DESIGN OF LOGIC NETWORK FOR GENERATING SEQUENCY ORDERED HADAMARD MATRIX H

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Abstract A logic network to produce the sequency ordered Hadamard matrix H based on the property of gray code and orthogonal group codes is developed. The network uses a counter to generate Rademacher function such that the output of H will be in sequency ordered. A general purpose shift register with output logic is used to establish a sequence of period P corresponding to a given value of order m of the Hadamard matrix. A logic network to generate sequency Hadamard matrix of order 2^3 was designed to illustrate the effectiveness of the procedure.

چکیده در این مقاله مدار منطقی جهت تولید ماتریس Hadamard بوسیله استفاده از حواص کدگری (Orthogonal) وکدگروهی ارتاگونال Gray Code طراحی گردیده بصورتیکه نتیجه بدست آمده دارای ترتیبی پیاپی میباشد . یک شمار گرویک ثبت کننده مخصوص کـه دارای خروجی منطقی میباشد جهت کنترل ترتیب مدار تولید کننده ماتریس Hadamard ساختـه شده و شبکهای بـرای مـاتریس Hadamard که دارای طول ۲۳ (هشت) میباشد طراحی و امتحان گردید تا صحت مقاله را اثبات کند .

INTRODUCTION

Hadamard transform, had found application in many areas, including signal processing, pattern recognition, image processing, digital filtering, communication theory and digital coding of waveform [1-5]. Much of the interest in this transform results from the computational advantages it offers over more conventional Fourier transform, and from the simplicity of solid state electronic hardware filters [2, 6]. Hadamard transform is produced by the discrete Walsh funciton. Their definitions are based on Rademacher of Harr functions. In 1923 walsh introduced a complete set of orthogonal functions. Their definitions are based on Rademacher or Harr functions. In 1969 Harmuth [7] discussed a diference equation for Walsh function as given in equaitons (1) and (2)

$$\mathrm{Wal}(0,m) \begin{cases} 1 & \text{for } -1/2 <= m < 1/2 \\ 0 & \text{for } m < -1/2 \text{ and } m >= 1/2 \end{cases} \tag{1}$$

and

$$\mathrm{Wal}\,(2j + k, m) = (-1)^{(j/2)+k}\ \mathrm{Wal}\,[\,j,\ 2(m + 1/4\,] + (-1)^{j+k}\,\mathrm{Wal}\,[j,\ 2(m - 1/4] \tag{2}$$

Where j = 0, 1, 2, ... and k = 0 or 1.

The discrete Walsh functions are sampled version of continous function as shown in figure 1 for an eight bit discrete walsh function. The N-length discrete Walsh funcitons can be defined for $N = 2^m$, where m is a positive integer.

The Hadamard matrix is a square array of plus and minus ones whose rows and columns are orthogonal to each other. Given a Hadamard matrix of order N wher N > 2 it is possible to produce Hadamard matrix of higher order, in this case if H is a Hadamard matrix of order N then a matrix of order 2N is produced by equation (3).

$$H = \begin{vmatrix} H & H \\ H & -H \end{vmatrix} \qquad (3)$$

Figure 2 shows several Hadamard matrices of order $N = 2^m$.

In this figure one may interchange rows, interchange columns and change the sign of every element in a row or in a column without disturbing the Hadamard property. In a normal form the first row and the first column of Hadamard matrices contain only plus ones. The rows of a Hadamard matrix are code words of an orthogonal code and conversely. Therefore, if a Hadamard matrix exists, then simplex codes also exist. Simplex code is one special code of transorthogonal code [8]. A code is called transorthogonal if the correlations between distinct code words are always negative. If W represents the word length of the code, then the correlation p(x, y) of two W-dimensional vectors x and y is given by equation (4)

$$p(x, y) = \frac{1}{W} \sum_{i=1}^{W} x_i y_i \qquad (4)$$

HADAMARD TRANSFORM

Let f be a column vector of dimension N that represents sampled values of an input signal, and let the column vector of dimension N, F, represent the desired Hadamard transform of f. Then F is given by equation (5)

$$[F] = H\,[f] \qquad (5)$$

where H is a Hadamard matrix of order $N = 2^m$.

