PHASE ONLY SYNTHESIS OF ANTENNA PATTERNS USING ITERATIVE RESTORATION METHODS

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Abstract In this work, the method of iterative Fourier transform phase reconstruction, conventionally used in holography and optical image reconstruction, is applied to phase only synthesis of antenna patterns. The method is applied to two types of pattern synthesis problems: "main lobe beam shaping" and "side-lobe-level reduction". The proposed method is most useful in the efficient employment of attainable radiation power of antenna array elements, when applied to uniform amplitude distribution of the aperture. Easy reconfiguration of phased arrays, with any desired amplitude distribution is also made possible through the application of the method. Simulation results demonstrate excellent control of side-lobe-levels as well as pattern shapes. The algorithm converges very rapidly, when applied to beam shaping, while, in side-lobe-level control, convergence is achieved at slower rates. Data displaying the resulting phase distribution of the apertures along with synthesized patterns and rates of convergence of various examples are presented and discussed.

Key Words Antenna Theory, Antenna Arrays, Numerical Techniques, Pattern Synthesis.

چکیده درمقاله حاضر، بکارگیری روش "تکرری بازیافت فاز تبدیل فوریه" که در هولوگرافی و بازیافت اپتیکی تصاویر بکار میرود، جهت سنتز نماد آنتنهای آرایه ای، تنها برمبنای فاز تحریک، پیشنهاد شده است. روش مزبور به دو نوع مسئله سنتز نماد تحت عناوین "شکل دهی گلبرگ اصلی" و "کاهش سطح گلبرگ فرعی" بکار گرفته شد. اعمال روش پیشنهادی به توزیع یکنواخت دامنهٔ تحریک، بکارگیری حداکثر توان تشعشعی قابل حصول از عناصر یک آرایه را برای هرشکل نماد دلخواه، میسر میسازد. همچنین امکان تغییر شکل دلخواه نماد آنتن توسط تغییر فاز عناصر با هر توزیع دامنه دلخواه، توسط این روش فراهم میگردد. مثالهای شبیه سازی شده نشان می دهند که هم "شکل گلبرگ اصلی" وهم "سطح گلبرگ فرعی" بخوبی توسط این روش قابل کنترل هستند. همگرایی این روش در مسائل شکل دهی گلبرگ اصلی بسیار سریع است ولی در مسائل کاهش سطح گلبرگ فرعی سرعت همگرایی زیاد نیست. در مثالهای ارائه شده، نتایج شامل توزیع فاز دریچه، نمادهای سنتز شده و چگونگی همگرایی الگوریتم هستند.

INTRODUCTION

In order to minimize the wasted transmitted power and achieve a required radiation coverage for an antenna, it is important to shape the antenna pattern. Efficient use of microwave amplifiers in a phased array antenna requires excitation of all amplifiers at their peak attainable power. This has led to much attention being paid to phase-only synthesis of radiation patterns [1,2]. In this paper we propose a numerical technique to synthesize any desired pattern by shaping relative phase of the radiating elements, allowing their amplitude to take on any desired distribution. The algorithm is based on Constrained Iterative Restoration (CIR) methods, originally used in holography and optical image restoration [3]. In applying the CIR algorithm, the amplitude of the desired pattern and the amplitude distribution of the aperture are assumed to be given, and through an iterative procedure, aperture phase distribution is determined. Excellent control of side-lobe-levels, as well as pattern shapes, result throughout procedure.

THE ITERATIVE METHOD

The basic form of the algorithm was originally used in electron microscopy the Gerchberg and Saxon [4]. The algorithm was then generalized by Fienup and called the "error-reduction" approach. In order to speed-up the convergence, he further modified the algorithm and called it the "input-output" approach [3]. The algorithms were mainly used in optical holography, spectral extrapolation, and removal of distortion in signal processing. Convergence of the algorithms, as subclasses of a broad class of restoration algorithms, was discussed by Shafer et. al. [5].

The method, as represented by Fienup, was applied to

Fourier transform pairs. Here, we will restate the procedure in a form that may be applied to antenna pattern synthesis problem. Adaptation of the procedure is made possible by the straightforward similarity between Fourier transformation and pattern-aperture relations.

