COMPUTER AIDED DESIGN FOR SINGLE-PHASE INDUCTION MOTORS BASED ON A NEW GEOMETRICAL APPROACH

J. Faiz
Department of Electrical Engineering
Faculty of Engineering, University of Tabriz
Tabriz, Iran

S. Honarparvar
Department of Electrical Engineering
Faculty of Engineering, University of Uromia
Uromia, Iran

Abstract Design of electrical motors normally involves two main stages: i) Preparation of the main dimensions and parameters. ii) Prediction of the performance. At the first stage the main dimensions of the motor, core stack L_{fe} and stator outer diameter D_o, must be chosen. A set of performance conditions such as breakdown torque, desired output and other important parameters must satisfy the international standard requirements or the specific requirements of the electric motor manufacturer. In order to meet the design objectives regarding the performance, the dimensions and the construction types are chosen, and then the cost within the constraints are imposed by manufacturing standard. The output of an electrical machine can be generally expressed as the product of L_{fe}, D_o and a coefficient C_o. at the beginning of the design process, the designer does not know a number of the required parameters, which are incorporated in C_o, but these parameters may be determined based on the performance calculations. The present paper considers the various factors affecting the choice of iron and copper volume in the motor. It shows how the geometrical parameters can alter the performance of the motor.

چکیده طراحی موتورهای الکتریکی معمولاً شامل دو مرحلهٔ اصلی است: i) محاسبهٔ ابعاد اصلی موتور و پارامترهای مربوطه. ii) پیش بینی کارآئی موتور. در مرحلهٔ اول، ابعاد اصلی موتور ازقبیل طول هسته Lfe و قطر خارجی استاتور D_o، بایستی انتخاب شوند. تعدادی از کمیت های مربوط به کارآئی موتور مانند گشتاور شکست، قدرت خروجی مطلوب و سایر پارامترهای مهم بایستی با استانداردهای بین المللی یا با احتیاجات ویژهٔ کارخانهٔ سازندهٔ موتور الکتریکی مطابقت کند. عموماً در فرآیند طراحی برای نیل به کارآئی موردنظر در موتور، ابعاد و نوع ساخت، انتخاب و سپس مسائل هزینه در چارچوب محدودیت های اعمال نیل شده توسط استاندارد کارخانهٔ سازنده، بررسی میگردد. توان خروجی یک ماشین الکتریکی را عموماً میتوان بصورت حاصلضرب L_{fe}، D_o، و ضریب C_o بیان نمود. در شروع فرایند طراحی تمامی پارامترهائی که در ضریب C_o دخالت دارد، برای طراح معلوم نبوده ولی بعداز محاسبات کارآئی قابل محاسبه میباشند. مقالهٔ حاضر عوامل مختلفی را مورد بررسی قرار میدهد که این عوامل روی حجم مس و آهن مصرفی تأثیر میگذارد. این مقاله چگونگی تأثیر ابعاد هندسی مختلف ماشین در تغییر کارآئی موتور را نشان میدهد.

INTRODUCTION

The simple and rugged construction of a squirrel cage induction motor is the major reason for the wide application of this type of motor. Design of the induction motors is commenced by determining the main dimensions of the motor. These dimensions are related with the motor rating, synchronous speed n_s and output coefficint C_o [1,2]. At the beginning of the design process, some parameters are unknown for the designer and an empirical output equation must be used. Generally, based on the well-known methods, the product $D_o^2 . L_{fe}$ depends upon the motor rating [1, 5]. This product has been modified by the authors and then the values of D_o and L_{fe} are used as the initial values forthe proposed CAD (Computer Aided Design) for the single-phase induction motor.

At the final stage of the motor design calculations, all required parameters of the motor are known for the designer. The calculated specifications of the motor such as pull-out torque, starting torque, output and efficiency are compared with the motor standard specifications such as IEC, VDE and NEMA. Even though the calculated values meet the pre-specified values and satisfy the required performance, it cannot be judged whether the iron volume of the designed motor is minimum or not.

