_5 = \frac{(1+\xi)(1-\eta)(1+\zeta)}{4(1-\xi)}$	$\frac{1}{12}[\frac{4}{(1-\xi)^2}-1](1-\eta)(1+\zeta)$
		$N_6 = \frac{(1+\xi)(1-\eta)(1-\zeta)}{4(1-\xi)}$	$\frac{1}{12}[\frac{4}{(1-\xi)^2}-1](1-\eta)(1-\zeta)$
		$N_7 = \frac{(1+\xi)(1+\eta)(1-\zeta)}{4(1-\xi)}$	$\frac{1}{12}[\frac{4}{(1-\xi)^2}-1](1+\eta)(1-\zeta)$
		$N_8 = \frac{(1+\xi)(1+\eta)(1+\zeta)}{4(1-\xi)}$	$\frac{1}{12}[\frac{4}{(1-\xi)^2}-1](1+\eta)(1+\zeta)$

$$x = \frac{-\xi(1-\eta)(1+\zeta)}{2(1-\xi)}x_1 + \frac{-\xi(1-\eta)(1-\zeta)}{2(1-\xi)}x_2 + \frac{-\xi(1+\eta)(1-\zeta)}{2(1-\xi)}x_3 + \frac{-\xi(1+\eta)(1+\zeta)}{2(1-\xi)}x_4 + \frac{(1+\xi)(1-\eta)(1+\zeta)}{4(1-\xi)}x_5 + \frac{(1+\xi)(1-\eta)(1-\zeta)}{4(1-\xi)}x_6 + \frac{(1+\xi)(1+\eta)(1-\zeta)}{4(1-\xi)}x_7 + \frac{(1+\xi)(1+\eta)(1+\zeta)}{4(1-\xi)}x_8 \quad (4)$$

where, x_1 to x_8 are the co-ordinates of the node 1 to 8, respectively. Now with

$$x_5 = 2x_1;\ x_6 = 2x_2\ x_7 = 2x_3 \text{ and } x_8 = 2x_4, \quad (5)$$

the inverse mapping can be written as follows

$$\xi = 1 - \frac{1}{2x}[(x_1+x_2+x_3+x_4) + \zeta(x_1-x_2-x_3+x_4) + \eta(-x_1-x_2+x_3+x_4) + \eta\zeta(-x_1+x_2-x_3+x_4)] \quad (6)$$

The condition given by Equation 5 indicates that the middle nodes, 5, 6, 7 and 8 should placed at a distance of twice the distance of nodes 1, 2, 3 and 4, respectively, from the reference pole. The values of ξ for all the nodes obtained via eqn. 6 are tabulated in Table 2. An identical expressionbe worked out for η and ξ. The same approach is followed for any other elements with 1/r and $1/\sqrt{r}$ types of decay.

TABLE 2: Values of ξ at Various Nodes.

Element Fig.	Node No.	x	η	ζ	ξ obtained fromeqn. 6
(1)	(2)	(3)	(4)	(5)	(6)
	1	x_1	-1	+1	-1
	2	x_2	-1	-1	-1
	3	x_3	+1	-1	-1
	4	x_4	+1	+1	-1
	5	$x_5=2x_1$	-1	+1	0
	6	$x_6=2x_2$	-1	-1	0
	7	$x_7=2x_3$	+1	-1	0
	8	$x_8=2x_4$	+1	+1	0

So, based on this concept the mapping and shape functions for 12 and 16 noded infinite elements can be workded out for both 1/r and $1/\sqrt{r}$ types of decay (Noorzaei [12, 13]).

PROGRAMMING FEATURES

In this study, an existing three dimensional finite element analysis program was modified to form a software with multi- element features. This was done by identifying each element by a particular code number. Based upon this assigned element code number, number of nodes per element, order of integration, size of elasticity [D] matrix, shape functions and their derivatives were picked up. The Jacobian and its inverse, elasticity [D] matrix, strain-displacement [B] matrix, and stiffness matrix were, therefore, computed automatically.