Suppose that F and I are two-dimensional Fourier transform pair sequences i.e.

$$I = \mathcal{F}[F]; \tag{1}$$

where F and I are complex functions such that

$$F = |F| \exp(j\varphi); \quad I = |I| \exp(j\psi). \tag{2}$$

Generally, in a reconstruction problem, only partial information is available in each domain, and the problem is to reconstruct I and F. In problems with which we are concerned, |F| and |I| are related to the pattern and aperture amplitudes and are assumed to be known, while ψ and φ are related to the aperture and pattern phase and are to be determined.

A block diagram of the error-reduction approach to solve this problem is shown in Figure 1.The procedure is as follows: F'_k an estimate of F at trial k is Fourier transformed, resulting in I'_k. It is then made to satisfy constraint C_1, to result in I_k. i.e

$$I'_k = |I'_k| \exp(j\psi_k) \tag{3}$$

$$I_k = C_1\{I'_k\} = |I| \exp(j\psi_k) \tag{4}$$

Here, C_1 replaces the amplitude distribution of I'_k by the desired form |I|. The next step is then to take the inverse Fourier transform of I_k, to result in F'_k. The last step in the iteration is to enforce the desired constraint on F'_k to result in F_{k+1}, i.e

$$F'_k = |F'_k| \exp(j\varphi'_k) \tag{5}$$

$$F_{k+1} = C_2(F'_k) = |F| \exp(j\varphi'_k) = |F_{k+1}| \exp(j\varphi_{k+1}) \tag{6}$$

or

$$|F_{k+1}| = |F|; \ \varphi_{k+1} = \varphi'_k$$

where, the amplitude is replaced by the desired form and the phase is left unchanged. Successive transformations and enforcement of constraints in both domains continue until constraints are satisfied in each domain, prior to being enforced. i.e

$$I'_N \approx I_N \ , \quad F_{N+1} \approx F'_N \tag{7}$$

Different methods of choosing an initial estimate have been proposed. Our simulation results, however, show that any initial estimate leads to the same quality of results at about the same speed. A random sequence as the initial guess for ϕ is as good as any other choice.

It is worth noting that there is no need to apply any constraints on the phase of the pattern (related to the phase of F). Thus, the solution to the problem is not unique and additional constraints may be enforced. One possible constraint that may be of interest is to force the aperture phase to behave smoothly. To achieve this, we may fit a low order polynomial to the phase distribution at every iteration, guiding the algorithm to converge to a solution with the desired property (i.e. smooth phase distribution.) However, it must be kept in mind that constraints must always be as relaxed as possible. Weaker constraints will result in faster convergence.

One criterion by which, convergence of the algorithm is measured, is the normalized mean-squared-error (MSE), which is defined in each domain by

$$E_I^2 = \frac{\left|\left|I_k - I'_k\right|\right|^2}{\left|\left|I'_k\right|\right|^2} \tag{8}$$

$$E_F^2 = \frac{\left|\left|F_{k+1} - F'_k\right|\right|^2}{\left|\left|F'_k\right|\right|^2} \tag{9}$$

where |x| is the Euclidian norm of x. It has been shown that the MSE can only decrease after each iteration, and thus convergence is guaranteed [3,5,6]. This is why the algorithm is called "error-reduction".

An alternate form of the algorithm, called the "input-

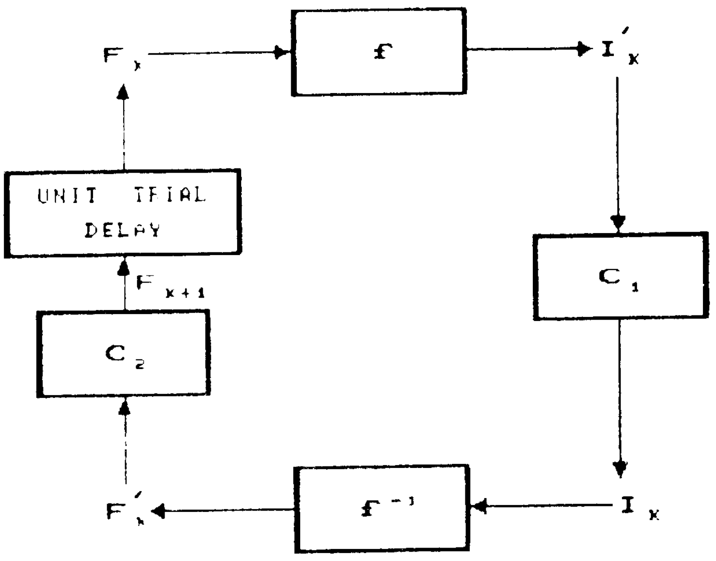


Figure 1. Error reduction approach.

