Compound Strip Method for Plane-Stress

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Abstract **The compound strip method (CSM) for plates is an expansion of the finite strip method (FSM) and was developed to incorporate the effects of the support elements in the analysis of linear elastic plate systems. In this paper the CSM is further expanded to analyze structures such as, stiffened plates under loads in the plane of the plate or the so called plane stress condition. Examples of these types of structures are retaining walls under gravity loads and stiffened plates used in tanks and containers. In this method of solution the stiffness of the stiffeners is added to the stiffness of the strip and summed over the entire structure. The displacement approach is used in formulation and resulting simultaneous equations are solved numerically. Two stiffened plate examples are solved using this apporach and the results are compared with the finite element method.**

چکیده روش نوار مرکّب برای ورقها توسعه یافتهٔ روش نوار محدود می‌باشد و در این روش اعضای تکیه گاهی مانند تیرها و ستونها نیز می‌توانند بصورت توأم با ورق مدل شوند و در واقع تحلیل روی ورق سخت شده انجام شود. در این مقاله روش نوار مرکّب در حالت تنش سطحی فرموله می‌گردد یعنی ورقی در حالت تنش سطحی با یک تیر به سختی معین در جهت طول دهانه یکجا مدل می‌گردد و تنش های موجود در آن با استفاده از روش تغییر مکانها محاسبه می‌گردند.

INTRODUCTION

The finite strip method (FSM) was developed by Cheung [1-8] as an alternative to the finite element method (FEM) [14] for the analysis of plate and shell type structures. The FSM has proven to be more economical by requiring less core storage and executing faster than the FEM. However, one major drawback of the FSM is its inability to incorporate supporting elements of the plate, such as beams and columns.

Although Cheung et al. [8], formulated a compatible girder at the location of a nodal line with the plate strip, other types of support were not discussed.

In 1986, Puchett and Gutkowdki [11] added the strain energy of the plate to the strain energy of supporting elements, such as transverse and longitudinal beams and columns, at the outset and minimized the total strain energy with respect to displacement parameters. The method is termed the "compound strip method" (CSM) and has successfully been applied to problems such as stiffened plates, free-vibration of plates, and continuous sector plates [11, 13, 12]. In 1987, Maleki [10], applied the CSM to folded plates and box girders with intermediate supports. In all cases the CSM compared favorably with other analytical and experimental methods.

The objective of this paper is to extend the capability of the CSM to analyze stiffened plates in plane stress condition. The plate is divided into a number of strips in the direction of the span and the longitudinal stiffener is modeled as a beam. The stiffness of this beam is written in terms of the displacement parameters of the plate strip to have compatibility at the junction of the two elements. The total stiffness of the structure is the sum of the stiffnesses of the plate and the stiffener. The resulting simultaneous equations are solved for the unknown displacements.

A fortran computer program is written for this purpose and is called "CFS" Program CFS is capable of compounding a longitudinal beam to each strip in plane stress at any desired location.

This method of solution is a new way of analyzing structures such as stiffened plates in plane stress, deep plate girders with stiffeners and similar types of structures.

In the following sections, the finite strip method is introduced first for the reader unfamiliar with the method and then the compound strip method is explained. To verify the results two stiffened plates are analyzed using the program CFS and the results are compared with the finite element method.

FINITE STRIP METHOD FOR PLANE - STRESS

The finite strip formulation for a plate in plane stress begins with an assumption for the displacement function which satisfies the simply supported condition at the ends of the strip. The simplest function to be used could have the following form:

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{m=1}^{r} [N]_m \{\delta\}_m \quad (1)$$

where u and v are the displacement functions in the x and y directions, respectively, and [N] and are the shape function and desplacement parameter matrices. The displacement function is in the form of a series which is added to the r term.

Different shape functions can be used to model the corresponding boundary conditions at the ends of the strip. Here the simply supported boundary condition is considered only, for which the above matrices are (Figure 1):

$$[N]_m = \begin{bmatrix} N_i \\ N_j \end{bmatrix}_m \quad (2)$$

$$= \begin{matrix} (1-{}^x/_b) & ({}^x/_b) \\ (1-{}^x/_b) & ({}^x/_b) \end{matrix} \begin{Bmatrix} Y_m \\ \frac{a}{m\pi} Y'_m \end{Bmatrix}$$

$$\{\delta\}_m = \begin{Bmatrix} \delta_i \\ \delta_i \end{Bmatrix}_m = \begin{Bmatrix} u_i \\ v_i \\ u_j \\ v_j \end{Bmatrix}_m \quad (3)$$

where, $Y_m = \sin m\pi\ y/a$ and $y'_m = (m\pi/a) \cos m\pi\ y/a$.

