

# REINFORCED CONCRETE COLUMNS IN TORSION

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**Abstract** ACI does not suggest any regulations for columns under torsion. However, by all means, the number of such columns used in steel and concrete structures are abundant. In this paper, using the results of laboratory tests on 35 square and rectangular columns with intermediate slender ratios, relations for determination of torsional strengths of reinforced concrete under axial compression have been developed. Compatibility of this relation with laboratory results have been examined.

**Key Words** Reinforced Concrete, Columns, Torsion

**چکیده** آئین نامه ACI در مورد ستونهای تحت پیچش ضوابطی ارائه نکرده است ولی چنین ستونهایی در سازه های بتن فولادی کم نیستند. در این مقاله با تکیه بر نتایج آزمایشگاهی ۳۵ ستون با مقطع مربع و مستطیل شکل و با لاغریهای متوسط رابطه ای جهت تعیین استحکام پیچشی ستونهای بتن آرمه تحت فشار محوری با مقاطع مربع یا مستطیل شکل ارائه شده و سازگاری پاسخ رابطه فوق با نتایج آزمایشگاهی سنجیده شده است.

## INTRODUCTION

Columns supporting beams or slabs in reinforced concrete buildings carry not only axial load but also significant torsion.

ACI [1] makes no provisions for the design of axially loaded members subjected to torsion. It only underlines the need of further tests for a better understanding of these elements.

Limited tests have been reported on rectangular short columns ( $KL/r < 22$ ) [2,3,4] none of which considers the influence of height and percentage of longitudinal steel on load carrying capacity of the columns subjected to combined axial load and torsion.

The test results of 35 square and rectangular intermediate columns ( $KL/r=32$ ) subjected to various combinations of axial load and torsion have been studied in this article and an equation is proposed to determine the ultimate load carrying capacity of (intermediate) columns subjected to axial load and torsion.

## EXISTING EQUATIONS

- 1- A. Bishara and J. C. Peir [2] have reported the experimental data obtained from tests of 25 rectangular short columns tested with different load - torque ratios. Figure 1 illustrates the dimensions of column specimens.

Bishara and Pier [2] have recommended the following expression for ultimate torsional strength:

$$T = \{ [0.656f'_c x^2 (y-x/3) / (\sqrt{f'_c} + 1.313) + (0.66m (f_{ly} / f_{sy}) (x_2 y_2 / x_1 y_1) + 0.33 y_1 / x_1) x_1 y_1 A_s f_{sy} / s] \cdot (\sqrt{1 + 12f_c / f'_c}) \} \quad (1)$$

in which

$m$  = ratio of the volume percentage of longitudinal and stirrup reinforcement =  $A_{st} s / (2(x_1 + y_1) A_s)$

$A_{st}$  = total area of longitudinal reinforcement

(For definition of other symbols see Figure 1 and Nomenclature)