output" approach, modifies the last step of each iteration as follows:

$$F'_k = |F'_k| \exp(j\varphi'_k) \tag{10}$$

$$F_{k+1} = F'_k + \beta_k \Delta F_d = |F_{k+1}| \exp(j\varphi_{k+1}) \tag{11}$$

where ΔF_d is an estimated correction term that must be added to F'_k to guide it towards the form that satisfies constraint C_2, and β_k is the convergence coefficient. Smaller values of β results in a slower rate of change in F_k, while large values of β causes the algorithm to end up in oscillations, undergoing overshoots and undershoots. "Error-reduction" approach is a special case of this algorithm in which $\beta=1$ and

$$\Delta F_d = |F| \exp(j\varphi'_k) - F'_k$$

A good choice for ΔF_d is suggested to be given as follows:

$$\Delta F_d = \frac{1}{2}\left[|F|\frac{F_k}{|F_k|} - F_k\right] + \left[|F|\frac{F'_k}{|F'_k|} - |F|\frac{F_k}{|F_k|}\right] =$$

$$|F| \exp(j\varphi'_k) - \frac{1}{2}\left[F'_k + |F| \exp(j\varphi_{k-1})\right] \tag{12}$$

Compared to the "error-reduction" approach, this choice of ΔF_d, gives rise to more cautious steps towards the desired solution. Here, the second term is taken to be the average between present solution and previous desired form. This choice of ΔF_d, reduces the amount of oscillations, causing the algorithm to converge faster.

Optimum choice of β_k at k th iteration is shown to be the expected value

$$\beta_k = E\left[\frac{|I_k|}{I}\right] \tag{13}$$

Our simulation results, however, show that setting β equal to unity results in about the same rate of convergence in pattern synthesis applications. This is due to the fast rate of convergence, resulting in the expected value becoming very close to unity after a few iterations.

LINEAR ARRAY SYNTHESIS

A linear array of N equispaced isotropic radiators is considered for synthesis. Ignoring mutual coupling the radiation pattern is given by:

$$F(u) = \sum_{i=0}^{N-1} I_i \exp\left(j\frac{2\pi}{N} u i\right) \tag{14}$$

where

$$u = \frac{Nd}{\lambda} \sin\theta \tag{15}$$

d is the element spacing, λ is the free space wavelength, θ is the viewing angle as measured from the array broadside normal, and I_{i-1} is the ith element complex excitation current. The above relation is the basis for using Fourier Transform relations in pattern synthesis problems. i.e.

$$F(n) = \mathcal{F}^{-1}\{I\} = \sum_{i=0}^{N-1} I_i \exp\left(j\frac{2\pi}{N} ni\right); \quad 0 \le n \le N-1 \tag{16}$$

where the operator $\mathcal{F}^{-1}$ represents an N point inverse discrete Fourier Transform and I_i is the sequence of complex excitation current of the array elements. Thus, the problem is readily formulated to the Fourier Transform pair synthesis problem discussed in the previous section. The only difference here, is that constraints are to be enforced only in the visible region of u(or n), corresponding to the values of n between Nd/λ and $-Nd/\lambda$. For positive values of n, this translates to two regions of $0 \le n \le Nd/\lambda$ and $N-Nd/\lambda \le n \le N$.

SIMULATED RESULTS

The input-output algorithm was applied to two types synthesis problems: "beam shaping" and "side-lobe-level control", on an array of 32 equispaced isotropic radiators. The length of the array was chosen to be 5λ. The criteria by which the quality of results is measured are two parameters of side-lobe-level (SLL) and ripple, as defined by

$$SLL = 20 \log \frac{\text{maximum pattern level at largest side lobe}}{\text{maximum pattern level}} \tag{17}$$

$$\text{ripple} = 20 \log \max \left[\left|\frac{\text{synthesized pattern}}{\text{desired pattern}}\right|\right] \tag{18}$$

where, ripple is measured only over the main beam where the pattern is to be shaped.