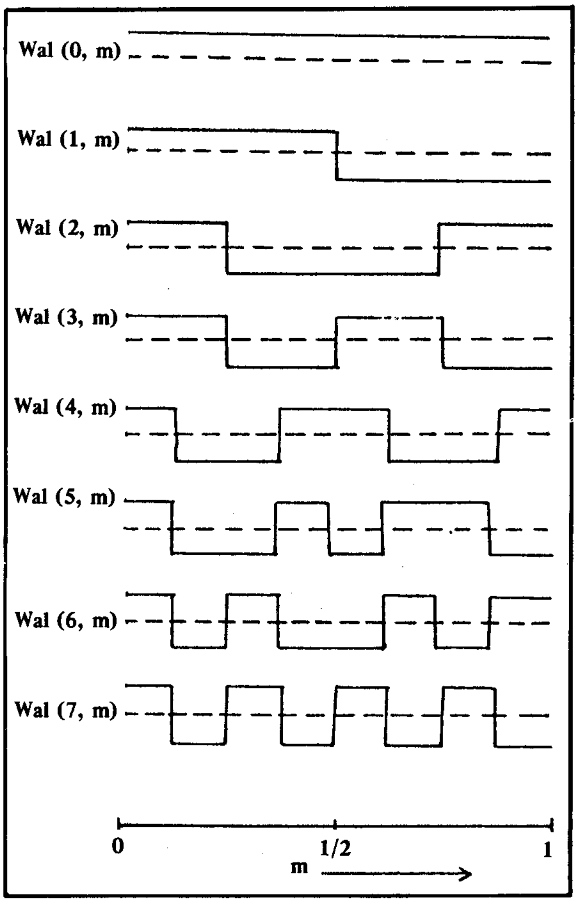

Figure 1. The Length Eight Discrete Walsh Function

A fast algorithm known as Fast Hadamard Transform algorithm can be used to calculate the Hadamard transform of a given function. In fact, if we replace the trigonometric multipliers in the standard fast Fourier transform routine by ±1, the Fast Hadamard Transform can be generated. This method is purely discrete and can be produced by using a digital computer. Carl and Swartwood [9] introduced a hybrid Walsh transform computer. Their spacial purpose computer is based on a recursive algorithm as described by Andrews and Caspari [10] and also by Goods [11]. In their result the analog computer was used and the output produced was discrete Walsh transform not in sequency ordered.

In this paper the Hadamard transform F of any function can be generated by first introducing a logic network to produce the Hadamard matrix H and using equation (5) to find F.

LOGIC NETWORK TO PRODUCE H

Hadamard matrix can be formed from orthogonal group codes which was defined in section I. Here if one let g_i be a binary vector of length $n=2^m$ and let G represent an orthogonal code containing N of these vector, then G is called a group if and only if a subset of $m=\log_2 N$ of the vectors g_i can be selected such that any of the N vectors g_i can be obtained from equation (6) [12]

$$g_i = \sum_{i=1}^{m} a_i g_i \qquad (6)$$

where a_i=0 or 1 and $\sum$ represents the modulo-two term by term sumation.

If we define the Hadamard matrix as H= Wal (m, n) then one can produce m by using a cyclic code [13] and the property of equation (6). Cyclic code is defined as any code in which, for every code word, an end-around shift operation yields another code word. The Gray code is one of the most common types of cyclic code. It has the characteristic that only one bit position will change at a time as the digits represented by the code are advanced from one number to the next [14]. Let $g_n g_{n-1} \cdots g_1 g_0$ denote a code word in the (n+1) st-bit gray code, and $b_n b_{n-1} \cdots b_1 b_0$ the corresponding binary representation of the code, then equation (7) gives the corresponding gray codes.

$$g_i = b_i \oplus b_{i+1} \qquad \text{for } 0 < i < n-1$$

$$g_n = b_n \qquad (7)$$

If in Hadamard matrix H, one replaces all −1 by 0 and all +1 by 1 and if in H=Wal(m, n) one replaces m by its equivalent gray code produced from the corresponding binary equivalent of m, then we can find a logic network to generate a Hadamard matrix. The logic network for a sequency ordered

1	1
1	−1

1	1	1	1
1	−1	1	−1
1	1	−1	−1
1	−1	−1	1

1	1	1	1	1	1	1	1
1	−1	1	−1	1	−1	1	−1
1	1	−1	−1	1	1	−1	−1
1	−1	−1	1	1	−1	−1	1
1	1	1	1	−1	−1	−1	−1
1	−1	1	−1	−1	1	−1	1
1	1	−1	−1	−1	−1	1	1
1	−1	−1	1	−1	1	1	−1

1	1	1	1	1	1	1	1
1	−1	1	1	−1	1	−1	−1
1	1	1	−1	1	−1	−1	−1
1	1	−1	1	−1	−1	−1	1
1	−1	1	−1	−1	−1	1	1
1	1	−1	−1	−1	1	1	−1
1	−1	−1	−1	1	1	−1	1
1	−1	−1	1	1	−1	1	−1

Figure 2. Several Hadamard matrices of order $N = 2^m$

Hadamard matrix of order 8 is shown in Figure (3).

Note that in this logic network the counter is the one that should be properly designed to generate sequency ordered Hadamard matrix. To produce a sequency ordered H, a special purpose autonomous synchronous machine must be designed. This machine can work as a binary counter, but not in an increasing order. The state diagram to generate a correct counter in order to find H for order of $N=2^3=8$ is shown in Figure (4).

The correct states of the state diagram to generate sequency ordered H can be found by looking at the output of H for a counter that increases each time by one and then find the correct change of states.