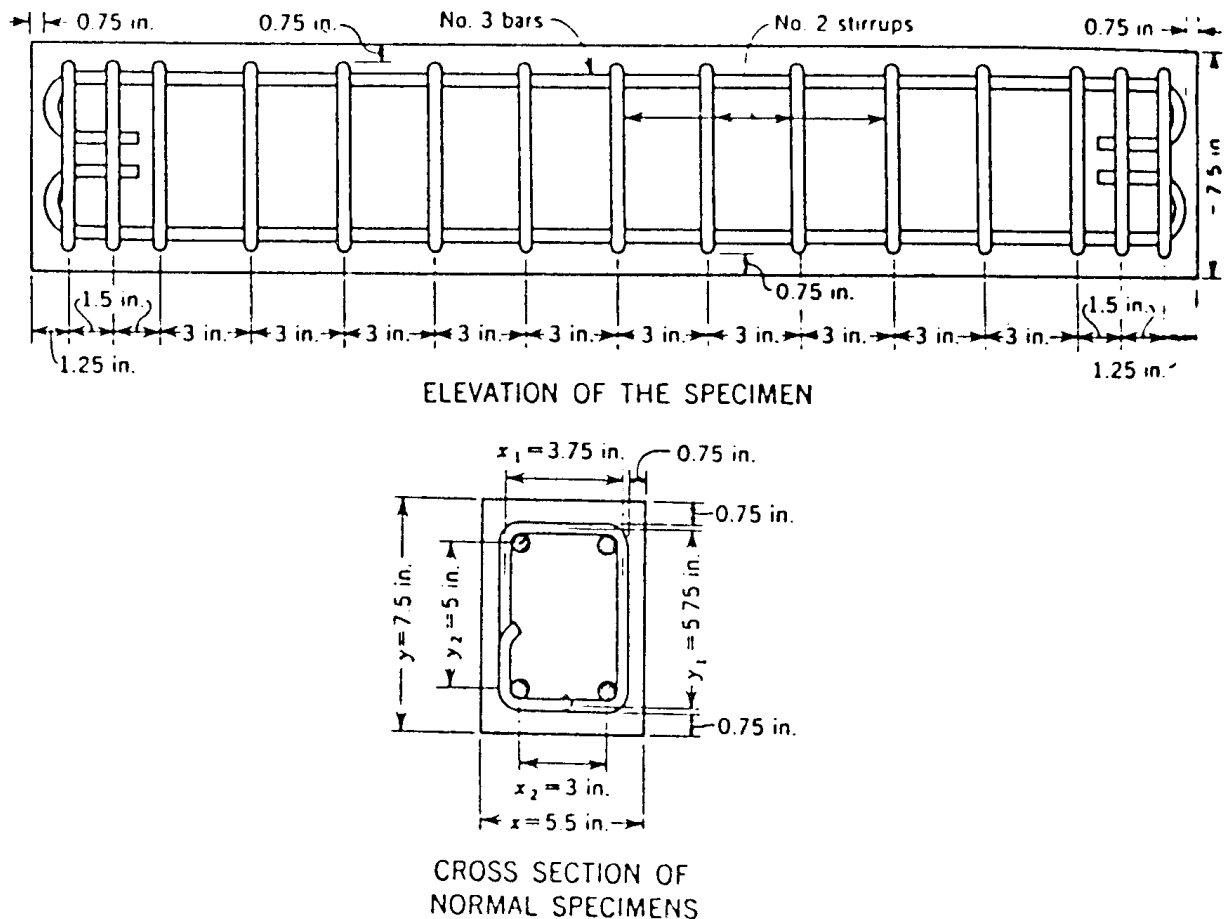
2- Thomas T. C. Tsu and E. L. Kemp [4] have proposed the following expression for ultimate torsional strength of prestressed members:

$$T_u = 0.21x^2 y \sqrt{f'_c} (2.5 \sqrt{1 + 10f_c / f'_c} - 1.5) + (0.66 + 0.33 y_1 / x_1) x_1 y_1 (A_s f_{sy} / s) \quad (2)$$

(For definition of symbols see Figure 1 and Nomenclature)

3- Ganpat S. Pandit and Manak B. Mawal [3] have derived the following equation for ultimate torsional strength of columns:

$$T_u = (1/3) x_2 y (f'_c / 10) (\sqrt{1 + 10f_c / f'_c}) R_1 R_2 (x_1 y_1 A_s f_{sy} / s) \quad (3)$$



-longitudinal reinforcement, four ACI No3 deformed bars ( $D=9.5$  mm) with  $f_{ly} = 47000$  psi ( $=3240$  bar)  
 -stirrups, ACI No2 ( $D=6.35$  mm) placed at 3 in ( $= 76.2$  mm) on centres

**Figure 1.** Details of Bishara and Peir specimens

in which

$R_1 = 1 - s/y$  and  $R_2 = (2p'_1 f_{ly}) / (p_s f_{sy})$  are the reduction factors with upper limit equal to one  
 $p'_1$  = percentage of longitudinal steel at the top or the bottom, whichever is smaller

$p_s$  = percentage of stirrup reinforcement

(For definition of other symbols see Figure 1 and Nomenclature)

## EXPERIMENTAL TESTS

### Test Set-Up

A total of 35 intermediate columns were tested for various combination of axial loads and torsions.

The first set consisted of 17 reinforced concrete rectangular columns (called R groups) in the dimensions of (5"×8"×48") 12.7×20.3×122 cm and a head of (12"×12"×12") 30.5×30.5×30.5 cm on the other side. The head was used to apply torque. These specimens were reinforced longitudinally with 6φ15.9 mm (ACI No 5) bars. Lateral reinforcement comprised φ = 9.5 mm (ACI No3) bars in the form of closed ties placed at 12.7 cm (= 5") center to center except at the junction of head and column. Details of the specimen are shown in Figure 2.

The second set (called S group) comprised 18 intermediate square columns of 12.7×12.7×122 cm (5"×5"×48") with a head of 30.5×30.5×30.5 cm (12"×12"×12") on the other side to apply torque.