The motor dimensions obtained from the traditional output equation are normally large and the volume of the iron is rather high. It means that in such a design the economical aspects have not been taken into account ant it is necessary to employ a suitable approach based on a new output equation in order to achieve an economical design. This new output equation, derived by Honsinger [3], is exploited in order to judge about the iron volume. The output equation is examined more deeply in the present work and it is used as a new design criterion in order to develop a CAD for a single-phase induction motor.

The importance of coefficients K_t and K_c (defined according to equations B.1. and B.2.), which depend upon the values of B_g, B_t and t, is pointed out and the range of the suitable values of these coefficients, in the motor design, is taken into consideration. For a standard punching t is constant, and only the change of B_g and B_t is possible. All important equations used in the present work, are given in Appendixes A and B.

The marked difference between the present work and that of the traditional method is the use of the new design criterion which leads to the optimum dimensions to some extent. In a further stage a normal optimization routine can easily and quickly be applied. In order to investigate how the iron volume may be reduced, a maximising function is included in the output coefficient. In the developed CAD for the single-phase induction motor the following points are emphasized:

1. The effect of the motor punching dimensions on the size of the motor.
2. The amount of the copper and steel used for an economical design.
3. The effect of the magnetic flux densities, B_t, B_c and B_g, and determination of the flux densities ratio for a better design.

STAGES OF THE DESIGN CALCULATIONS

The main calculation stages which are employed in the proposed CAD for a single-phase induction motor, are described as follows [6-8].

Step 1. The product $D_o^2L_{fe}$ is determined based on the general specifications such as output power, pole number, frequency, type of motor and duty cycle. The individual values of D_o and L_{fe} are then determined using the proper methods [1,5,10,11]. The obtained values of D_o and L_{fe} are normally larger than the expected values. Thus the existing and single-phase induction motors data employed and the output coefficient factor is modified to obtain more realistic dimensions for the designed motor.

Step 2. There are two cases for the choice of the punching. In the first case, punching design is based on the general specifications and the flux densities in the various parts of the motor. In the second case, selecting the standard punching to match the guessed D_o. For the first case, the empirical curves for B_g are used [1]. The designer chooses B_T and B_c based on the maximum allowable flux density for the lamination. In a general purpose motor the flux densities, particularly B_t, must be so chosen that the selected steel does not operate in the highly saturated region of the magnetisation characteristic. However in the short hour motor it is desirable that B_t is chosen in the saturated region of the magnetisation characteristic. Considering these guidlines the punching calculations are carried out and the slot geometry is determined. The second case is usually used in practice and all data relevant to the slots of the motor are defined by the standard. The core or tooth flux density is entered to the program based on the skill and experience of the designer.

Step 3. Since the main dimensions of the punching and the core stack have already been determined, main winding should first be designed before any attempt at the design of the auxiliary winding. When the main winding has been designed satisfactorily, it is essential to design the best possible starting winding. Many designers consider only a few possible standard windings for the starting winding. Among them the best possible design which satisfies the required specifications must be taken. The size of the wire for the windings is calculated in this stage.

Step 4. The magnetic circuit and constants of the motor equivalent circuit are calculated and finally the motor performance is predicted. The performance parameters such as torque, output power, efficiency and power factor are compared with those of the standard or manufacturer requirements.

FLOW-CHART OF THE COMPUTER DESIGN PROGRAM

In Figure 1, block No. 1 includes all initial inputs required for initiating the design process. Knowing the motor type (permanent-capacitor or start-capacitor) and duty cycle, appropriate codes are stored in the program. Block No. 2 comprises the estimation of D_o and L_{fe} and D_i can be determined from the D_i/D_o ratio, which is one of the important and well-known ratios for the designers. The ratio for various pole numbers, taken from IEC standard punchings, are tabulated in Table 1, in which the average value of D_i/D_o for each pole number is also included.

TABLE 1. Variation of D_i/D_o Based on IEC Standard ($=\lambda$).

