A STUDY OF THE DESIGN OF HORIZONTAL AXIS WIND TURBINE

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Abstract In this using paper a method is presented for the aerodynamic and structural analysis of a horizontal axis wind turbine using simplified methods. In the first part of the program the optimum rotor configuration for twist and chord is determined using the momentum and blade element theories for a rotor without coning or tilting and assuming zero drag average wind velocity. Then coning angle and mass distribution are determined from an iterative procedure considering the effect of drag. The equations have further been extended to include the influence of tilting angles and several wind conditions, such as wind gradient, tower shadow and wind shifts. Numerical results obtained with the present method agree well experimental and other numerical data.

Key Words Wind Turbine, Drag, Aerodynamics, Turbine Design

چکیده دراین مقاله روش ساده ای برای تحلیل اثرودینامیکی و سازه ای توربین بادی با محور افقی ارائه شده است. درقسمت اول برنامه، با استفاده از نظریه ه ی مومنتم و اجزاء پرهٔ یک روتور بدون ایجاد حالت مخروطی یا خم شدن و با فرض کشش صفر و درسرعت متوسط باد، آرایش بهینهٔ روتور ازنظر پیچش و وتر (Chord) تعیین شده است. سپس با استفاده از روش تکرار با درنظر گرفتن اثر کشش زاویهٔ مخروطی شدن و توزیع جرم معلوم گردیده است. این رابطه ها برای دربرگرفتن اثر زاویه های خم شدن و شرایط مختلف باد مانند گرادیان، جابه جایی و سایهٔ برج بسط داده شده اند. نتایج عددی حاصل از روش حاضر با داده های تجربی و روشهای عددی دیگر تطابق خوبی نشان می دهد.

INTRODUCTION

This paper present a method for calculating the overall design, performance and the dynamic response of a horizontal axis wind turbine. Blade design and performance are calculated based on a combination of momentum theory and blade element theory. The influence of wind shear, tower shadow and wind shift on the cyclic aerodymamic loads and on the wind turbine performance has been extensively investigated. In order to balance aerodynamic and centrifugal forces, coning angle and mass distribution are determined by an iterative procedure. Stresses in the blade due to the aerodynamic, gravity and centrifugal forces are also calculated and compared with limiting allowable stresses of the blade materials. The bending natural frequencies of the rotating blade are estimated using Lagrange equations and these frequencies are compared to the exciting frequencies to determine the resonance regions. In the present analysis, the performance of three types of blade shapes have been considered: optimum-chord optimum-twist, linear-chord linear-twist and linear-chord zero-twist.

VELOCITY COMPONENTS AT THE BLADE

For calculating the forces and moments on a blade, the velocity components of the air flow relative to any point on the blade and also the induced velocity components have to be known. Several reference frames are considered to include the effect of wind shift, tilt, azimuth and coning. This has been discussed in Appendix A. In the fixed reference frame S_0 the wind velocity can be expressed as [2]

$$\overline{V}_{s0} = V_{oc} \begin{vmatrix} \cos\gamma \\ \sin\gamma \\ 0 \end{vmatrix} \tag{1}$$

where $\gamma = 90° - \gamma^*$
and γ^* = Yaw angle.

Expressed in the local coordinate S_3 attached to a point on the blade, the wind velocity components can be described as [2]

$$\overline{V}_{s3} = [K_\beta]^T [K_\theta]^T [K_T]^T \overline{V}_{s0} \quad (2)$$

The above equation can be written as

$$\overline{V}_{s3} = [K_\beta]^T [K_\theta]^T [K_T]^T \begin{vmatrix} \text{Cos}\gamma \\ \text{Sin}\gamma \\ 0 \end{vmatrix} V_{oc} \quad (3)$$

This leads to the following expression:

$$\overline{V}_{s3} = \begin{vmatrix} \text{Cos}\gamma\,\text{Cos}\theta_k \\ + \text{Sin}\gamma\,\text{Sin}\theta_k\,\text{Sin}\alpha_T \\ \text{Sin}\gamma\,\text{Cos}\alpha_T\,\text{Cos}\beta + \text{Cos}\gamma\,\text{Sin}\beta\,\text{Sin}\theta_k \\ - \text{Sin}\gamma\,\text{Sin}\alpha_T\,\text{Cos}\theta_k\,\text{Sin}\beta \\ -\text{Sin}\gamma\,\text{Sin}\beta\,\text{Cos}\alpha_T + \text{Cos}\gamma\,\text{Sin}\theta_k\,\text{Cos}\beta \\ - \text{Sin}\gamma\,\text{Cos}\theta_k\,\text{Sin}\alpha_T\,\text{Cos}\beta \end{vmatrix} V_{oc} \quad (4)$$