The columns were reinforced longitudinally by 4φ15.9 mm (ACI No5) bars and laterally in the form of closed ties by φ = 9.5 mm (ACI No3) bars placed at 12.7 cm (= 5") center to center except at the junction of the head and the column.

Details of the specimen are shown in Figure 3.

Concrete from ready mix concrete plant was used in the experiment and the steel reinforcement consisted of ACI grade 40 steel bar ( $f \approx 2760$ ). The percentage of the total longitudinal steel is 4.66% for rectangular columns and 4.96% for square columns. The percentage of steel  $\rho = A_{st} / A_g$  is higher than the 1% minimum and less than the 8% maximum required by the ACI code 3 [1].

The axial load was applied at the top of the specimen by means of hydraulic ram fixed to the assembly frame, and the twisting moment was applied by means of two equal and opposite horizontal forces which would eliminate the effect of shear on the column.

The first set of columns were tested by applying only axial load (P). A fraction of the ultimate load was first applied for the succeeding sets, maintaining the axial load constant, and the twisting moment was applied until the column failed.

The last set was subjected to only twisting moment (T) until it failed failure.

### Test Results

The test results on rectangular and square columns are presented in Table 1. The values recorded for compressive strength is an average of three 6"×12" (ACI standard) concrete cylinders.

## PROPOSED EQUATION FOR ULTIMATE TORQUE

As proposed by References 3 and 4 the ultimate torque of a reinforced concrete column may be expressed by:

$$T_u = T_{uc} + T_{us} \quad (4)$$

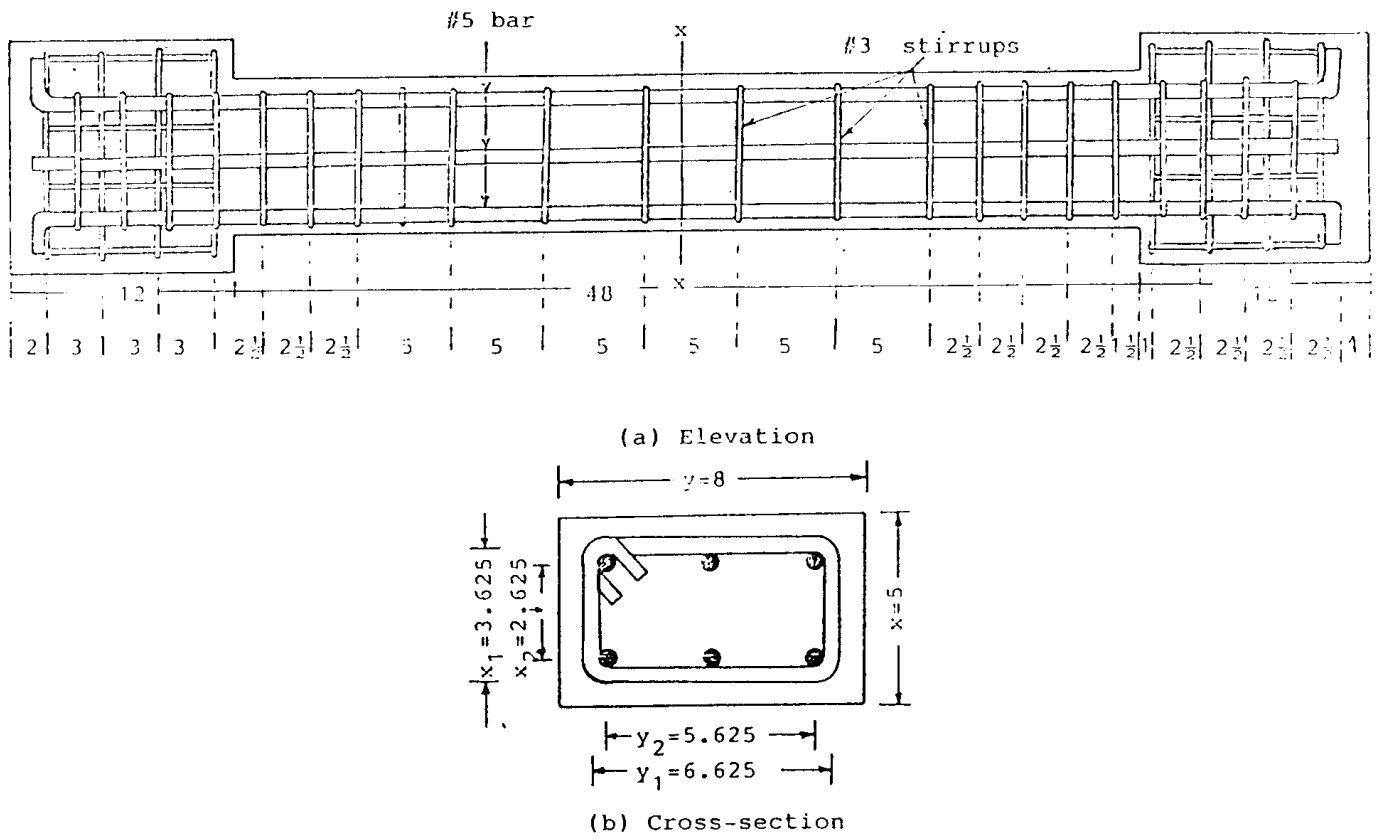


Figure 2. Reinforcement details of rectangular column (all dimensions are in inches)

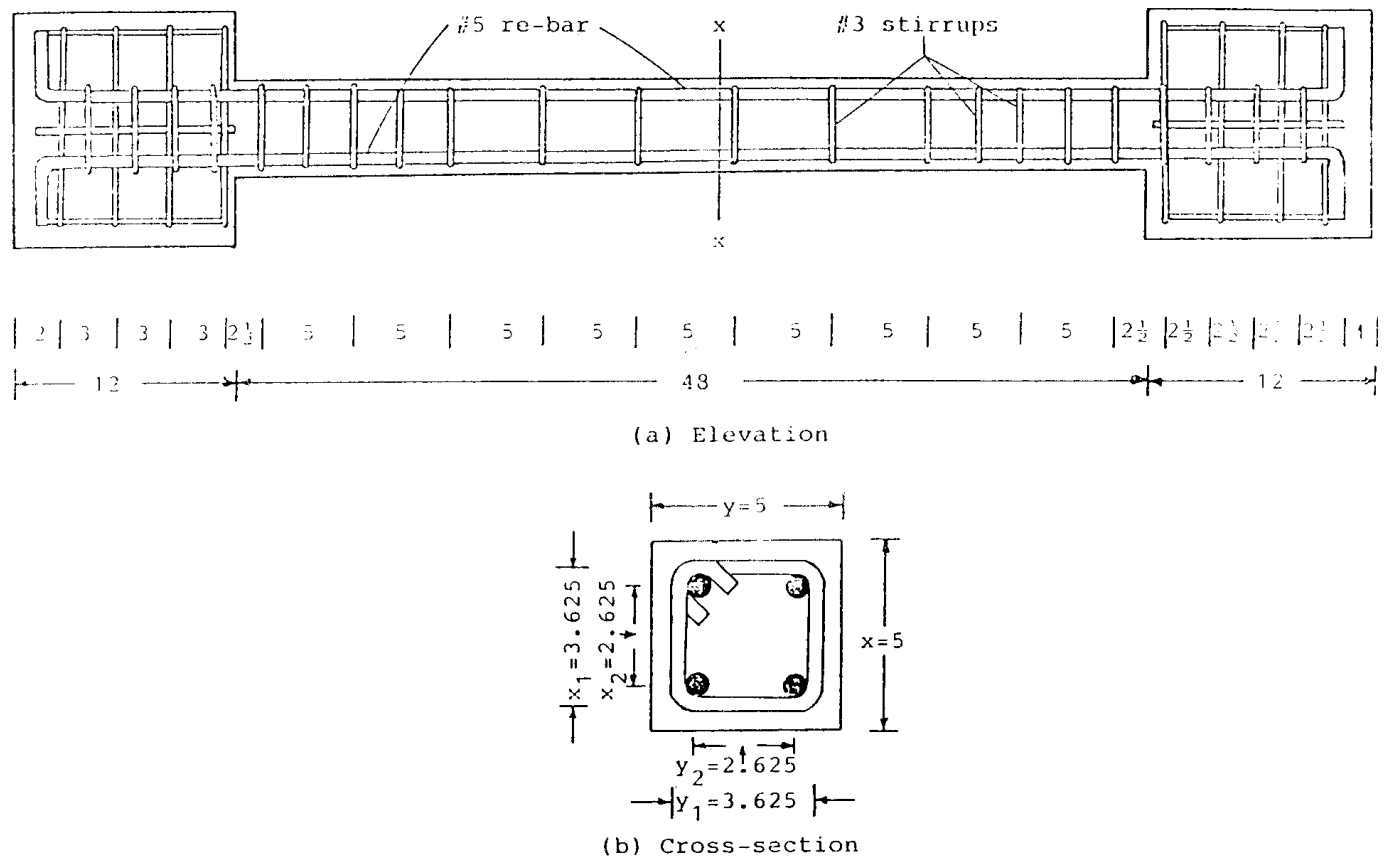


Figure 3. Reinforcement details of square column (all dimensions are in inches)