APPLICATION TO RAFT-FOUNDATION SOIL SYSTEM

Definition of the Problem

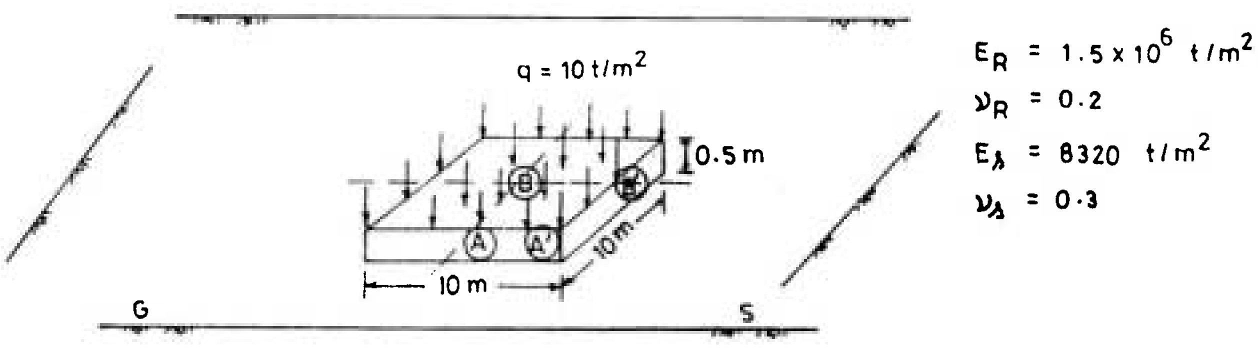
A square raft resting on soil mass (adopted by Buragohain and Shah, [14]) as shown in Figure 1a is taken to study the behaviour of the three dimensional infinite elements. The data regarding geometry, loading, and material properties are also presented in the Figure 1a.

Finite-Infinite Element Discretization

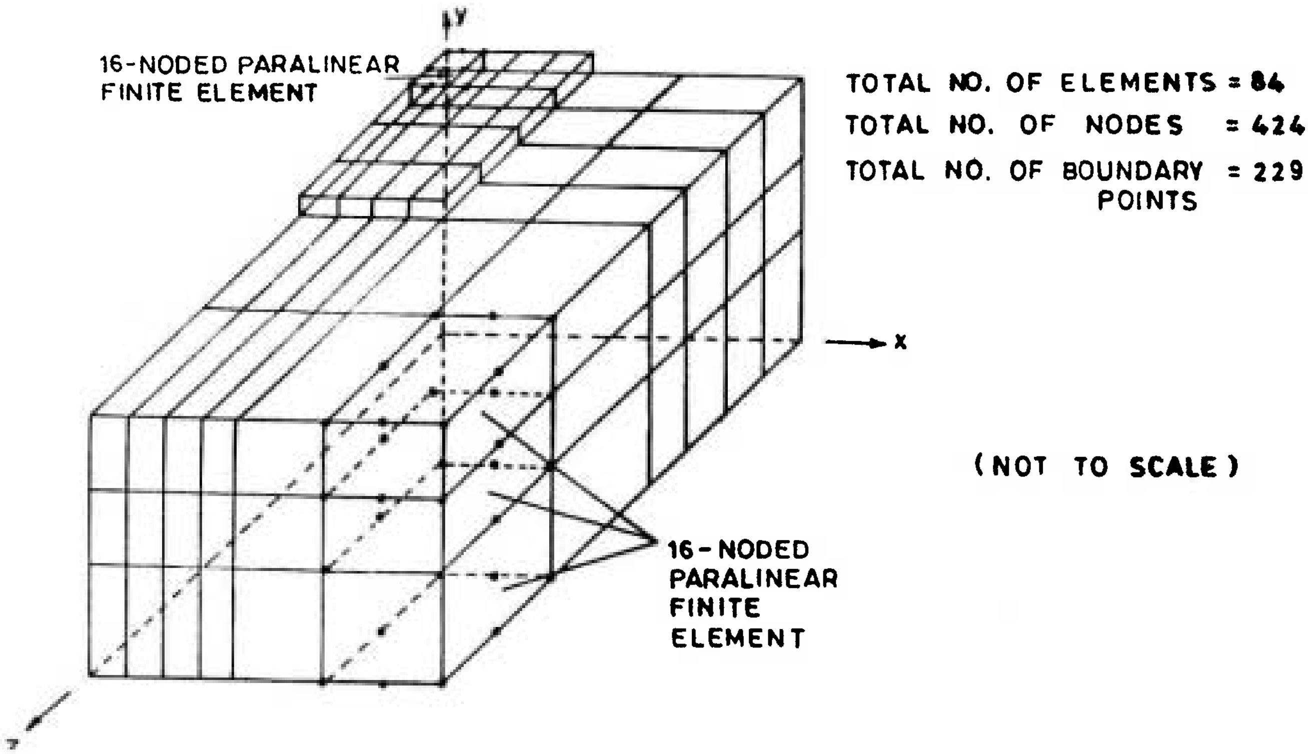
In order to highlight the efficiency of the three dimensional infinite elements, the problem of the square raft-soil system is solved using the following discretizations:

Case (i) A fully finite element idealization with three layers of soil as shown in the Figure1b.