Standard code	Two pole	Four pole	Six pole	Eight pole
56	0.50	0.56	0.62	0.62
63	0.50	0.55	0.61	0.61
71	0.51	0.58	0.62	0.62
80	0.51	0.58	0.66	0.66
90	0.51	0.59	0.66	0.66
100	0.53	0.60	0.68	0.68
112	0.54	0.60	0.67	0.67
132	0.55	0.62	0.67	0.70
160	0.56	0.62	0.68	0.70
180	0.55	0.62	0.66	0.70
200	0.56	0.63	0.66	0.71
225	0.55	0.63	0.70	0.70
250	0.57	0.64	0.69	0.72
280	0.57	0.64	0.69	0.72
315	0.58	0.65	0.75	0.75
Average	0.520	0.65	0.663	0.681

Estimation of flux densities is performed in block No. 3. This step is taken in both the standard punching and designed punching. It means that in this stage two options are arbitrarily accessible to the designer. In the first option the designer may enter B_t from the keyboard. In the second option B_g is firstly taken from the empirical curves [1]; then by considering the motor type, service duty cycle and limitations of K_t and K_c, the values of B_c and B_t are obtained. For a standard punching value of B_g is given by the software and value of B_t must be provided by the designer. The requied data for other calculations are determined and stored in the data files. Blocks No. 4 and 5 are allocated to do the job.

The next stage is to design a proper main winding (block No. 6). It is desirable to have a sinusoidal flux density waveform in the airgap and this may be approximately achieved by altering the number of conductors in the slot. It is noted that there are three general methods for finding the number of conductors.

In the winding design, a harmonics ratio and a winding distribution factor are taken into account. When the magnetic circuit has been evaluated the saturation factor, which is necessary in the performance calculation, is obtained. The magnetisation characteristic (BH) and thickness of the lamination are stored in a data file.

After calculation the motor equivalent circuit constants, performance of the motor can be predicted (block No. 8). Such performance prediction for the single phase induction motor is possible using the revolving field theory or crossfield theory. It has been generally proved that both methods yield very close results. When the predicted performance of the motor does not satisfy the desired specifications, the design process must be repeated until the best results are attained within the specified constraints.

In block No. 11, the previous input data is modified. It is noted that a number of iteration loops between the blocks of Figure 1 modifies the assumed values such as the flux density ration.

Block No. 10 is performed if the results are satisfactory. Equation A.3 (or A.5) is established and the output is computed. If the calculated items can lead to the desired output, it can be assured that the proposed motor has an optimum iron volume. If discrepancy between the estimated values and required values (block No. 10) is larger than the required value, the main dimensions of the motor (D_o, D_i and L_{fe}) must be changed. For the lower error only L_{fe} may be altered. This stage is controlled by the proper constraints in the program. Use of a standard punching for an appropriate motor allows the designer to compulsorily change the assumed B_g and K_t. Block No. 12 can be substituted by an optimization routine, which enables the achievement of the final design specifications in a short