**TABLE 1. Test Results**

Rectangular Columns				Square Columns			
Sp. No.	Axl Load P kN	Torque T kNcm	tw agl $\theta \times 10^{-3}$	Sp. No.	Axl Load P kN	Torque T kNcm	tw agl $\theta \times 10^{-3}$
R-1-1	0	940	-	S-1-1	0	470	-
R-1-2	0	980	-	S-1-2	0	366	-
R-1-3	0	1490	3.7	S-1-3	0	350	-
R-2-1	885	-	-	S-2-1	533	0	-
R-2-2	885	1810	-	S-2-2	747	0	-
R-2-3	885	2010	1.6	S-2-3	578	0	-
R-3-1	733	2070	-	S-2-4	378	0	-
R-3-2	733	2060	1.2	S-2-5	600	0	-
R-4-1	533	2110	2.	S-3-1	138	717	3.3
R-4-2	533	1980	2.4	S-3-2	133	890	3.1
R-5-1	356	1800	3.3	S-4-1	254	761	3.5
R-5-2	365	1970	1.8	S-4-2	240	917	2.7
R-6-1	182	1740	-	S-5-1	311	930	2.2
R-6-2	187	1920	1.9	S-5-2	320	1322	2.6
R-7-1	458	2340	3.2	S-6-1	360	975	1.5
R-7-2	445	2090	2.1	S-6-2	369	1015	3.1
				S-7-1	480	788	0.8
				S-7-2	480	1050	1.3
f = 321 bar				f = 345 bar			

Based on skew bending theory developed in Reference 5 Pandit proposes Equation 5 for the ultimate torque capacity of plain concrete of a short column:

$$T_{uc} \approx (1/3) x^2 y (f'_c / 10) \sqrt{1 + (10f_c / f'_c)} \quad (5)$$

This article suggests Equation 6 instead of Equation 5 for intermediate columns with a moderate reinforcement:

$$T_{uc} \approx I_1 x^2 y (f'_c / 10) \sqrt{1 + 10f_{ci} / f'_c} \quad (6)$$

$$I_1 (\text{empiric factor}) = 0.37 - 0.22 (f_{ci} / f'_c) + 0.09 (f_{ci} / f'_c)^2$$

$f_{ci}$  = (compressive stress of concrete in column)  
 $= P / (A_g + A_{st} (E_s / E_c - 1))$   
 (For other definitions see Figure 1 and Nomenclature)

For the contribution of reinforcement in ultimate torque capacity ( $T_{us}$ ) Equation 7 is suggested:

$$T_{us} = I_2 I_3 (x_1 y_1 A_s f_{sy} / s) \quad (7)$$

where  $I_2 = 2-s/y$  and  $I_3 = (p_l / p_s) (f_{ly} / f_{sy})$ .  $p_l$  = percentage of longitudinal steel and  $p_s$  = percentage of stirrup reinforcement as defined in

Reference 1; and  $f_{ly}$ ,  $f_{sy}$  = yield stress of longitudinal and stirrup reinforcement respectively.

It should be noted that References 3 and 4 have proposed a similar relation for the contribution of reinforcement in ultimate torque capacity of short columns, but Equation 7 correlates better with experimental results of intermediate columns.

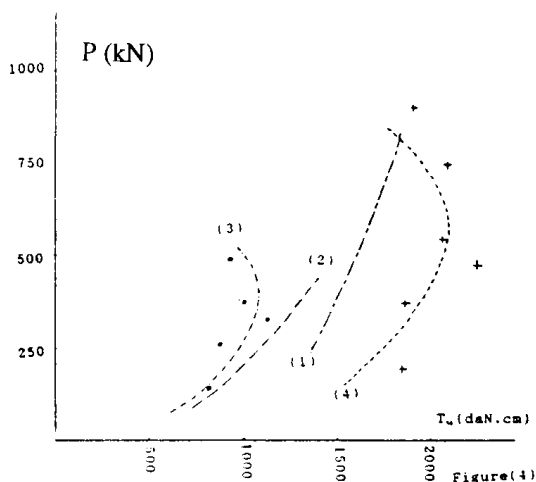
Substituting from Equations 6 and 7 into Equation 4, the ultimate torque capacity of an intermediate column with moderate reinforcement can be expressed by Equation 8:

$$T_u = I_1 x^2 y (f'_c / 10) \sqrt{1 + 10f_{ci} / f'_c} + I_2 I_3 (x_1 y_1 A_s f_{sy} / s) \quad (8)$$

### COMPARISON WITH TESTS

Figure 4 shows that Equation 8 has a very satisfactory agreement not only with test results of square columns but also with those of rectangular ones.