Case (ii) A coupled finite-infinite element



(a)



(b)

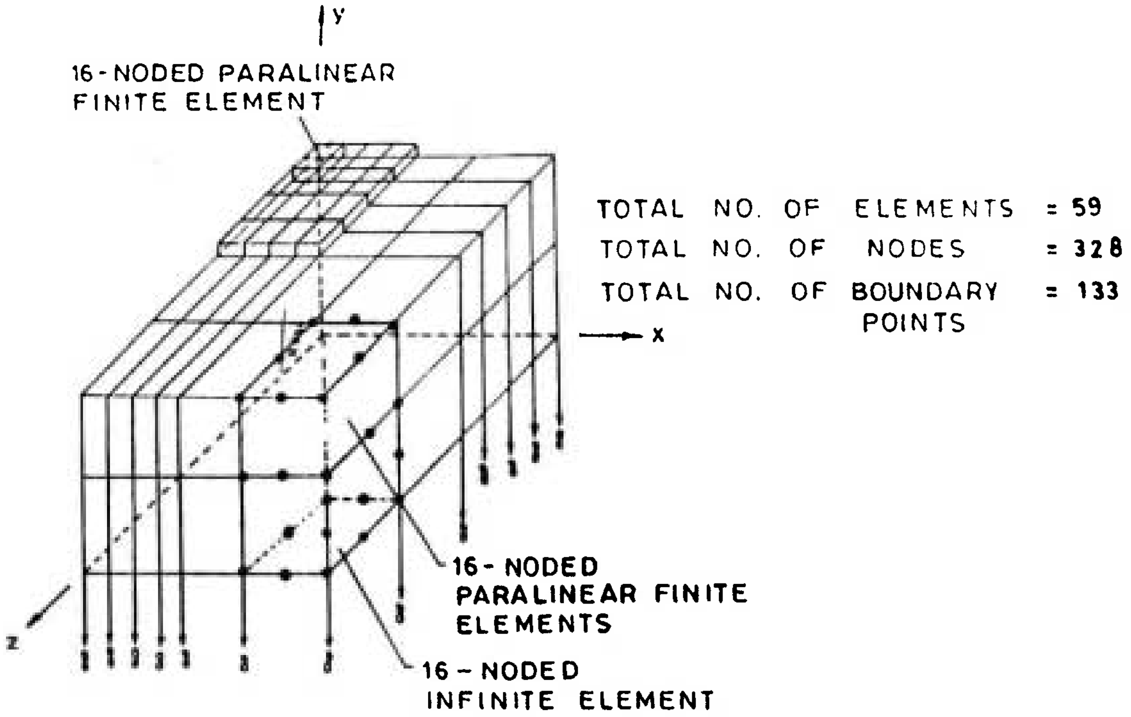
Figure 1. (a) Square raft (10 ×10 × 0.5 m) resting on soil surface. (b) Three dimensional fully finite element discretisation of quarter portion of raft-soil system.

idealization with two layers of soil, one layer of finite element and one layer of infinite element as shown in Figure 2a.

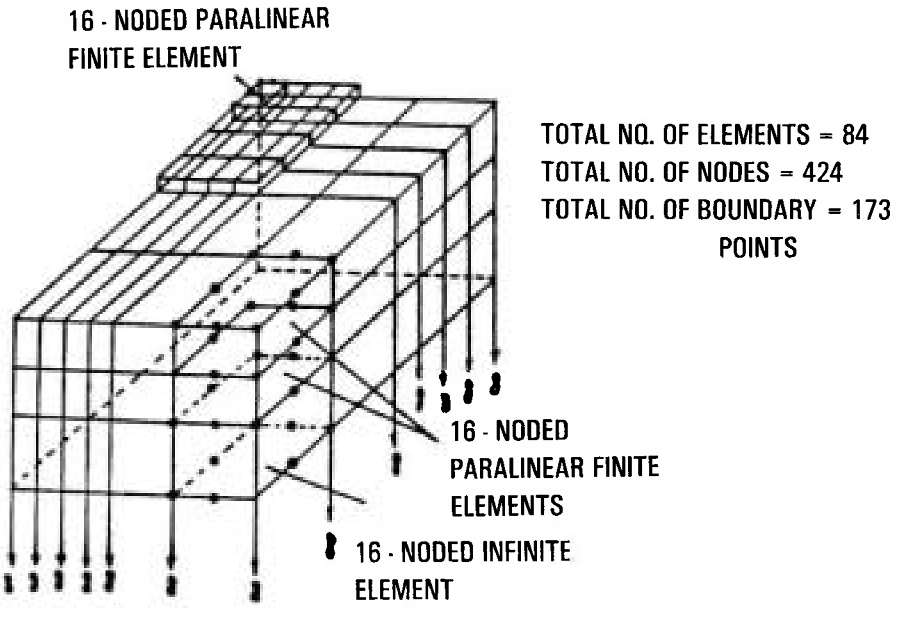
case (iii) A coupled idealization of the raft and soil system in which the soil mass has been idealized by two layers of finite elements, followed by one layer of infinite elements as shown in Figure 2b.