Formulating the strain energy of the plate, u, with the above matrices results in [7]:

$$u = \frac{t}{2}\sum_{m=1}^{r}\{\delta\}_m^T \int_0^a\int_0^b B]_m^T[D][B]_m\, dxdy\{\delta\}_m$$

$$-\sum_{m=1}^{r}\{\delta\}_m^T \int_0^a\int_0^b N]_m^T\, q\} \quad (4)$$

where [B] and [D] are strain and elastic matrices for a plate with thickness t under plane loading (q). Now by minimizing U with respect to displacement parameters the stiffness relationship is obtained:

$$\sum_{m=1}^{r} t\int_0^a\int_0^b [B]_m^T[D][B]_m\, dxdy\, \{\delta_m$$

$$\sum_{m=1}^{r} \int_0^a\int_0^b [N]_m^T \{q\} dxdy \quad (5)$$

or in simplified form:

$$[S]_m\{\delta\}_m = \{F\}_m \quad (6)$$

where [S] and (F) are the stiffness and force matrices, respectively.

COMPOUND STRIP METHOD FOR PLANE STRESS

The compound strip method [10, 11, 12, 13] is an extension of the FSM and incorporates the support elements of the plate such as intermediate beams and columns in the analysis. In this method, the strain energy of the supporting member is written in terms of the displacement parameters, , of the plate and then added to the strain energy of it. This way, the compatibility is assured at the intersenction of the two.

In this paper a longitudinal beam with stiffness "EI" and span "a" is considered for compounding (Figure 1). The beam can be located anywhere in the transverse (x) direction. The strain energy of this beam in terms of nodal displacement in the x direction, u, is:

$$u = \frac{EI}{2} \int_o^a \left(\frac{\partial^2 u}{\partial y^2} \right)^2 dy \tag{7}$$

where u was given in equation (1) and is repeated here:

$$u = [(1-{}^x/_b) u_{im} + (\frac{x}{\cdot}) u_{jm} \quad \frac{m^2 \pi^2}{a^2} \sin \frac{m\pi y}{a} \tag{8}$$

Substituting for u in the strain energy equation and minimizing it with respect to four displacement parameters of a strip, gives the stiffness matrix for the longitudinal beam:

$$K = \frac{EIm^4 \pi^4}{2a^3} \begin{bmatrix} (1-{}^x/_b)^2 & 0 & \frac{x}{b}(1-{}^x/_b) & 0 \\ 0 & 0 & 0 & 0 \\ (1-{}^x/_b){}^x/_b & 0 & ({}^x/_b)^2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

Similary, the strain energy of the longitudinal beam in terms of the nodal displacements in the "y" direction (v) is:

$$u = \int_o^a ½ K_a V^2 \tag{10}$$

where Ka is the axial stiffness of the beam and "v" from equation (1) can be written in the following form:

$$V = [(1-{}^x/_b)V_{im} + {}^x/_b V_{jm}] \cos \frac{m\pi y}{a} \tag{11}$$

Substituting the above equation in the strain energy equation and minimizing with respect to four displacement parameters of a strip give the stiffness matrix for the beam:

$$K = \int_o^a K_a \cos \frac{m\pi y}{a} \cos \frac{n\pi y}{a} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & (1-{}^x/_b)^2 & 0 & (1-{}^x/_b)({}^x/_b) \\ 0 & 0 & 0 & 0 \\ 0 & (1-{}^x/_b)\frac{X}{b} & 0 & ({}^x/_b)^2 \end{bmatrix} \tag{12}$$

The above formulations are incorporated in a comuter program called CFS. In the section to follow two examples will be solved using this program and the results compared with the finite element method.