Beam shaping

An example for beam shaping, the desired pattern was chosen to be in the form of CSC, between $\Theta=3°$ and $\Theta=65°$. Outside the shaped region, the desired pattern was set to satisfy side-lobe-level requirements. Aperture amplitude distributions of uniform, and cosine squared on a pedestal were used as examples and in each case the phase distribution was synthesized. Results show that excellent fit to the desired patterns may be achieved in every case. The rate of convergence is very fast for regions where the desired pattern is down to about -10 to -15 dB. Generally, less than 10 iterations results in satisfactory fits of main lobe down to-10 dB regions. Regions of the pattern with smaller values are fitted in a slower manner, requiring up to 100 iterations. Figures 2 and 3 show the results for uniform and cosine squared on a pedestal distribution, applied to the algorithm for CSC beam shaping. Figures

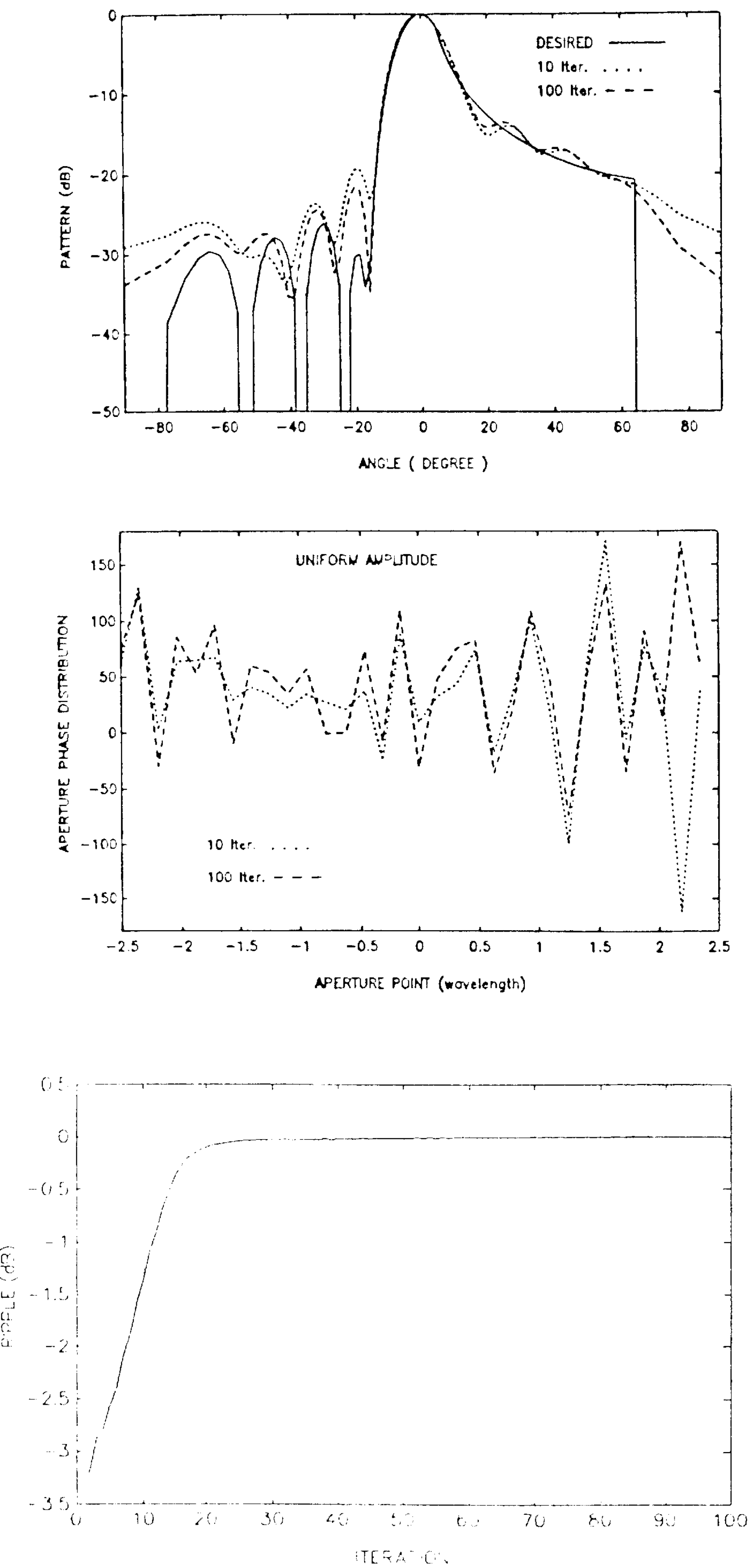


Figure 2. CSC pattern synthesized by uniform amplitude distribution. (a) pattern, (b) aperture phase distribution, (c) convergence.