Behaviour of Raft-Soil System

Infinite elements with 1/r type of decay were used. The order of integration of 3 by 3 for the finite elements and 2 by 2 for the infinite elements was employed. The results obtained using the above three discretization patterns named as case (i), case (ii), and case (iii), are discussed for such parameters like total and



(a)



(b)

Figure 2. **(a)** One layer of finite and one layer of infinite. **(b)** Two layer of finite and one layer of infinite elements.

differential settlements and contact pressure distribution below the raft and compared with the results obtained by Buragohain and Shah [14].

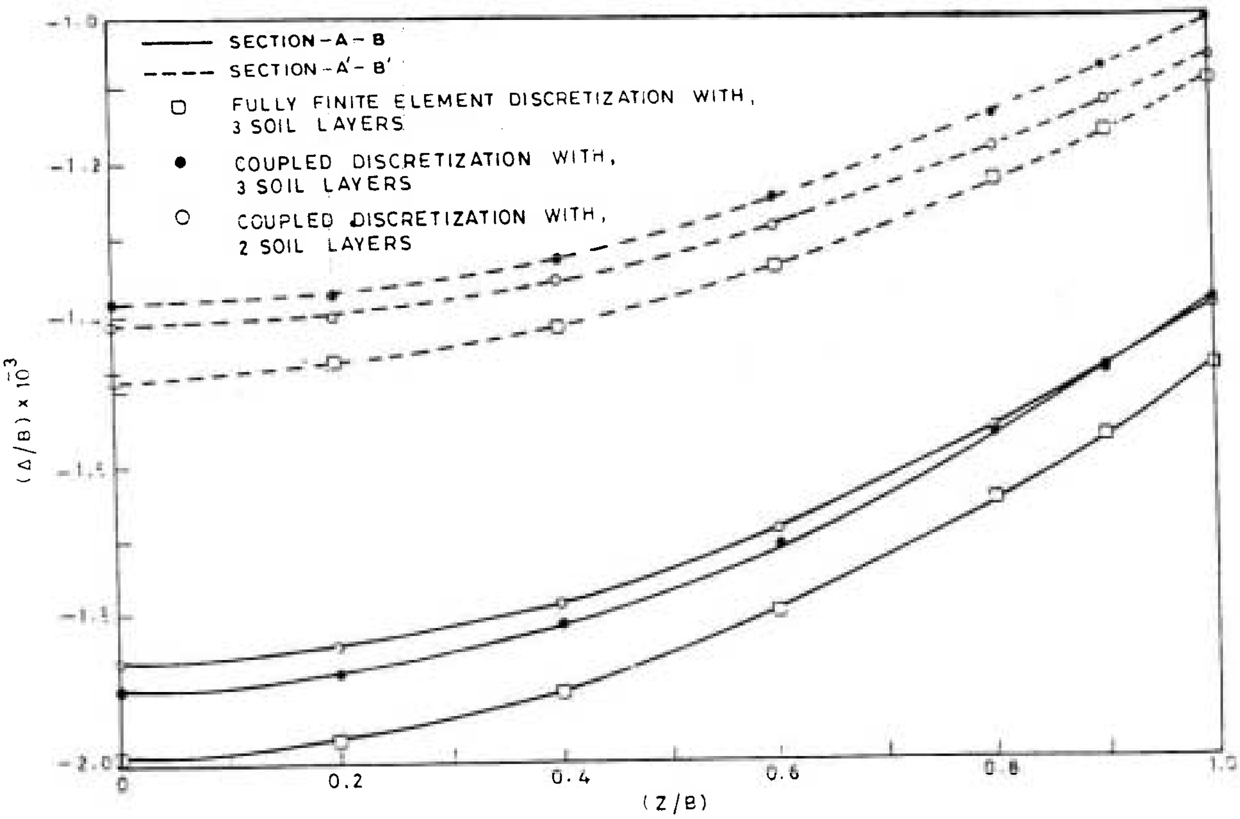


Figure 3. Vertical deformation profile of the raft along section - AB and A′B′.

Vertical Deformation Along the Base of the Raft

Figure 3 shows the defomed profile of the raft along section AB and A 'B' (Figure1a) obtained on the basis of three different meshes. The plots show that the deforma-tions obtained from using fully three dimensional finite element idealization and those obtained from using a coupled finite-infinite element idealization with three finite layers of soil agree well with each other.

Table 3 provides information to compare values ob-tained for total settlements at the centre, mid-side and comer of the raft with those of Buragohain and Shah [14] in which an infinite element with an exponential decay function was used. It can be concluded that the values obtained by Buragohain and Shah [14] are overestimated by 10-15 percent.

Differential Settlement Below the Raft

Table 4 presents the values of differnetial settlements between the centre and mid-side of the raft and also between the centre and comer of the raft obtained on the basis of three discretization pattems and their comparison with the values obtained by Buragohain and Shah [14]. It can be observed that the differential settlements obtained using fully three dimensional finite element analysis and coupled finite-infinite element analysis with three layers of soil agree very well. Although coupled finite-infinite element analysis with two layers of soil gives lower values, the difference is negligible. The values reported by Buragohain and Shah are on the higher side.