ILLUSTRATIVE EXAMPLE

In this section a 4 meter by 1 meter steel plate is considered for analysis. The plate is 10 centimeters thick and has simple supports at the 1 meter edges. The analysis is performed in two stages using the finite element method (FEM) and the finite strip method (FSM). Figure 2(a) illustrates the finite element mesh used which consists of 45 nodes and 32 elements. Program SAP84 version 84.03 is used for the FEM analysis. The loading

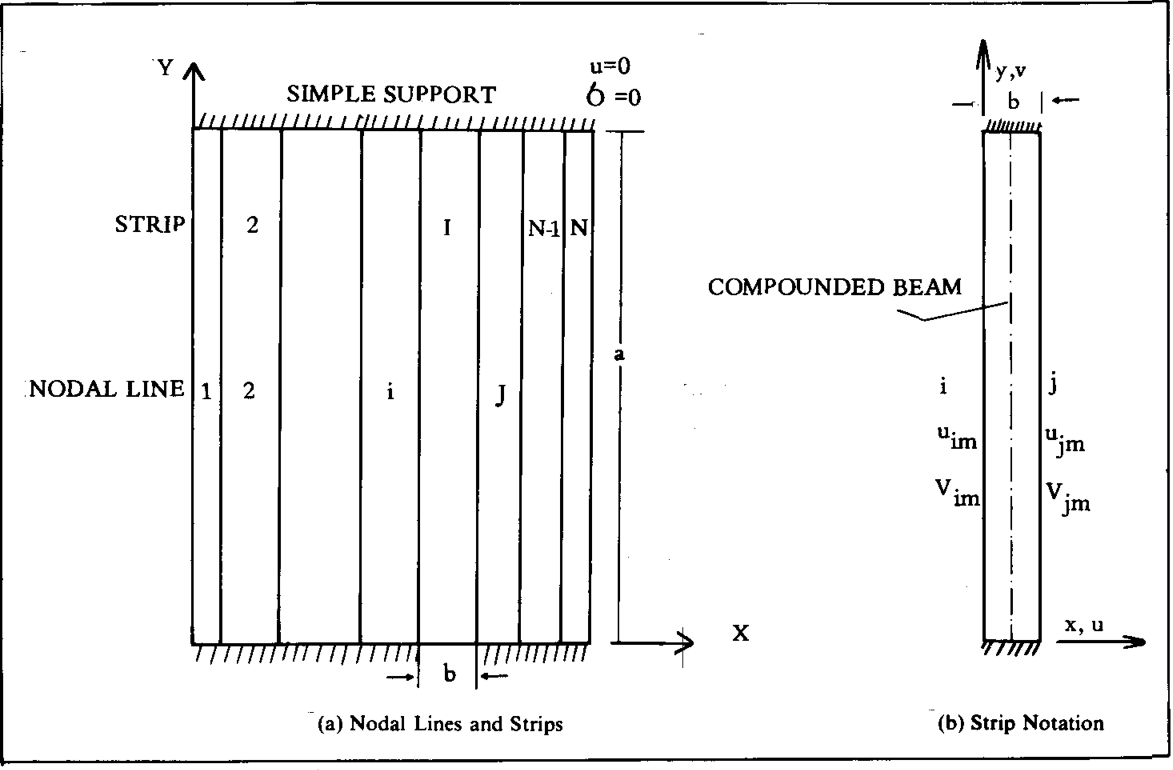

Figure 1. Typical plane - stress strip

consists of the weight of the plate applied in the x direction. Figure 2(b) shows the finite strip model for the same example. Four equal strips with 5 nodal lines are used in the model. Program CFS analyzes this model in a few second. The output of each program is shown in the appendix. The results for these two analyzes are compared in Table 1. The agreement is acceptable for all the values reported. It should be mentioned that closer agreement can be achieved using more strips in the FSM model. The CFS program executes faster than SAP84 and requires less input and output time. This is the main advantage of the finite strip method over the finite element method.