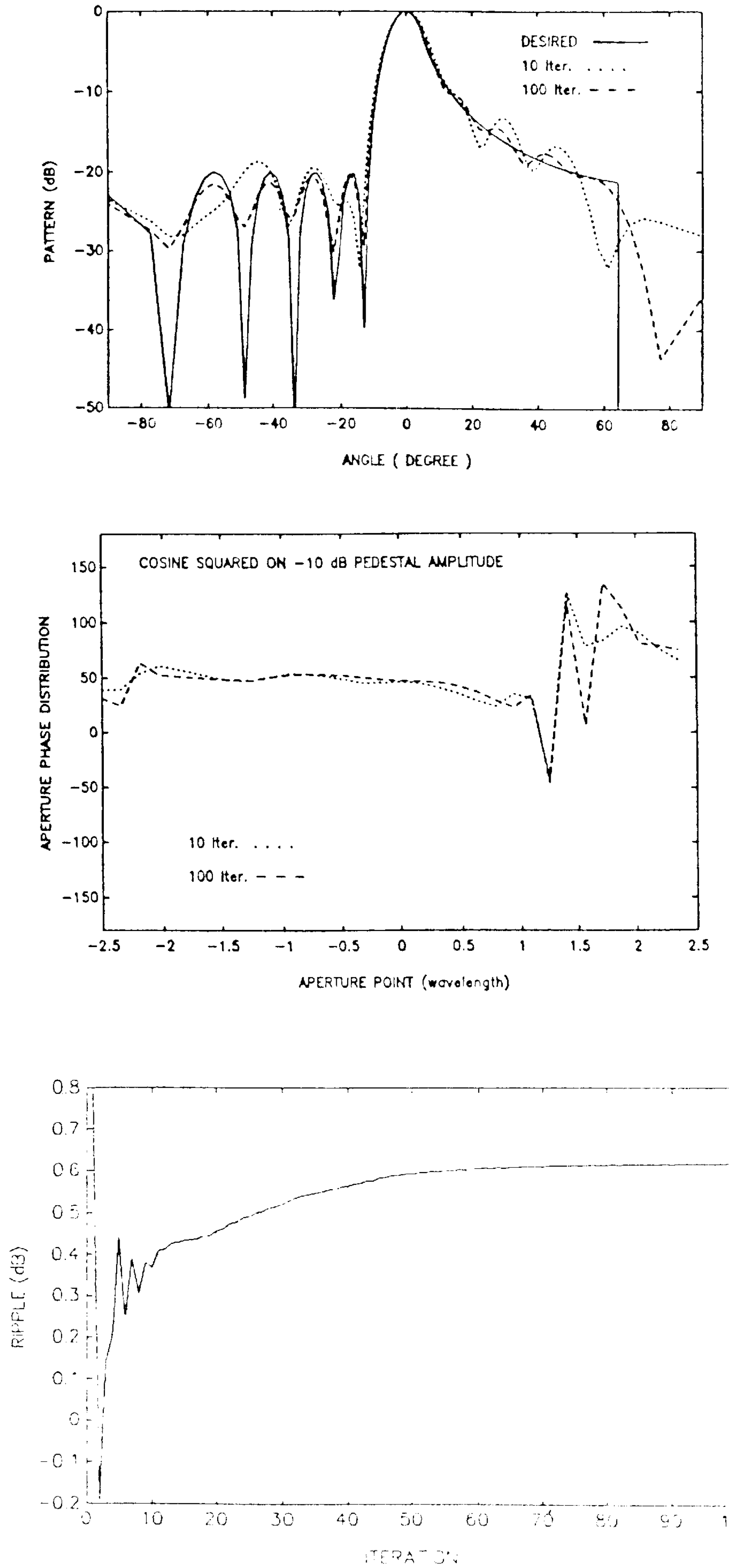