Application to Space Frame-Raft-Soil System

In order to further emphasize the importance of the coupled formulation, an attempt was made to analyse a problem of space frame supported by raft soil system. A fully finite element analysis of the same problem was available in the literature (King et.al. [15, 16, 17]).

TABLE 3. Comparison of Total Settlements (mm) of Centre, Mid-side and Corner of the Raft for Various Discretisations.

Item	Case (i)	Case (ii)	Case (iii)	Buragohain & Shah (1.1)
(1)	(2)	(3)	(4)	(5)
Centre of raft	9. 937	9.319	9.491	10.470
Mid-side of raft	7.389	6.996	6.994	7.250
Corner of raft	5.486	5.314	5.037	4.580

TABLE 4. Comparison of Differential Settlements (mm), Between Centre, Mid-side and Corner of the Raft for Various Discretisations.

Item	Case (i)	Case (ii)	Case (iii)	Buragohain & Shah (1.1)
(1)	(2)	(3)	(4)	(5)
Between centre and mid-side of the raft	2. 548	2.323	2.547	3.220
Between centre and corner	4.451	4.005	4.454	5.890

Definition of the Problem

The proposed coupled finite-infinte physical model is used for the interactive analysis of a four storey, five bay by three bay, space frame-raft soil system (King et. al. [15, 16, 17]).

Figure 4a shows the isometric view of the space frame-raft-soil system. The layout details of the frame are shown in Figure 4b. Figures 4c and 4d show the front and side elevations of the frame along with other geometrical details. All the floors are

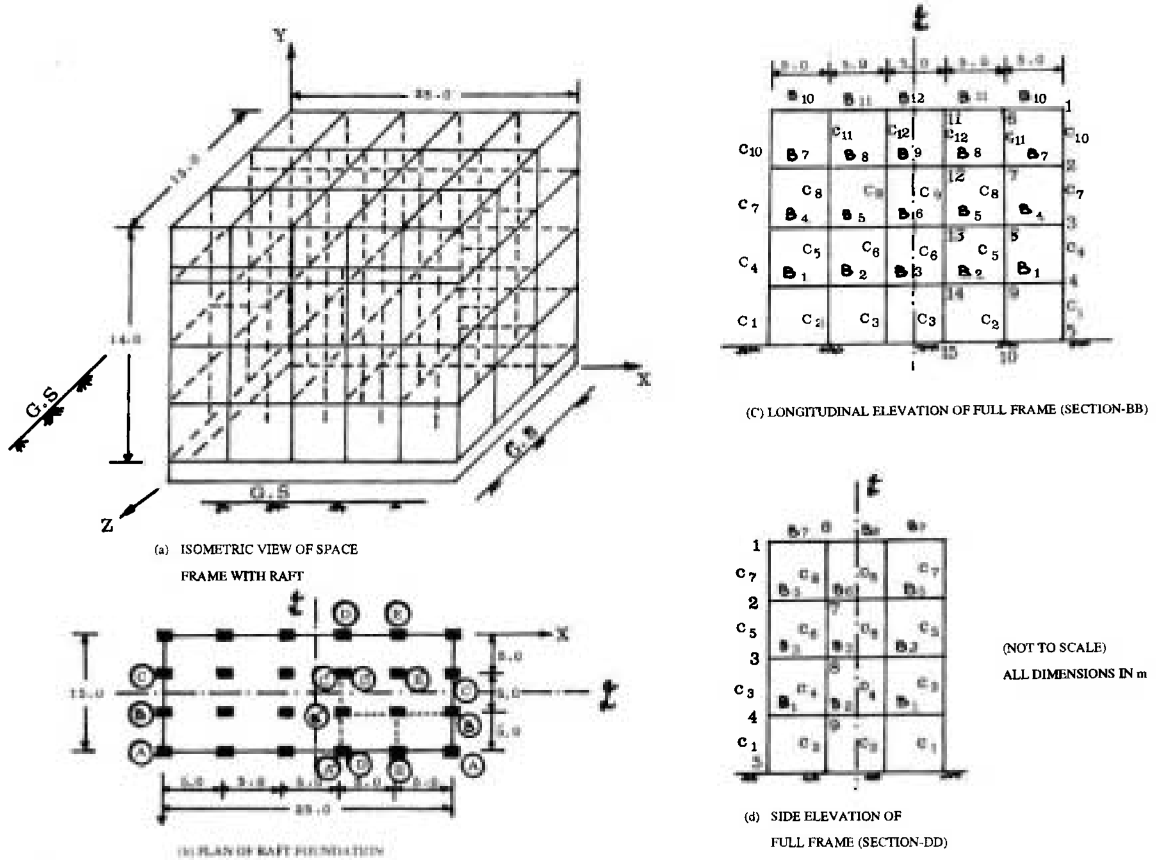


Figure 4. Geometrical details of the space frame.