The rotational motion of the blade will add velocity components Ωr Cosβ in the total velocity vector. Introducing the induced velocity in S_3 co-ordinate system as [2]

$$\overline{V}_{s3} = \begin{vmatrix} a'\Omega r\,\text{Cos}\beta \\ V_{oc}\,a\,\text{Cos}\beta\,\text{Cos}\,\alpha_T\,\text{Sin}\,\gamma \\ V_{oc}\,a\,\text{Sin}\,\beta\,\text{Cos}\,\alpha_T\,\text{Sin}\,\gamma \end{vmatrix} \quad (5)$$

The components of local velocity W can be expressed in the local reference system as [2]

$$W_x = V_{oc_o}\,\text{Cos}\gamma\,\text{Cos}\theta_k + V_{oc_o}\,\text{Sin}\gamma\,\text{Sin}\theta_k\,\text{Sin}\alpha_T - \Omega r\,\text{Cos}\beta\ (1+a') \quad (6)$$

$$W_y = V_{oc_o}\ \text{Sin}\gamma\,\text{Cos}\alpha\,\text{Cos}\beta\,(1\text{-}a) + \text{Sin}\beta\ \text{Sin}\theta_k\,\text{Cos}\gamma - \text{Sin}\gamma\,\text{Sin}\beta\ \text{Sin}\alpha_T\,\text{Cos}\theta_k \quad (7)$$

The local angle of attack α is defined as

$$\alpha = \phi - \beta_T = \tan^{-1}\frac{W_y}{W_x} - \beta_T \quad (8)$$

the velocity components acting on a blade element rotating at a radius r are shown in Figure 1. By equating the torque and thrust from momentum theory and blade element theory the expressions for interference factors a and a' are obtained.

$$a(1\text{-}aF) = \frac{1}{8}\sigma\frac{W^2\,\text{Cos}\phi\,C_L}{\text{Cos}\beta\,\text{Cos}^2\alpha_T\,\text{Sin}^2\gamma\,V^2_{oc_o}\,F} \quad (9)$$

$$a'(1\text{-}aF) = \frac{1}{8}\sigma\frac{W^2\,\text{Sin}\phi\,C_L}{\text{Cos}\alpha_T\,\text{Sin}\gamma\,\text{Cos}^2\beta\,V_{oc_o}\,F} \quad (10)$$

The drag terms have been omitted in Equations 9 and 10, on the basis that the retarded air due to drag is confined to thin helical sheets in the wake and have little effects on the induced flows [1].

FORCES AND MOMENTS

The major forces which dictate the performance of a horizontal axis wind turbine are the aerodynamic, gravity, centrifugal and gyroscopic forces which act on the rotor and the tower. In the present analysis, gyroscopic force has been neglected because of its smaller value in comparison to other forces for rigid tower.

All the forces in the local S_3 co-ordinate system can be

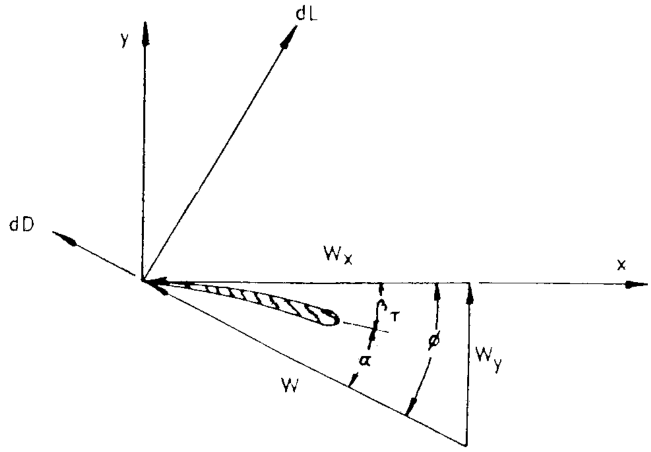

Figure 1. Velocity diagram for a rotor blade element

expressed as [2]

$$\bar{F}_{S_3} = \begin{vmatrix} F_{x3} \\ F_{y3} \\ F_{z3} \end{vmatrix} \tag{11}$$