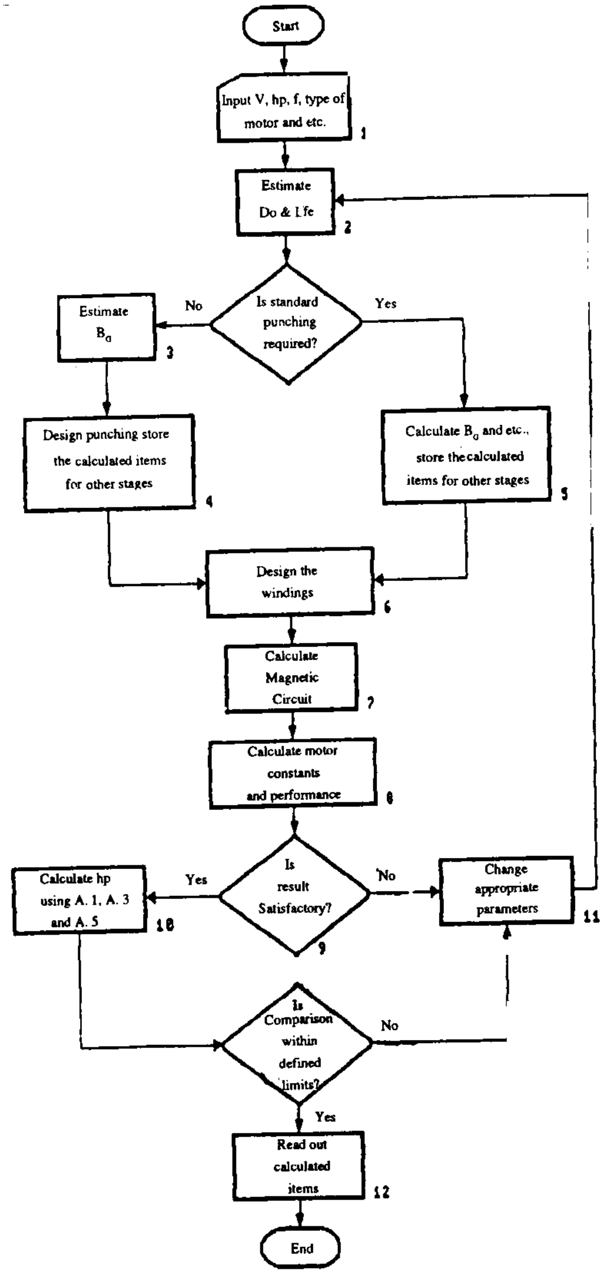

Figure 1. Flow-chart of the CAD of single-phase induction motor.

time. This is one of the major features of the outlined method for the motor design.

IMPORTANCE OF FLUX DENSITY RATIO

There are two important functions, f_{cu} (λ) and f_o (λ), in the new output equation, which both depend on λ. Values of λ due to the different standard codes and number of poles are listed in Table 1. As Table 1 shows, λ is normally between 0.5 and 0.7.

In order to optimize the core dimensions of the motor (minimizing core volume), maximizing fuction f_o (λ) is employed. For applying the constraints in the proposed CAD, f_o (λ) versus λ is plotted for various k_t. Intersection of the locus of the maximum λ and f_o (λ) curves for the range of λ between 0.5 and 0.7 gives k_t between 0.2 and 0.5.

B_t and therefore B_c are generally selected by the designer and the value of k_t is required for the design. By the proper choice of flux density ratios, the desired specifications of the motor can be achieved more quickly. The rate of the convergence to the ultimate design depends mainly on the skill and experience of the designer. In addition it depends on the severity of the performance requirements.

In most cases the designer has some knowledge about the motor punching or is obliged to utilize a particular one on which he desires to build any special motor to meet the given specifications.

However no motor manufacturer would require to use different punchings and various mechanical parts. The importance of the punching design should not be minimized. It is a complex problem involving a number of physical dimensions, all are more or less independent variables.

Since K_t value depends inversly upon B_t, and tooth width determines B_t values, by proper selection of t and k_t it is possible to find a sensible value for B_t and hence to achieve a satisfactory design. It can be found that for any proper punching it is possible to choose the boundary values for k_t and hence converge to the solution quickly. Values of k_t are chosen between 0.2 and 0.6 and these are used as a criterion to lead the whole program, with respect to flux density selection, as well as to the final design. Therefore the effect of the saturation, particularly on the motor tooth, is implicity included in the design calculation by k_t coefficient. However, the effect of the tooth flux density upon the output power of the designed motor will be discussed later.

The geometry of the magnetic circuit is such that the gap flux density is likely to remain substantially constant from the smallest to the largest sizes for similar motors. The flux densities depend on the tooth width and the most important design variable is ratio of slot width to tooth width. The density ratios equations are given in Appendix B. Generally speaking for any given limited value of the tooth flux density, there is a corresponding maximum permissible flux density for the airgap which is determined by the chosen ratios of slot width to tooth width [8]. Equations B.3-B.5 show the influence of flux density in maximizing function f_o (λ).

Another impotant matter with respect to f_{cu} (λ) is its relation with the copper cost. k_t value indirectly denotes the slot width to tooth width ratio and therefore the volume of the copper in the slot.

The general behaviour of f_o (λ) and f_{cu} (λ) can be studied by varying λ and keeping K_t constant or in contrast, varying K_t and keeping λ constant. However the ratio of the iron and copper is fully under control of λ and K_t values.