transversed by the main beams (0.3 × 0.6 m) in the longitudinal as well as transverse directions. The interior columns of ground and first floor are 0.4×0.4 m in size and the remaining columns are of 0.36 × 0.36 m. The thickness of the raft is 0.4 m. The modulus of elasticity of 1.4 t/m² and poisson's ratio of 0.3 are used for concrete in the analysis. The modulus of elasiticity and poisson ratio's of soil have been taken as 1000 t/m² and 0.45 respectively. The loading pattern on the structure and the raft are shown in the Figure 5.

Finite Infinite Element Discretization

The finite element discretization is attempted using beam bending element with six D. O. F. (u, v, w, θ_x, θ_y, θ_z) per node to represent the structural members. The plate bending element with five D. O. F. (u, v, w, θ_x, θ_z) per node is used to represent the raft and the

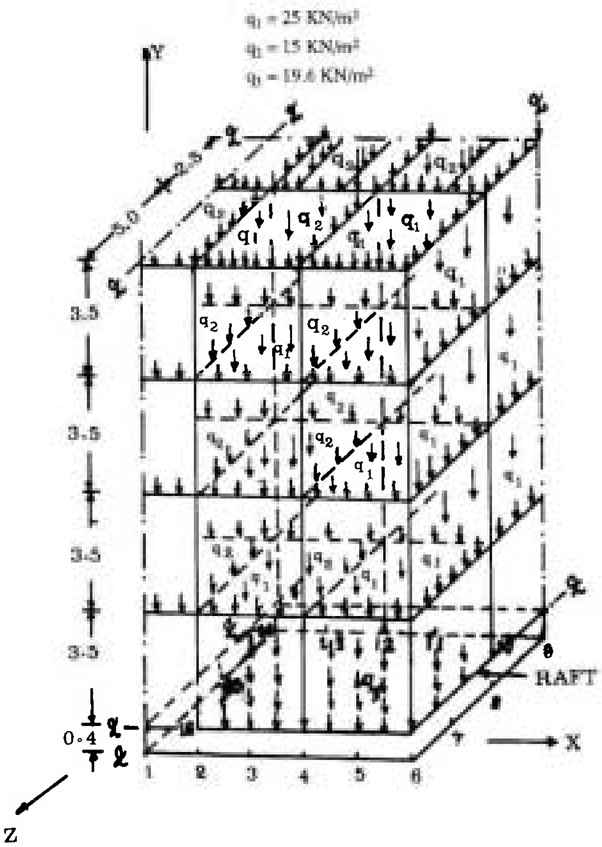


Figure 5. Loading details of the space frame.

soil mass is idealized by three dimensional coupled finite infinite elements. Figure 6 shows the coupled finite-infinite discretization of the entire system using a quarter symmetry. Since, the present idealisation involves varieties of the elements with different D. O. F. per element in the domain of the interest, it is discussed elsewhere (Godbole, et. al. [18], and Noorzaei, [12]).

Comparison of Finite Element Modelling

King et. al. [15, 16] discretized superstructure, raft and soil system by means of conventional two noded beam element with six D. O. F. per node, conventional plate element with three D. O. F. per node and eight noded brick element with three D. O. F. per node respectively. Instead, in the present study, the three noded isoparametric beam bending element with six D. O. F. per node and eight noded isoparametric plate bending element with five D. O. F. per node are employed to represent the structure members, slab and raft respectively. The soil mass below the raft, where stress concentration is predominant, is represented by coupled sixteen noded isoparametric finite-infinite elements.

For the rest of soil mass, coupled eight noded isoparametric finite-infinite elements are used (Figure 6).

In soil-structure interaction analysis using finite element technique, one of the major issues is the number of soil layers needed to give the best possible physical representation, especially for the far field, with minimum computational cost. In the interactive analysis by king, et. al. [15, 16], seven and nine layers of finite elements were used in soil in vertical and horizontal directions, respectively. However, in the present analysis using coupled finite-infinite element formulation, only two layers of finite elements and one layer of infinite elements are found to be satisfactory in soil modelling in either direction