Considering the non-rotating system S_1 attached to the hub, the equation of forces becomes

$$\bar{F}_{S_1} = [K_\theta]\,[K_\beta][\bar{F}_{S_3}] \tag{12}$$

The following equation yields from Equation (12)

$$\bar{F}_{S_1} = \begin{vmatrix} F_{x3}\,Cos\theta + F_{y3}\,Sin\,\theta\,Sin\,\beta + F_{z3}\,Sin\,\theta\,Cos\beta \\ F_{y3}\,Cos\beta - F_{z3}\,Sin\,\beta \\ -F_{x3}\,Sin\,\theta + F_{y3}\,Sin\,\beta\,Cos\theta + F_{z3}\,Cos\theta\,Cos\beta \end{vmatrix} \tag{13}$$

At the tower top forces can be written as [2]

$$\bar{F}_{S_0} = [K_T][K_\theta][K_\beta][\bar{F}_{S_3}] \tag{14}$$

Equation 14 can be expressed as

$$\bar{F}_{S_0} = \begin{vmatrix} F_{x3}\,Cos\theta + F_{y3}\,Sin\theta\,Sin\beta + F_{z3}\,Sin\theta\,Cos\beta \\ F_{x3}\,Sin\,\alpha_T\,Sin\theta + F_{y3}\,(Cos\beta\,Cos\alpha_T - Sin\alpha_T\,Sin\beta\,Cos\theta) \\ - F_{z3}\,(Cos\alpha_T\,Sin\beta + Sin\,\alpha_T\,Cos\beta\,Cos\theta) \\ -F_{x3}\,Cos\alpha_T Sin\theta + F_{y3}\,(Cos\beta\,Sin\alpha_T + Cos\alpha_T\,Sin\beta\,Cos\theta) \\ + F_{z3}\,(Cos\alpha_T\,Cos\beta\,Cos\theta - Sin\beta\,Sin\alpha_T) \end{vmatrix}$$

(15)

At the blade supporting point the expression for moment for a differential element can be written as [2]

$$\bar{dM}_{S_3} = \bar{dF}_{S_3} \times \bar{r}_{S_3} \tag{16}$$

This equation can be expressed as

$$\bar{dM}_{S_3} = \begin{vmatrix} i_3 & j_3 & k_3 \\ dF_{x3} & dF_{y3} & dF_{z3} \\ 0 & 0 & r_3 \end{vmatrix} \tag{17}$$

where r_3 is the distance from the blade root along Z_3 direction.

Equation 17 can be reduced as

$$\begin{vmatrix} dM_{x3} \\ dM_{y3} \\ dM_{z3} \end{vmatrix} = \begin{vmatrix} r_3\,dF_{S_3} \\ -r_3\,dF_{S_3} \\ 0 \end{vmatrix} \tag{18}$$

The equation for total moments in different directions for one blade can be written as follows:

$$\text{Flapwise moment, } M_{x3} = \int_0^R r_3\,dF_{y3} \tag{19}$$

$$\text{Edgewise moment, } M_{y3} = \int_0^R r_3\,dF_{x3} \tag{20}$$

At the tower top coresponding to inertial system S_o, the moment can be expressed as [2]

$$\bar{M}_{S_o} = \bar{F}_{S_o} \times \bar{r}_{S_o} \tag{21}$$

The following equation yields from Equation 21

$$\begin{vmatrix} M_{x_o} \\ M_{y_o} \\ M_{z_o} \end{vmatrix} = \begin{vmatrix} i_o & j_o & k_o \\ F_{x_o} & F_{y_o} & F_{z_o} \\ X_o & Y_o & Z_o \end{vmatrix} \tag{22}$$

where X_o, Y_o and Z_o are the moment arms in the respective coordinate system. Equation (22) can be expressed as

$$\begin{vmatrix} M_{x_o} \\ M_{y_o} \\ M_{z_o} \end{vmatrix} = \begin{vmatrix} Z_o F_{y_o} - Y_o F_{z_o} \\ X_o F_{z_o} - Z_o F_{x_o} \\ Y_o F_{x_o} - X_o F_{y_o} \end{vmatrix} \tag{23}$$

DETERMINATION OF CONING ANGLE AND MASS DISTRIBUTION

Mass distribution is calculated by balancing the centrifugal force with the aerodynamic lift force considering a certain coning angle. According to Figure 2, for a small differential element, the relation between elementary lift dL and differential mass dm can be obtained by the following equation [2]

$$\frac{dL}{dr} = \Omega^2 r \cos\beta \sin\beta \frac{dm}{dr} \tag{24}$$

It is assumed that the blade characteristics, number of blades and airfoil sections are known beforehand.