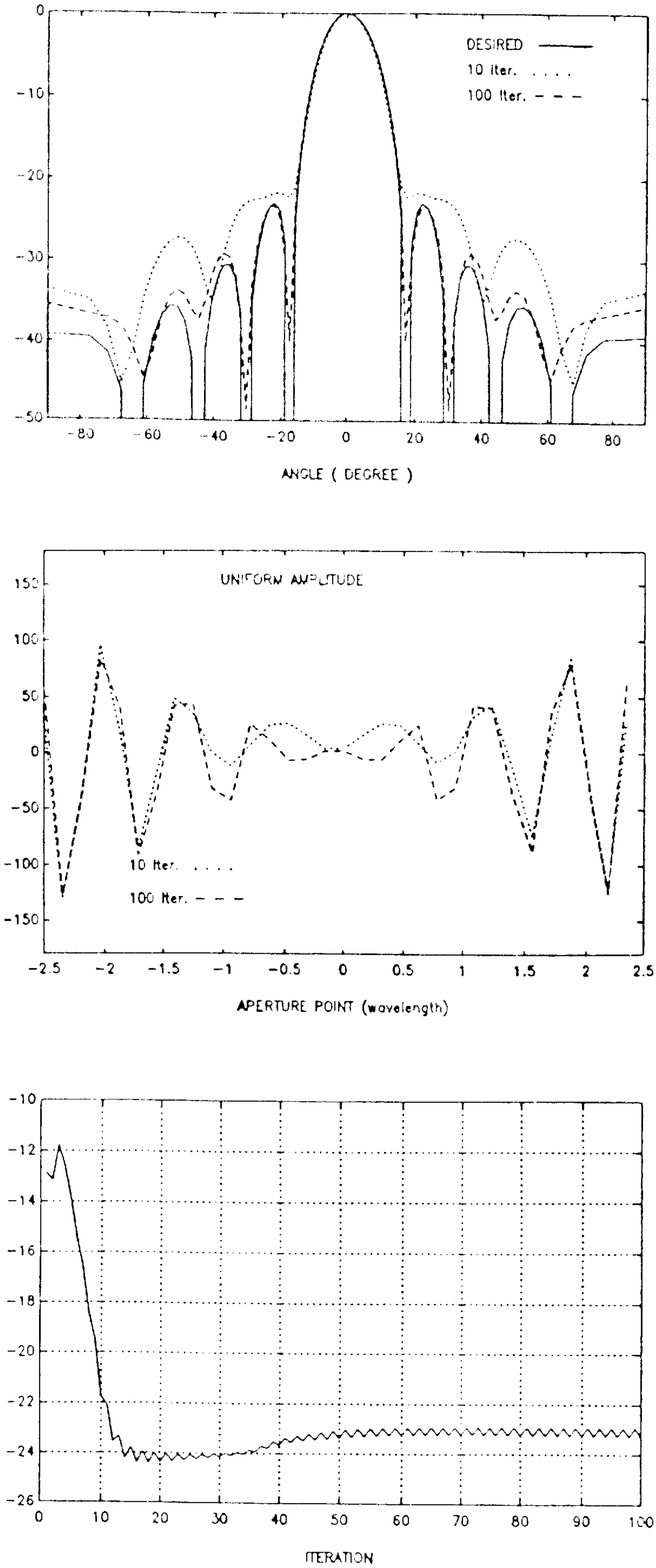