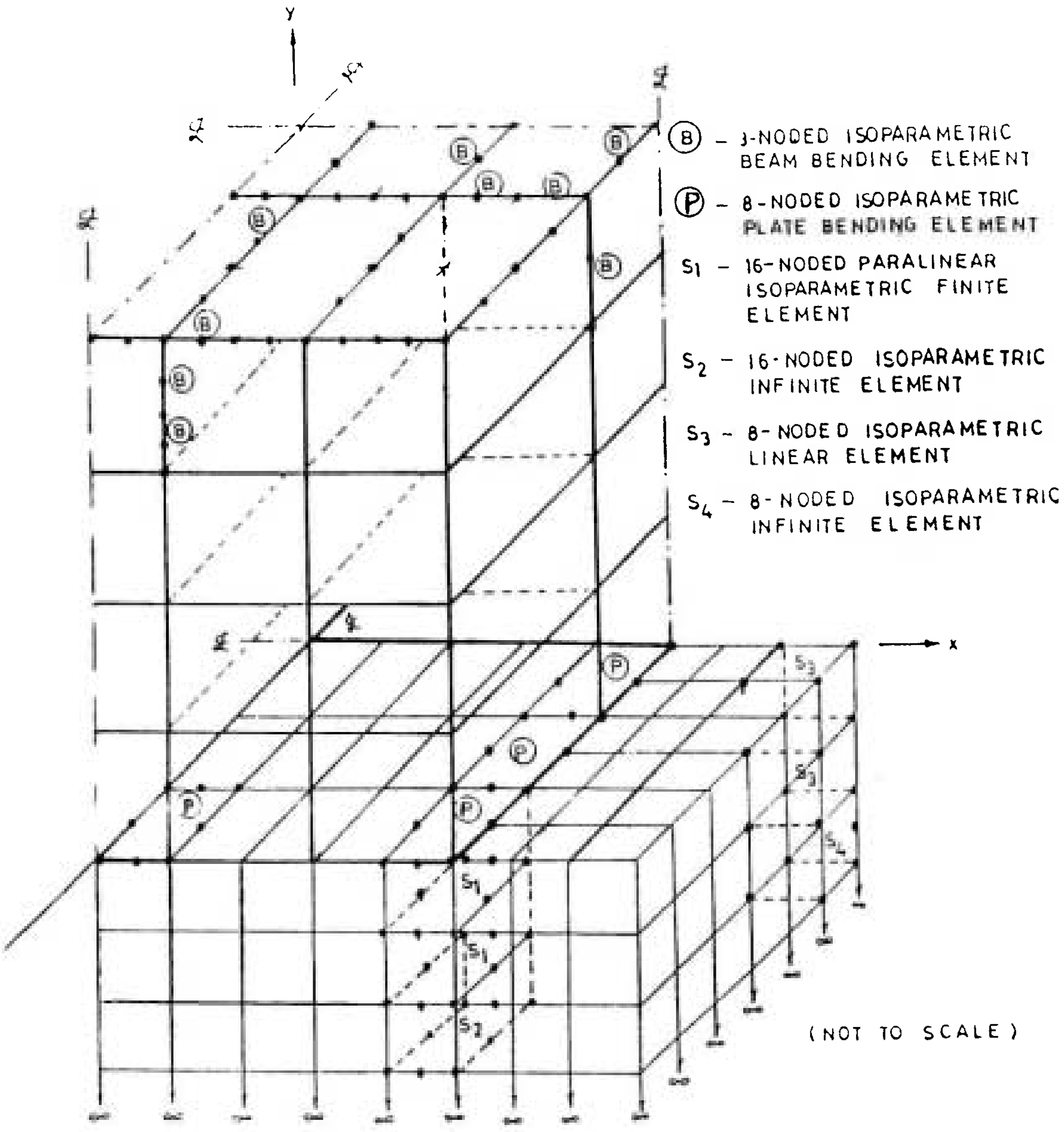


Figure 6. Multi-element discretisation of space frame raft-soil system.

(Figure 6).

Raft Settlements

Table 5 shows the settlements of the raft obtained from the proposed analysis and their comparison with those reported by King and Chandrasekaran et. al. [15, 16]. There is a good agreement between the values of the settlements. Moreover, in view of the discretization presented in this study with that of king et. al. [15, 16], it can be concluded that there is a significant reduction in computational cost while improving the physical modelling aspects of the problem. this signifies the superiority of the couple finite-infinite element formulation.

Comparison of Contact Pressures

The comparison of the contact pressure distribution along section BB' (Figure 4) is presented in Figure 7.

TABLE 5. Comparison of Settlements (mm) of the Raft.

		Settlements			
Points*	King and Chandrasekaran [15, 16] Analysis	Present Analysis	Points*	King and Chandrasekaran [15, 16] Analysis	Present Analysis
(1)	(2)	(3)	(4)	(5)	(6)
1	34.00	36.70	9	32.50	33.40
2	36.50	39.30	10	40.00	39.30
3	35.76	35.76	11	43.00	44.80
4	35.00	35.87	12	45.50	47.80
5	28.00	29.20	13	47.50	49.70
6	27.00	27.50	14	49.50	50.00
7	28.00	29.13	15	45.00	48.50
8	34.00	35.50	16	43.00	43.30

* for points, refer to Figure 5.