EXAMPLE OF GENERATING H OF ORDER $2^M=8$

Hadamard matrix of order 8 can be generated by the use of logic circuit shown in Figure (3), and the synchronous machine designed from the state diagram of Figure (4).

The output of H is shown in table 1. Note that in this logic network a D-type flipflop is used. In logic circuit shown in Figure (3) a general shift register with output logic is used.

The principle advantage of this configuration is that the internal logic $f(x_1, x_2, x_3, ..., x_n)$ in conjunction with the n-position register can be used to establish a period P, and the output logic $f^*(x_1, x_2, x_3, ..., x_n)$ can modify the sequence of period P already generated to any other sequence of period P corresponding to a given value of m. Where m is defined in Wal (m, n).

For higher order m it is necessary to change the general shift register to a shift register of order m and our counter to generate c_0, c_1, ... c_m, here we should be very careful to lay out the correct state diagram in order to get Hadamard matrix of correct sequency order.

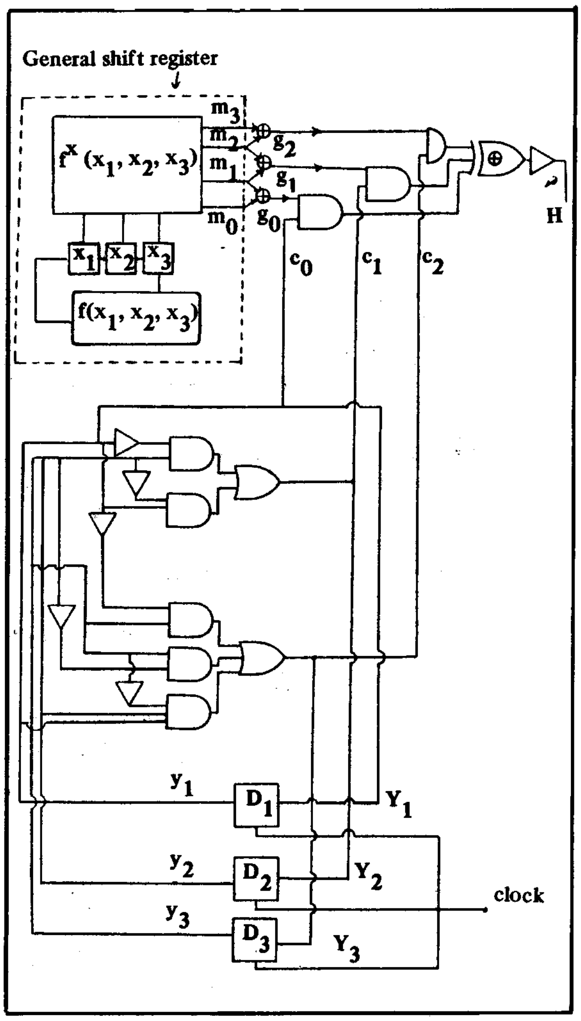

Figure 3. Logic network for a sequency ordered Hadamard matrix of order 8.

CONCLUSIONS

In this paper a special logic network was designed in order to produce a sequency ordered Hadamard matrix.

The properties of orthogonal codes, simplex codes and gray codes were used to design the necessary hardware needed to produce H. A general purpose shift register with out-

Table 1 **Output of logic circuit shown in figure (3). For H of order $2^3 = 8$**

	Binary representation of G								
Count	000	001	010	011	100	101	110	111	Sequency
000	1	1	1	1	1	1	1	1	0
100	1	1	1	1	0	0	0	0	1
010	1	1	0	0	0	0	1	1	2
110	1	1	0	0	1	1	0	0	3
001	1	0	0	1	1	0	0	1	4
101	1	0	0	1	0	1	1	0	5
011	1	0	1	0	0	1	0	1	6
111	1	0	1	0	1	0	1	0	7

put logic is introduced to generate sequence of period P corresponding to a given value for m as defined by equations (1) and (2).

The logic network was used as a test for finding Hadamard matrix of order 2^3=8. The output of the logic network is shown in Table

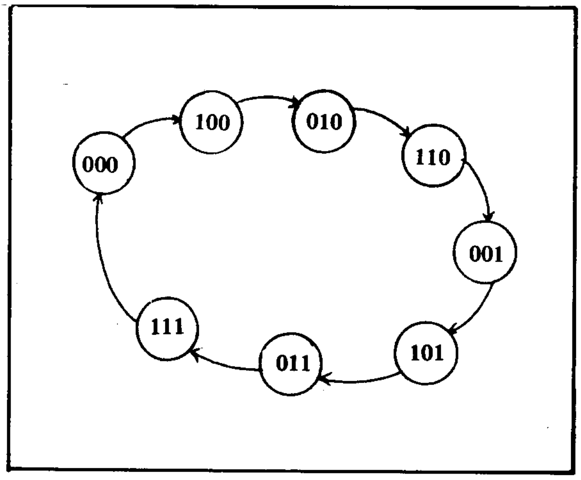

Figure 4. State diagram for designing counter to produce sequency order H.

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