Figure 3. CSC pattern synthesized by cosine squared on a pedestal amplitude distribution. (a) pattern, (b) aperture phase distribution, (c) convergence.

ı) and 3(a) demonstrate the resulting patterns after 10 d 100 iterations, and Figures 2(b) and 3(b) present the ıthesized phase distributions. Part (c) of the figures plot ples, as defined by (18) versus the number of iterations. The later figures demonstrate that the algorithm progresses very fast at the starting iterations, but small offsets are corrected slowly thereafter.



ure 4. Cosine pattern synthesized by uniform amplitude distribution. (a) pattern, (b) aperture phase distribution, (c) convergence.

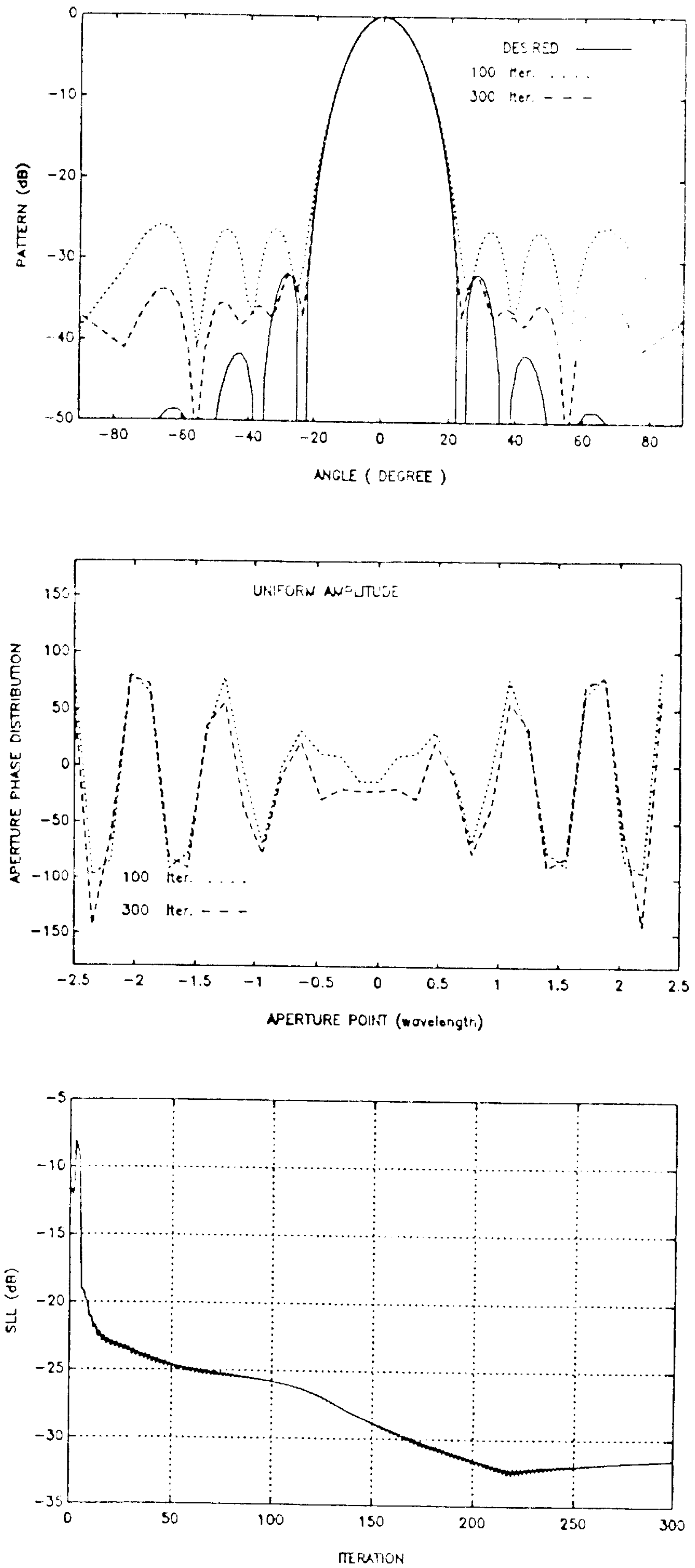


Figure 5. Cosine squared pattern synthesized by uniform amplitude distribution. (a) pattern, (b) aperture phase distribution, (c) convergence.

Side-lobe-level control

Uniform amplitude distribution was applied to the algorithm to synthesize patterns produced by cosine distibution, cosine squared distribution, and Chebychev optimum equiripple distribution with -20 dB SLL. Results are presented in Figures 4,5 and 6. Part(c) of the figures plot SLL versus the number of iterations. Slower rate of convergence compared to beam shaping is clearly demonstrated in these figures. Results, however, suggests that radiation efficiency of the array may be greatly improved by the resulting relative phase of the radiators, where, without the need for taper in amplitude distribution., side-lobe-levels of down to -30 dB are achieved in the case of Figure 5, and optimum equiripple is very closely produced in Figure 6.