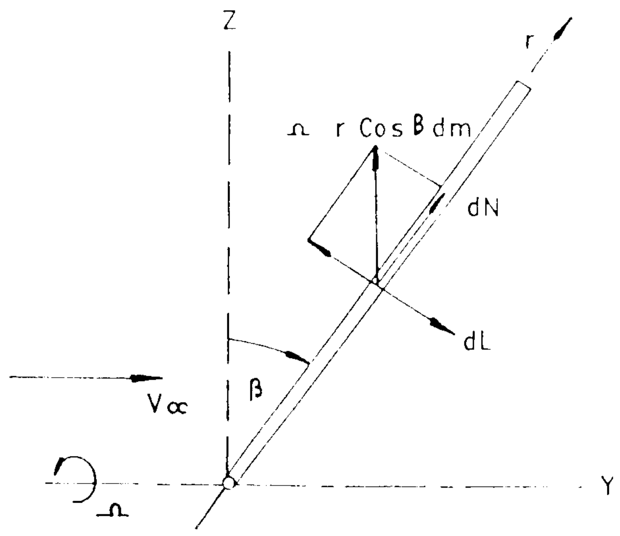

Figure 2. Lift and centrifugal forces on a blade element

BLADE NATURAL FREQUENCIES

The natural frequencies of the blade are determined considering the method of assumed modes and using Lagrange's equation. The blades are assumed to be rotating at a constant angular frequency Ω. It is also considered that the vibratory blade does not dissipate energy, i.e. its maximum potential energy equals maximum kinetic energy.

Considering the lagging plane the equation of natural frequency is give by [2]

$$\sum_{j=i}^{n} [B_{ij} - (\lambda_j^2 + \Omega^2) A_{ij}] C_j = 0 \tag{25}$$

where λ_j = ratio of the blade natural frequency to the shaft rotational frequency for the jth mode shape.

For the flapping plane the equation of natural frequency is given by

$$\sum_{j=i}^{n} (B_{ij} - \lambda_j^2 + A_{ij}) C_j = 0 \tag{26}$$

In the present analysis, the following four functions [2] are assumed to calculate the mode shape and the natural frequencies of the blade:

$$\gamma_1 (x) = x^4 - 4x^3 + 6x^2 \tag{27}$$

$$\gamma_2 (x) = 3x^5 - 10x^4 + 10x^3 \tag{28}$$

$$\gamma_3 (x) = 2x^6 - 6x^5 + 5x^4 \tag{29}$$

$$\gamma_4 (x) = x^{10} - 2.5x^9 + 1.607143\, x^2 \tag{30}$$

The accuracy of the mode shapes and frequencies improves with the number of functions chosen.

RESULTS AND DISCUSSION

The results are presented for a two-bladed 47 m wind turbine with a tip speed ratio 8, constant rpm and variable pitch. Throughout theoretical studies NAC4 4418 airfoil and Prandtl's tip loss correction were used. The results are calculated for wind power law exponent of 1/6 and for the present analysis the hub height is considered as reference height.

Figures 3 and 4 show the variations of pitching moment and yawing moment for two blade. At zero coning angle wind shear produces a periodic wind load with a frequency of twice per revolution. The increase of coning angle reduces the effective radius of the blade which ultimately reduces the pitching moment and yawing moment. Figure 5 and 6 show the variations of power and thrust with azimuth at different tilt angles. When the tilt angle is zero

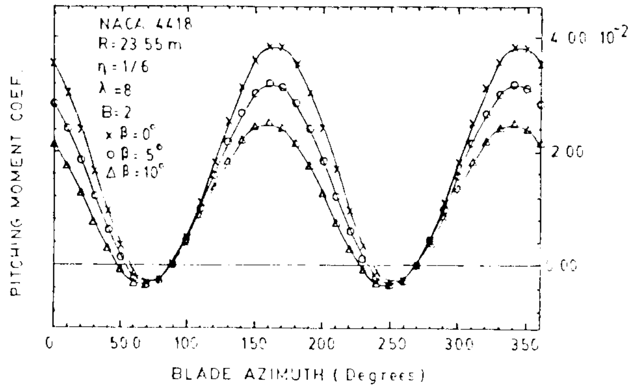

Figure 3. Variation of pitching moment during one revolution at different coning angles

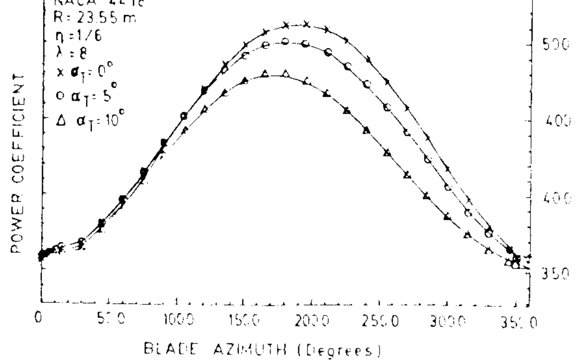

Figure 5. Variation of power coefficient with angular position at different tilt angles