SAMPLE DESIGNS AND DISCUSSION

One of the three output equations A.1, A.3 and A.5 are used as a criterion for judging about the obtained iron volume. These output equations are expressed in terms of the L_{fe} and D_o which are determinant values for motor volume. The output equations also contain the output coefficients C_{os}, C_{or} and C_t, so the designer can alter all parameters involved in these coefficients. After a proper number of iterations, the calculated output approaches the desired output. The data given in reference [12], for a single-phase capacitor-run motor, is used to design this motor by the proposed CAD. The given data and calculated output using Equations A.1, A.3 and A.5 are tabulated in Table 2.

Figure 2 shows the torque-speed characteristics for the designed motor in which curves No. 1 and No. 2 have been taken from reference [12]. The size of the designed motor using the developed CAD is smaller than those introduced in reference [12]. The starting torque and the full-load torque are almost the same in the three cases but T_m/T_{fl} for the designed motor is slightly smaller than the other two. It must be noticed that this ratio remains in the range of acceptable ratio for the motor designed.

Table 3 summarizes the general specifications and

TABLE 2. Data and Calculated Results for a Sample Single Phase Induction Motor.

Required output = 1/3hp $\quad B_g = 0.68$ T
No. of poles = 6 $\quad B_t = 1.40$ T
Frequency = 60HZ $\quad B_c = 1.38$ T

Line voltage = 110 V

$D_0 = 120$ mm, $D_i = 80$ mm, $L_{fe} = 58$mm $\lambda = 0.67$

$K_t = 0.511$ $\quad a = 0.229$ $\quad f_0(\lambda) = 0.125$
$K_c = 0.173$ $\quad b = 0.684$ $\quad f_{cu}(\lambda) = 0.186$

1	hp=0.3399	Based on Eqn. A. 3
2	hp=0.3287	Based on Eqn. A. 5
3	hp=0.3240	Based on Eqn. A. 1

TABLE 3. Data Calculated Results for a Capacitor-Start Single-Phase Induction Motor.

General specifications of the motor

Required output = 0.496 hp $\quad B_g = 0.602$ T
No. of poles = 4 $\quad B_t = 1.500$ T
Frequency = 50 HZ $\quad B_c = 1.230$ T

Line voltage = 220 V

$D_o = 120$ mm, $D_i = 70$ mm, $L_{fe} = 77$mm $\lambda=0.58$
$K_t = 0.4520$ $\quad a = 0.1600$ $\quad f_o(\lambda) = 0.2301$
$K_c = 0.2542$ $\quad b = 0.7062$ $\quad f_{cu}(\lambda) = 0.3967$

Calculated output power

1	hp= 0.530	Based on Eqn. A. 3
2	hp= 0.491	Based on Eqn. A. 5
3	hp= 0.474	Based on Eqn. A. 1

calculated values for capacitor-start single phase induction motor.

Since the test results and the complete specifications of a capacitor-start single phase induction motor was available, its performance was compared with a similar motor designed by the proposed CAD. Final results for the designed motor are summarized in Table 3. This table reveals that even by use of Equation 1 (corresponding to the largest volume), it is not possible to obtain the expected output power (0.496 hp). Flux density, particularly at the tooth, has been chosen as large as possible. The magnetisation characteristic of the manufactured motor is saturated for $B_t = 1.6$ T, and $B_t = 1.5$ T is taken as the largest possible value.

Two upper and lower values of the above mentioned tooth flux density, $B_t = 1.75$ T and $B_t = 1.4$ T, are chosen and the output powers of the motor for different values of the tooth flux density are estimated and summarized in Table 4.