In this figure we have plotted the results obtained by using the two other expressions proposed in References 2 and 3.



**Figure 4.** Comparison of the theoretical and the experimental data

- (1) Equation 3 for rectangular column
- (2) Equation 1 for square column
- (3) Equation 8 for square column (article)
- (4) Equation 8 for rectangular column (article)
- + Test's average result for R group specimens
- Test's average result for S group specimens

### CONCLUSION

In this paper we have reported the test results on intermediate reinforced concrete columns subjected to torsion. Based on test results and theoretical studies available in the literature, this article proposes an equation considering the design of intermediate moderately reinforced rectangular on square concrete column subjected to axial and torsional moment.

Recognizing the paucity of torsion tests on axially loaded concrete members, the writers emphasizes the need for further tests for better understanding of torsioned columns with different dimensions or percentage of longitudinal and lateral reinforcements. Equation 8 can only lead to satisfactory results if the column has not an unusual character (moderate reinforcement and  $y < 2x$  is needed).

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### NOMENCLATURE

- $A_g$  = cross-sectional area,  $A_g = xy$  (cm<sup>2</sup>)
- $A_s$  = cross-sectional area of one stirrup leg (cm<sup>2</sup>)
- $A_{st}$  = total cross-sectional area of symmetrically placed longitudinal bars (cm<sup>2</sup>)
- $E_s$  = modulus of elasticity of steel reinforcement (daN/cm<sup>2</sup>)
- $E_c$  = modulus of elasticity of concrete (daN/cm<sup>2</sup>)
- $f_c$  = compressive stress due to axial load,  $f_c = P/A$  (daN/cm<sup>2</sup>)
- $f'_c$  = cylinder compressive strength of concrete (daN/cm<sup>2</sup>)

$f_{ci}$  = compressive stress of concrete due to axial load,  
 $f_{ci} = P / (A_g + (E_s / E_c - 1) A_{st})$   
 $f_{ly}$  = yield strength of longitudinal bars (daN/cm<sup>2</sup>)  
 $f_{sy}$  = yield strength of stirrups (daN/cm<sup>2</sup>)  
 $I_1$  = empiric factor  
 $I_1 = 0.37 - 0.22 (f_{ci} / f'_c) + 0.09 (f_{ci} / f'_c)^2$   
 $I_2$  =  $2 - s/y$   
 $I_3$  =  $p_l f_{ly} / (p_s f_{sy})$   
 $m$  = ratio of volume of longitudinal bars to volume of stirrups  
 $m = A_{st} s / (2A_s (x_1 + y_1))$   
 $P$  = axial load on column (daN)  
 $P_l$  = percentage of total longitudinal steel  
 $P'_l$  = percentage of longitudinal steel at top or bottom whichever is smaller =  $p_l / 2$  for symmetrical sections  
 $P_s$  = percentage of stirrup reinforcement  
 $R_1$  =  $1 - s/y$   
 $R_2$  =  $2p'_l f_{ly} / (p_s f_{sy})$   
 $s$  = spacing of stirrups in direction parallel to longitudinal axis of member (cm)  
 $T_u$  = ultimate torque capacity under combined axial load and torque (daN.cm)  
 $T_{uc}$  = ultimate torque resisted by concrete (daN.cm)  
 $T_{us}$  = ultimate torque resisted by reinforcement (daN.cm)  
 $x$  = smaller overall dimension of rectangular cross section of column (cm)  
 $y$  = larger overall dimension of rectangular cross section of column (cm)  
 $x_1$  = smaller center-to-center dimension of closed rectangular stirrup (cm)

$y_1$  = larger center-to-center dimension of closed rectangular stirrup (cm)  
 $x_2$  = center-to-center distance between longitudinal corner bars in shorter direction of cross section (cm)  
 $y_2$  = center-to-center distance between longitudinal corner bars in longer direction of cross section (cm)

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