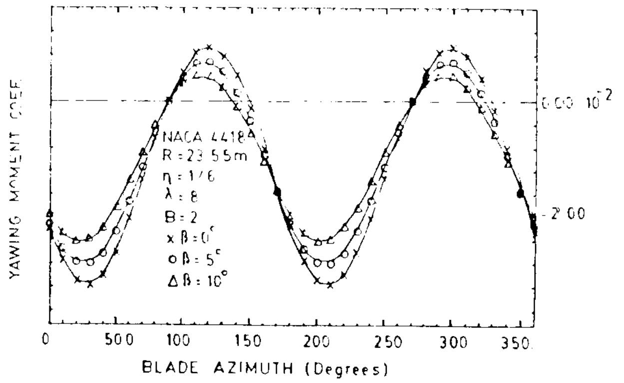

Figure 4. Variation of yawing moment during one revolution at different coning angles

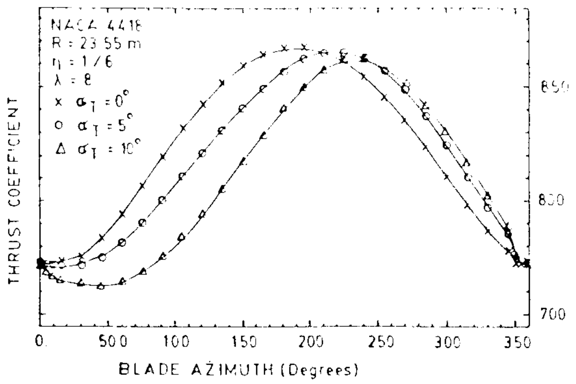

Figure 6. Variation of thrust coefficient with angular position at different tilt angles

the variation will be only due to wind shear. When the blade becomes in a vertical position the resultant wind velocity and the angle of attack will increase. Due to the variation in angle of attack normal and tangential forces will also change with azimuth. As the tilt angel increases the maximum amplitude will be phase shifted as the blade will be in a favourable angle of attack at the shifted position.

The aerodynamic forces on a blade will vary during a revolution in the case where the rotor axis is not parallel to the wind direction even though wind speed is constant. This results from changes in both magnitude and direction of the resulting local wind speed for the profile, which alters with the varying moment of the blade with and against the wind direction. The power and thrust coefficients produced by the wind turbine at various yaw angles are presented in Figures 7 and 8. It can be concluded that the rotor can be yawed into and out of the wind either to maintain power level as wind varies or to unload the rotor for shutdown. The changes in both wind speed and direction give rise to change in blade performance with azimuth. In Figur 9, the percentage decrease of power due to wind shear and tower shadow is presented. For a wide range of tip speed ratio the power deficit remains almost constant. However, power deficit only due to wind shear is very small. About 7% decrease of power may take place due to wind shear and tower shadow.

The cantilever blade frequency from 0 to 50 rpm is presented in Figure 10 of MOD-0[3] wind turbine. The two blades of MOD-0 configuration are designed to produce 133 kW of shaft power, the reuslting electric power being 100kW. The calculated values of blade frequencies are for the fully-coupled flapping and lead-lag bendings. But in the present analysis, flapping and lead-lag bendings are considered separately for simplicity. There are some difference of results for higher modes. The accuracy of the results depends on the number of functions chosen for assumed modes. The measured values are for non-rotating conditions only. However, the results of the present method are in good agreement with the given results.

For the three blade configurations considered, it has been observed that a linear-chord linear-twist blade is comparable to the optimum designed blade, whilst offering a considerable reduction in manufacturing time and cost.

CONCLUSIONS

For the design of a horizontal axis wind turbine the combined influence of coning, tilting, yawing and different wind conditions such as, wind shift, wind shear and tower shadow should be considered. Successful large, reliable, low-main-tenance wind turbines must be designed with full consideration for minimizing dynamic response to aerodynamic, inertial and gravitational forces.