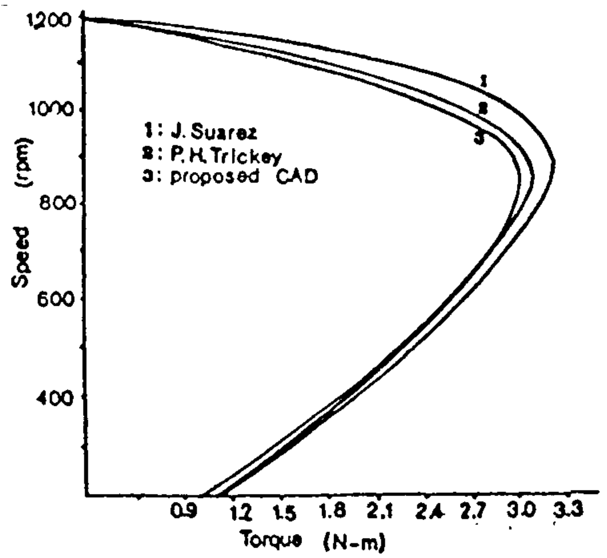

Figure 2. Torque-speed characteristics of the motors

TABLE 4. Motor Output Powers for Different Tooth Flux Density.

B_t	Eqn.	1.4 T	1.5 T	1.75 T
Output power hp	A. 3	0.525	0.530	0.500
	A. 5	0.488	0.491	0.491
	A. 1	0.461	0.474	0.489

Table 4 indicates that the increase of the tooth flux density can change the output power and the rising or falling of the output power depends upon the used output equation.

Table 5 shows the general specifications of three capacitor-start single phase induction motors manufactured by different manufacturers. Comparison of Tables 4 and 5 indicates the superiority of the designed motor by the proposed CAD. It should be noted that the third motor shown in Table 5 is overheated under full load condition because its core losses are high.

Again the comparison shows that the designed motor is smaller than the other two. The magnetization characteristics of the selected lamination is given in Figure 3 [9]. The standard punching [10] for this particular motor is represented in Figure 4.

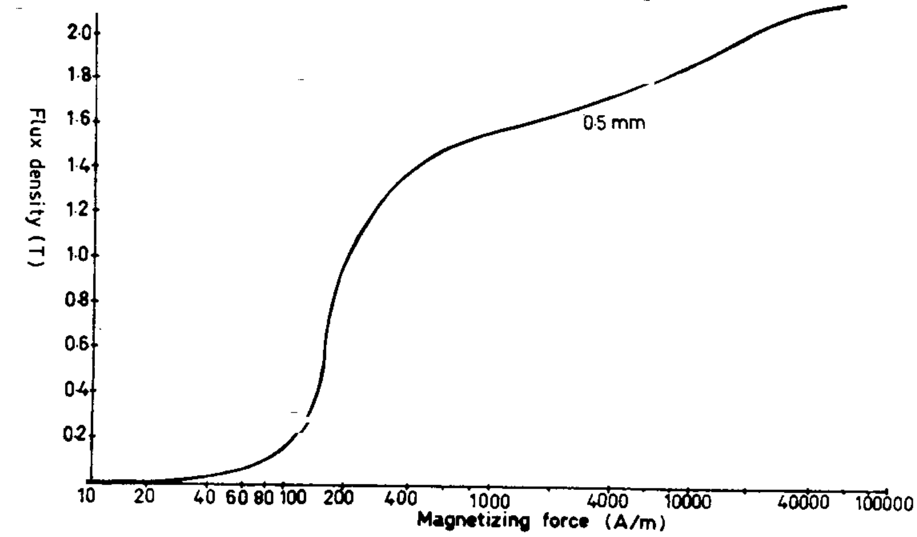

Figure 3. Magnetization characteristics of the lamination steel.

TABLE 5. Comparison with the Manufactured Motors.

Type	Output power hp	Speed rpm	D_r mm	L_{fe} mm
ELCO ESK 804 B	0.496	1430	69.94	86.0
ELPROM EPOK-80 d4A-B3	0.496	1425	73.50	81.5
MOTOGEN 804 A	0.496	1420	69.30	77.0

The important parameters, which can be changed by the designer, depend upon various factors. As it has been indicated in block No. 10 (Figure 1) the calculated output could be larger or smaller than the desired output. In this case, the iron volume is very large or very small and L_{fe} and D_o or both must be changed; but it must be noted that D_o has more significant effect on the output. Changing L_{fe} or some parameters included in the output coefficients such as flux density values (if it is permissible) and hence $f_o(\lambda)$ may lead to the desired design.