In the second example the same plate is stiffened with a beam along the span. This

Table 1. Comparison of FEM and FSM Results

Longitudinal Stresses at Midspan for the Plane Stress Plate without Stiffener

NODE NUMBER	FEM ANALYSIS	FSM ANALYSIS	PERCENT DIFFERENCE
5	-0.94×10^5	-0.96×10^5	2
14	-0.44×10^5	-0.45×10^5	2
23	0.0	0.0	0
32	$+0.44\times10^5$	$+0.45\times10^5$	2
41	$+0.94\times10^5$	$+0.96\times10^5$	2

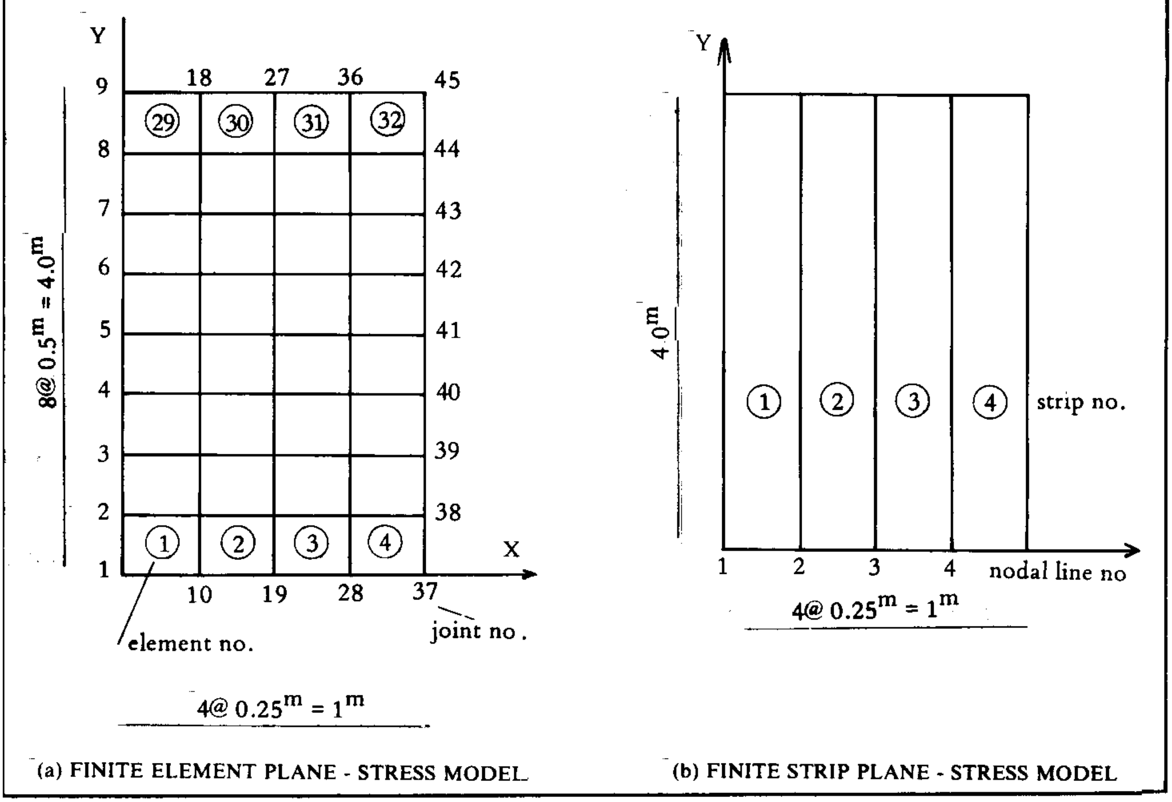

Figure 2. Simply supported deep beam example

beam is located on nodes 19 through 27 in the FEM model and on nodal line 3 in the FSM model. The modulus of elasticity for the longitudinal beam is 2038 kg/m^2, the moment of inertia is 1472 m^4 and it has an area of 10 m^2.

Table 2 compares the results of the FEM and the FSM which uses a compounded member as described in the previous section. Considering the differences in the FEM and the FSM analyzes reported in Table 1, it is concluded that the compound strip method (CSM) can be used in plane-stress analysis without a substantial error.

Table 2. Comparison of FEM and FSM Results

Longitudinal Stresses at Midspan for the Plane stress Plate with Longitudinal Stiffener:

NODE NUMBER	FEM ANALYSIS	FSM ANALYSIS	PERCENT DIFFERENCE
5	-0.92×10^5	-0.94×10^5	2
14	-0.43×10^5	-0.44×10^5	2
23	0.0	0.0	0
32	$+0.43\times10^5$	$+0.44\times10^5$	2
41	$+0.92\times10^5$	+0.94x105	2

CONCLUSION

The compound strip method can be easily expanded for plane-stress analysis and the illustrative example indicates that the results are comparable with the finite element analysis. The CSM has the advantage of faster execution time and a lesser input/output volume for the same problem.

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