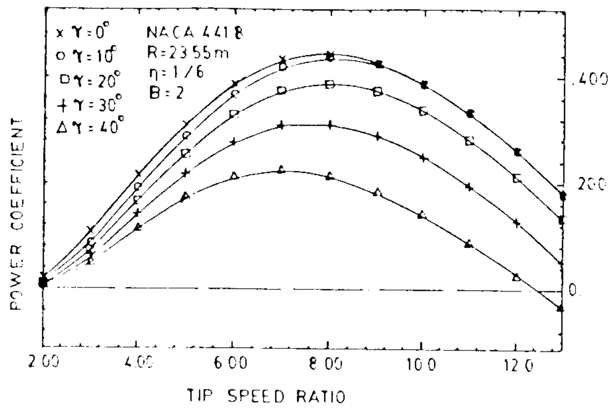

Figure 7. Effect of yaw angles on power coefficient

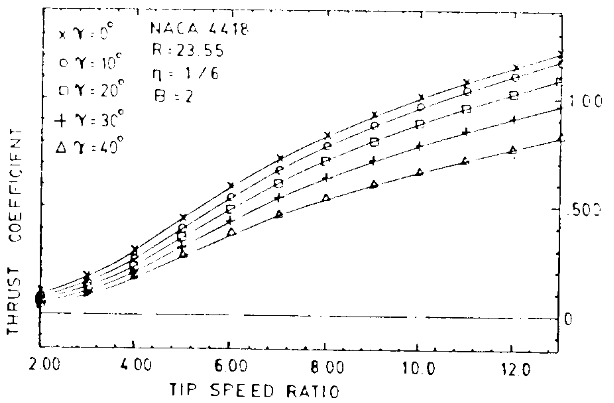

Figure 8. Effect of yaw angles on thrust coefficient

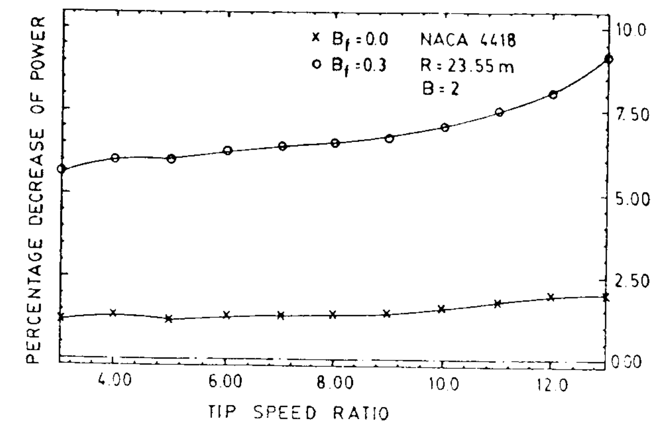

Figure 9. Percentage decrease of power due to tower shadow

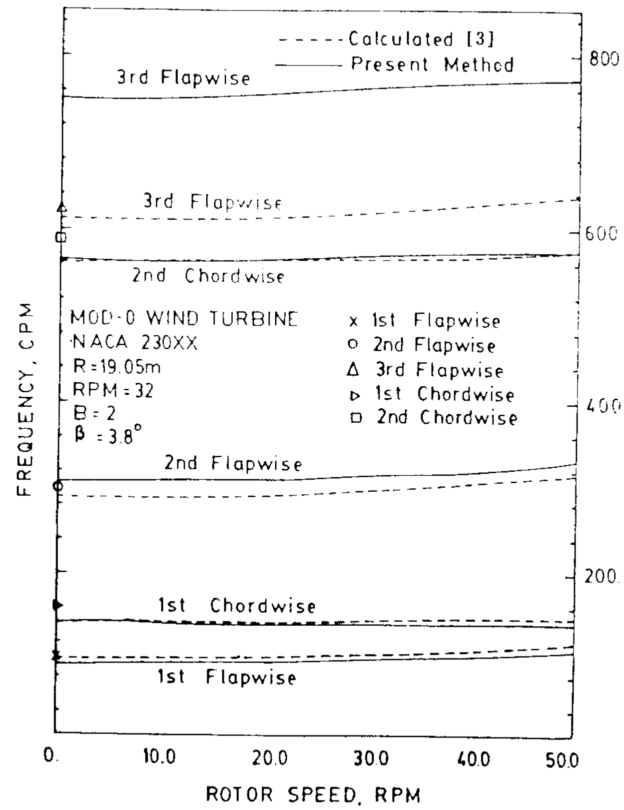

Figure 10. Cantilever blade frequency spectrum of MOD-0 wind turbine

NOMENCLATURE

a	axial interference factor
a'	tangential interference factor
E	modulus of elasticity
F	tip loss factor
F_{x_0}	force acting along x-direction in S_0 co-ordinate system
F_{x_3}	force acting along x-direction in S_3 co-ordinate system
F_{y_0}	force acting along y-direction in S_0 co-ordinate system
F_{y_3}	force acting along y-direction in S_3 co-ordinate system
F_{z_0}	force acting along z-direction in S_0 co-ordinate system
F_{z_3}	force acting along z-direction in S_3 co-ordinate system
i,j,k	units vectors along x-y-, and z-directions respectively
K_θ	transformation matrix for azimuth
K_β	transformation matrix for coning
K_T	transformation matrix for tilting
L	lift force
r	local blade radius
R	rotor radius
S_0	fixed reference co-ordinate system
S_1	co-ordinate system considering tilt angle
S_2	co-ordinate system considering blade azimuth
S_3	co-ordinate system considering blade coning
V_{oc}	undisturbed wind velocity
W	relative wind velocity
X_0	distance along x-axis in S_0 reference system
Y_0	distance along y-axis in S_0 reference system
Z_0	distance along z-axis in S_0 reference system
α_T	tilt angle
θ	azimuth angle
β	coning angle