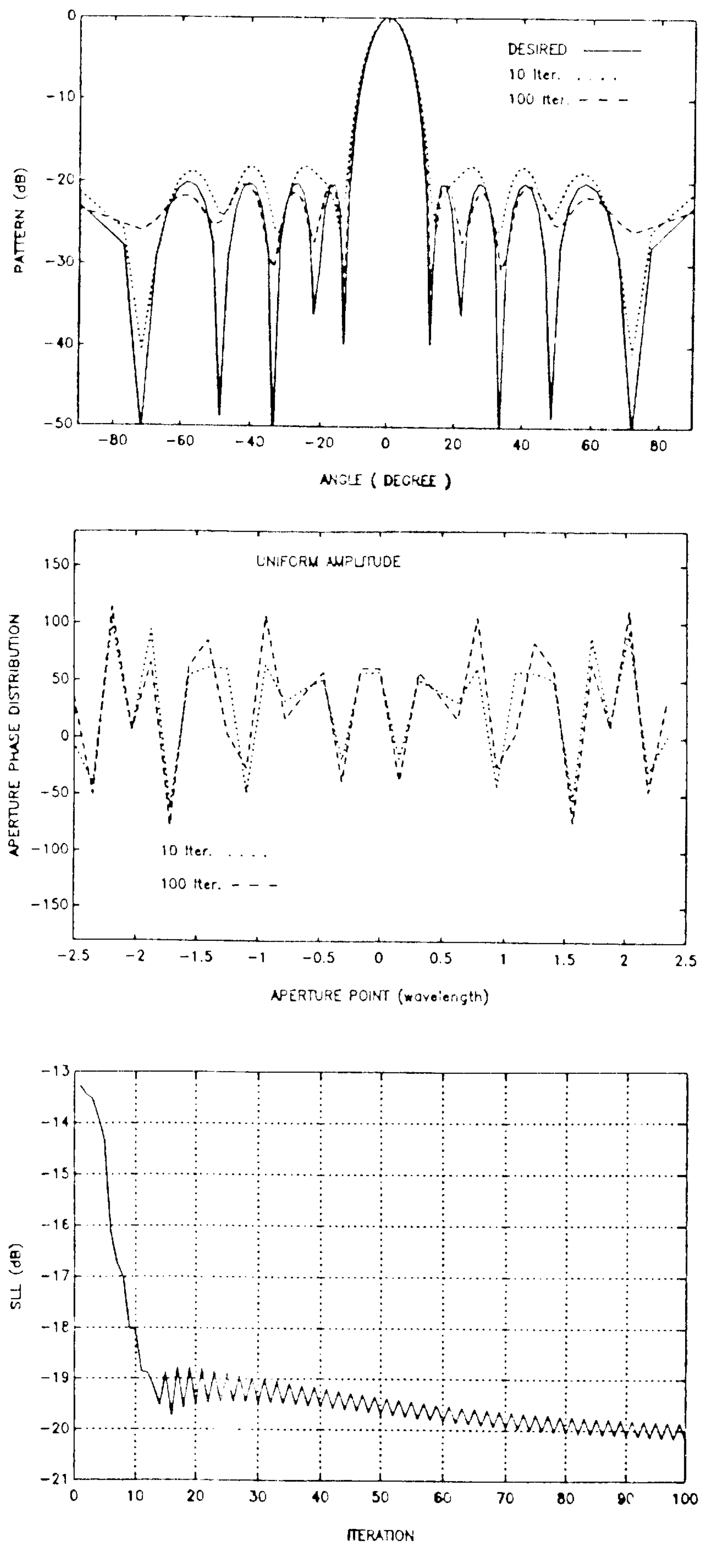


Figure 6. -20 dB SLL Chebychev pattern synthesized by uniform amplitude distribution. (a) pattern, (b) aperture phase distribution, (c) convergence.

CONCLUSION

It is possible to obtain aperture phase distribution for virtually any aperture amplitude distribution in order to approximate any desired pattern. In this paper we have demonstrated that the iterative Fourier transform phase reconstruction method is a very simple and efficient algorithm to determine such a phase distribution. A 32 element array of length 5λ for CSC beam shaping and side-lobe-level control of uniform amplitude distribution, were chosen as examples to demonstrate some numerical results. Very small ripples in CSC beam shaping (less than 1 dB) were achieved after less than 100 iterations. In side-lobe-level control, SLL's of down to -30 dB and an optimum equiripple pattern of -20 dB SLL were achieved for uniform amplitude distribution.

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