The calculated output values in Table 2 indicate that if Equation A.3 is chosen as a design criterion, the motor will have a small size in expense of high temperature rise due to saturation. If Equation A.1 is used the dimensions of the motor must be increased. This implies that the motor size must be greater than the designed motor. When Equation A.5. is utilized for judging the designed motor size, an intermediate size motor is obtained; and so based on the motor service hour, two Equations A.3 and A.5 can be extensively used as a criterion in the design process.

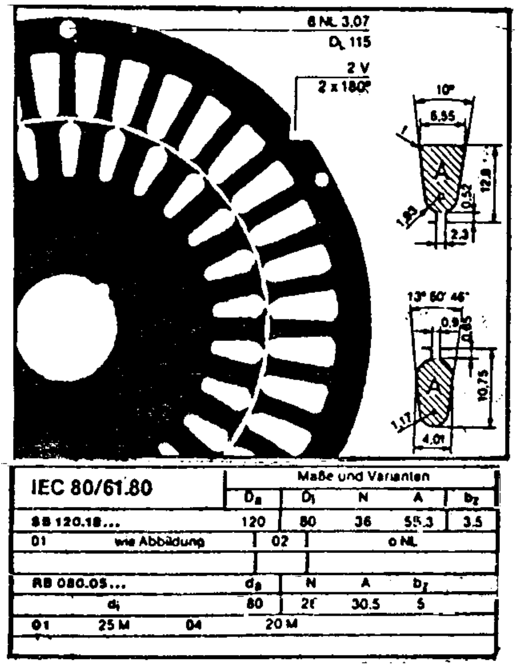

Figure 4. A standard punching used for the designed motor.

A BRIEF COMPARISON

In the traditional method for single-phase induction motor design, if the predicted performance of the designed motor is not satisfactory (block number 8 in Figure 1) the designer must change some preassumed values. In such a case skill and experience of the designer is the key issue for the design process. The reason is a very complicated relationship among many parameters which may be altered. In the proposed CAD the latest output Equations [3] are used and a new method for evaluating and comparing the predicted results is given.

It is clear from block number 10 of Figure 1 that the output power of the motor is calculated based upon three different equations. For short hour motors, Equation A.3 is used, because it leads to a smaller motor and saturation of the magnetic circuit of the motor is significant. Designing the motor which is suitable for general purposes. The old motor based upon Equation A.5 usually gives a moderate size motor which is suitable for general purposes. The old Equation A.1, which is used for many years, is not convenient. The constraints for minimizing iron volume and therefore the cost of the motor are discussed in Appendix B.

CONCLUSIONS

The paper has described a computer package developed based on a new geometrical approach for the design of the single- phase induction motors, which leads to the minimum iron volume compared with the existing motors in the market. This is performed using the limited range of coefficients k_c and k_t. The advantage of the design method has been shown by comparing the output of the designed motor, using the developed CAD, with those designed by other methods.

List of principal symbols

B_g = magnetic flux density in airgap, T
B_t = magnetic flux density at tooth, T
B_c = magnetic flux density in yoke, T
L_{fe} = core stack, mm
D_o = stator outer diameter, mm
D_i = stator inner diameter, mm
D_r = rotor outer diameter, mm
S_1 = stator slot numbers
A_s = stator slot area, mm^2
$J_s = CI/\pi D_r$ surface current density, A/mm
$J_v = CI/S_1A_s$ volume current density, A/mm^2
K_w = winding factor
V/E = supply voltage/ induced voltage
K_s = slot shape coefficient
hp = output power in horse-power
C = total number of conductors
n_s = synchronous speed, rpm
C_{os} = stator output coefficient
C_{or} = rotor output coefficient
C_t = total output coefficient
$\lambda = D_i / D_o$
I = stator current, A
STF = stacking factor
t = tooth width, mm
$\chi = K_w (V/E) \eta \cos \phi$
K_t = tooth flux density ratio
K_c = core flux density ratio
$f_o(\lambda) = \lambda f_{cu}(\lambda)$ maximizing function
$f_{cu}(\lambda)$ = ratio of copper volume to iron volume
p = number of poles