β_T	twist angle
ρ	air density
λ	tip speed ratio
σ	solidity
Ω	angular velocity of rotor
ϕ	angle of relative wind velocity

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APPENDIX A

Local Reference Frames

To calculate the aerodynamic forces acting on the rotor several coordinate systems are introduced in the present analysis. These frames include a reference frame S_0 fixed at the top of the tower of the wind turbine with Z_0 being the vertical axis and (X_0, Y_0) forming the horizontal plane. A second non-rotating frame S_1 fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance and a rotation of tilting angle α_T around the X_0 axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle β. the relationships between the reference frames can be expressed as

$$S_1 = [K_T]\, S_0 \;,\; S_2 = [K_\theta]\, S_1 \;,\; S_3 = [K_\beta]\, S_2$$

The transformation matrices are

$$\text{for tilting angle, } [K_T] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\,\alpha_T & -\mathrm{Sin}\,\alpha_T \\ 0 & \mathrm{Sin}\,\alpha_T & \mathrm{Cos}\,\alpha_T \end{vmatrix}$$

$$\text{for azimuth angle, } [K_\theta] = \begin{vmatrix} \mathrm{Cos}\,\theta_k & 0 & \mathrm{Sin}\,\theta_k \\ 0 & 1 & 0 \\ -\mathrm{Sin}\,\theta_k & 0 & \mathrm{Cos}\,\theta_k \end{vmatrix}$$

$$\text{for coning angle, } [K_\beta] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\,\beta & -\mathrm{Sin}\,\beta \\ 0 & \mathrm{Sin}\,\beta & \mathrm{Cos}\,\beta \end{vmatrix}$$

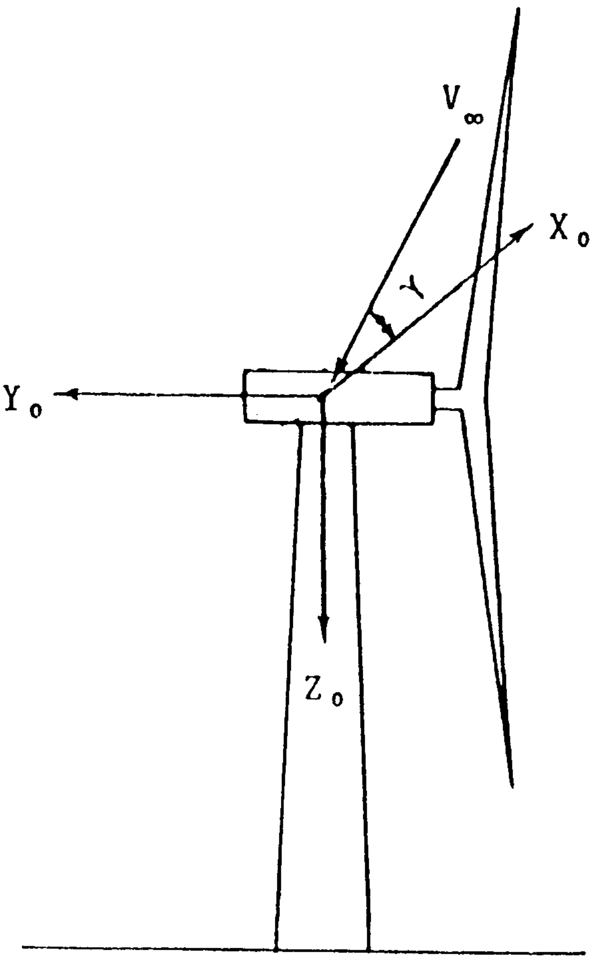

Figure A1. Reference Frame S_0 (Coordinate System S_0)