The contact pressure distribution obtained for both of the analyses follows the same trend. Moreover, the values of contact pressures at various points along the section are nearly the same except at the edge of the raft. King et. al. [15, 16] give the value of contact pressure at the edge of raft which is 1.6 times as high as the value obtained in the present analysis. This difference may be due to the fact that King

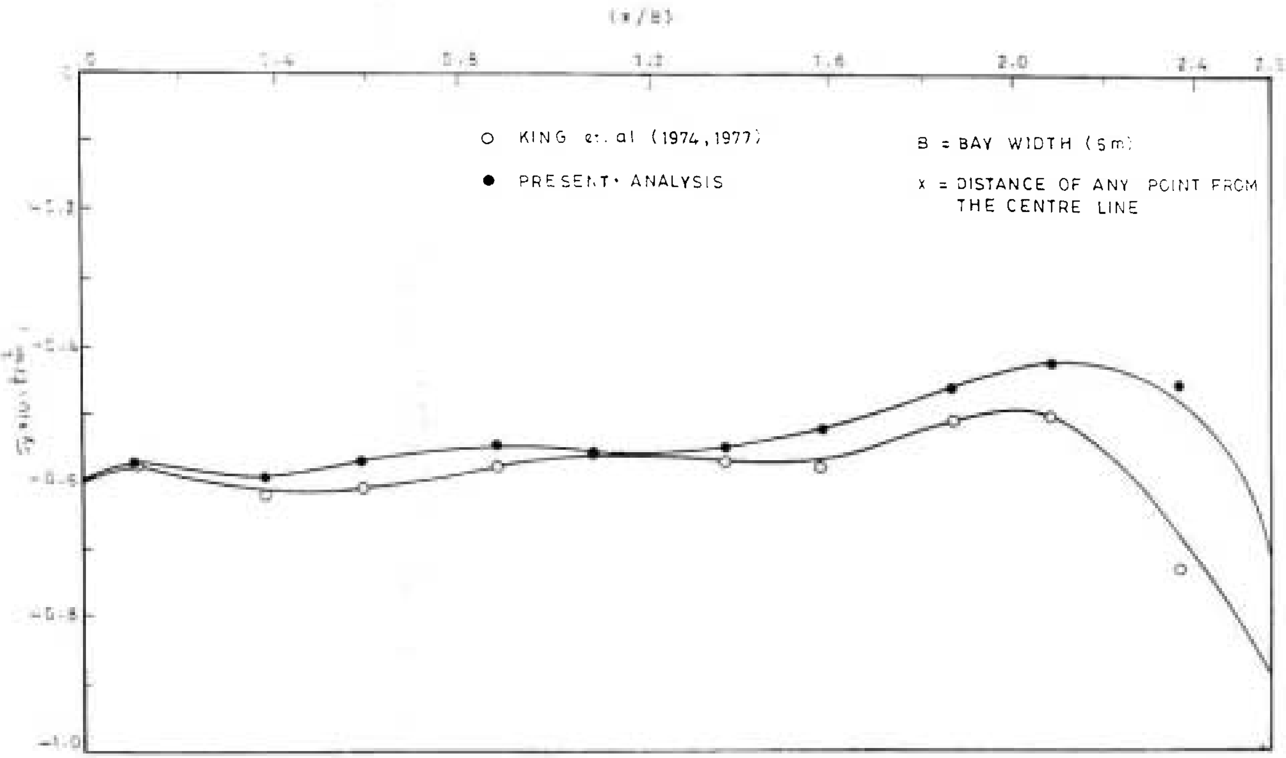


Figure 7. Contact pressure distribution below the raft (Section - BB′).

and Chandrasekaran [15, 16] have considered a linearly increasing soil modulus with depth whereas in the present study, an average value of soil modulus is assumed in each of the soil layers discretised.

CONCLUDING REMARKS

Based on the findings of the present study, the following observation can be made:

i) The infinite elements, coupled with the conventional finite elements, and their behaviour with regard to the far field representation and computational cost have been studied by solving two three dimensional geomechanical problems.

ii) The idealization with one layer of finite elements followed by a layer of infinite elements (Figure 2a) gives almost the same results as those given by either fully finite element idealization (Figure 1b) or by using two layers of finite elements followed by a layer of infinite elements (Figure 2b).

iii) Three dimensional infinite elements are quite general in nature in the sense that the 16-noded infinite elements can be attached to 20 noded finite elements. As such, the 12 and 16 noded infinite elements developed in this study can be attached to 8, 12, 16 or 20 noded finite elements in any combination (Noorzaei, [12, 13]).

iv) There has been a good agreement between the proposed model of space frame-raft-soil system via coupled formulation with that reported in the literature (King et. al. [15, 16]) with respect to settlements and contact pressures. But the proposed coupled finite-infinite elements model yields a significant reduction in effort of data preparation and the computational cost.

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