REFERENCES

1. P. H. Trickey: *Product Eng*., December (1946).
2. J. C. H. Bone: *Electric Power Applications,* Vol. 1, No. 1, (1978).
3. V. B. Honsinger: *IEEE Transactions on Energy Conversion,* Vol. EC-2, No. 1, (1987).
4. B. Schwarz: *IEE Proc*, Vol. 113, No. 3, (1966).
5. C. G. Veinott: "Theory and Design of Small Induction Motors" McGraw-Hill, New-York, (1959).
6. C. G. Veinott: "Computer-Aided Design of Electric Machinery" MIT Press, (1972).
7. G. W. Herzog, *et al*: *AIEE Transactions,* Oct. (1959).
8. C. G. Veinott: *AIEE Transaction,* April (1966).
9. Kawazaki Co., Japan: Nonoriented Sheet Steel for Magnetic Applications, (1989).
10. *IEC* Publication 404-8-4: Magnetic Materials-Specifications for Cold-Rolled Non-Oriented Magnetic Steel Sheet and Strip (1986).
11. H. Vickers: "The Induction Motor", 2nd Ed., Pitman Pub. Co., N. Y., (1950).
12. J. Buckman, *et al.*: *IEEE Transactions on Power Apparatus and Systems,* Vol. 4, No. 4, (1978).

APPENDIX A

Output Equations

Motor rating may be defined according to some variables such as D_o, L_{fe}, J_v (or J_s) and B_g. The classic output equation [4] relating these variables with the output power is revised in [3] and is given as follows:

$$hp/n_s = C_{or}\, D_r^2\, L_{fe} \qquad (A.\ 1)$$

where

$$C_{or} = 0.1558.\ B_g.\ J_s \times 10^{-9} \qquad (A.\ 2)$$

This equation may be used to examine the effect of the various parameters, such as per unit resistance and per unit reactance (and their ratio), on the motor performance.

Bone [2] has assumed a constant current density for the conductors and then he has obtained the permissible conductor current as a function of the cube of the motor diameter. Honsinger [3] has also used the same assumption and developed a new sizing equation. It is primarily based on the slot geometries as follows:

$$hp/n_s = C_{os}\, D_o^3\, L_{fe} \qquad (A.\ 3)$$

'here

$$C_{os} = 0.039\ B_g\ J_v\ f_o(\lambda)\ \chi[1-(k_s / D_o^2) / (f_{cu}(\lambda)]\ 10^{-9} \qquad (A.\ 4)$$

Function $f_{cu}(\lambda)$ is defined in Appendix B, and the expression inside the bracket depends upon the slot shape and usually equals one. A more suitable formula and perhaps a better one for an economical design can be derived by multiplying the two sizing equations A. 1 and A. 3 and then taking the square root. The resultant equation arranged in terms of D_o is [3] as follows:

$$hp / n_s = C_t\, D_o^{2.5}\ L_{fe} \qquad (A.\ 5)$$

where

$$C_t = \lambda\sqrt{C_{or}.\ C_{os}} \qquad (A.\ 6)$$

APPENDIX B

Flux density ratios and maximizing functions

Flux density values in variou: portions of motor essentially influence the performance. The designer is to determine B_t and B_g values and from these the punching geometry will be known. Here the important relationshi's which are used in proposed CAD have been introduced.

The tooth flux density and the core flux density ratios can be defined respectively as follows:

$$K_t = \frac{B_g}{B_t.\ STF} \qquad (B.\ 1)$$

$$K_c = \frac{2B_g}{B_c.\ STF.\ P} \qquad (B.$$

K_t and K_c are directily proportinoal to maximizing function $f_o(\lambda)$ as described in detail by Honsinger [3]. Based on the above values $f_o(\lambda)$ can be expressed as follows:

$$(\lambda) = \lambda \quad (\lambda) = \lambda\,(a\lambda^2 - 2b\lambda + \qquad (B.\ 3)$$

where

$$a = (K_t + K_c)^2 - (1 - K_t)^2 \qquad (B.\ 4)$$

$$b = K_t + K_c \qquad (B.\ 5)$$

Function $f_{cu}(\lambda)$ denotes the ratio of the copper volume to the iron volume and it has a valuable interpretation as shown in the recent work [3].