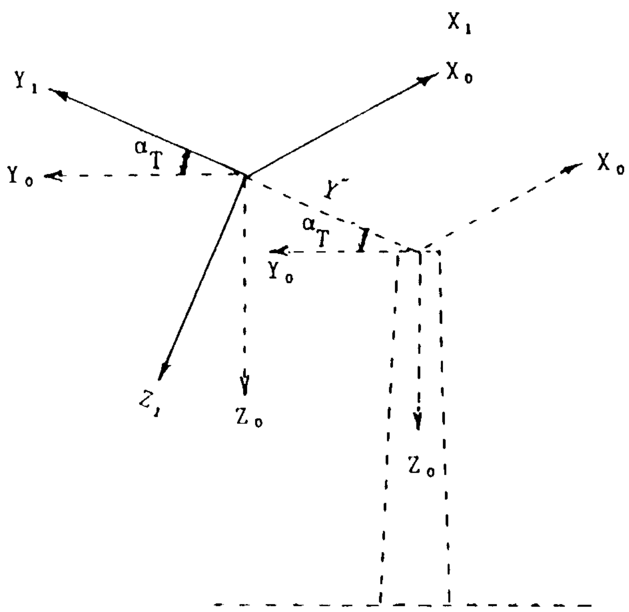

Figure A2. Translation Over Y' and Rotation About X_0 by Angle α_T (Coordinate System S_1)

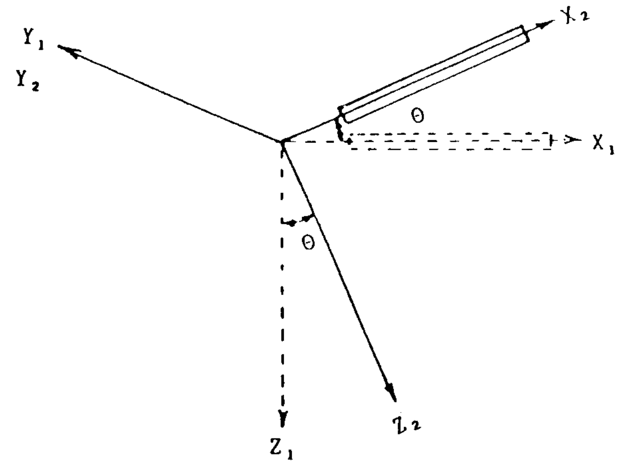

Figure A3. Rotation About Y_1 by Angle θ (Coordinate System S_2)

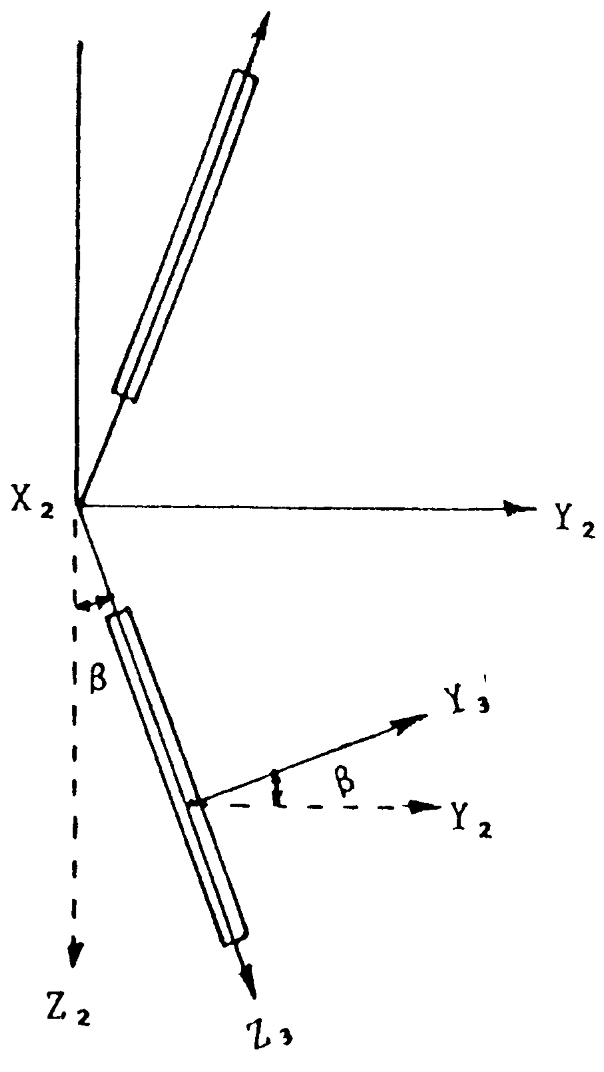

Figure A4. Rotation About X_2 by Angle β (Coordinate System S_3)