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Mathematical Model for Estimation of Return to Scale in Four-Level Green Supply Chain by using Data Envelopment Analysis

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ABSTRACT

Today, focusing on gaining a competitive edge in the global business market lies in enhancing supply chain performance. This study endeavors to examine the attainment of Returns To Scale (RTS) within a four-level green supply chain framework through the application of Data Envelopment Analysis (DEA). To achieve this objective, the Banker, Charnes, and Cooper (BCC) multiplicative model are employed to determine the return to scale at each level within the complete supply chain, ultimately culminating in estimating the overall return to scale for the entire supply chain. The statistical population for this applied research, aligned with its objectives, comprises 42 cement companies. The assessment of returns to scale in these companies, featuring a four-level chain encompassing suppliers, manufacturers, distributors, and customers, is measured. The outcomes of the model reveal that return to scale remains constant in 28 companies, exhibits a decreasing trend in 14 companies, and conversely demonstrates an increasing trajectory in 2 companies within the supplying sector, one company in manufacturing, and 14 companies in distribution. The findings underscore that an increasing return to scale renders the expansion of Decision-Making Units (DMUs) economically viable. Conversely, a diminishing return to scale suggests a rational limitation of DMUs.

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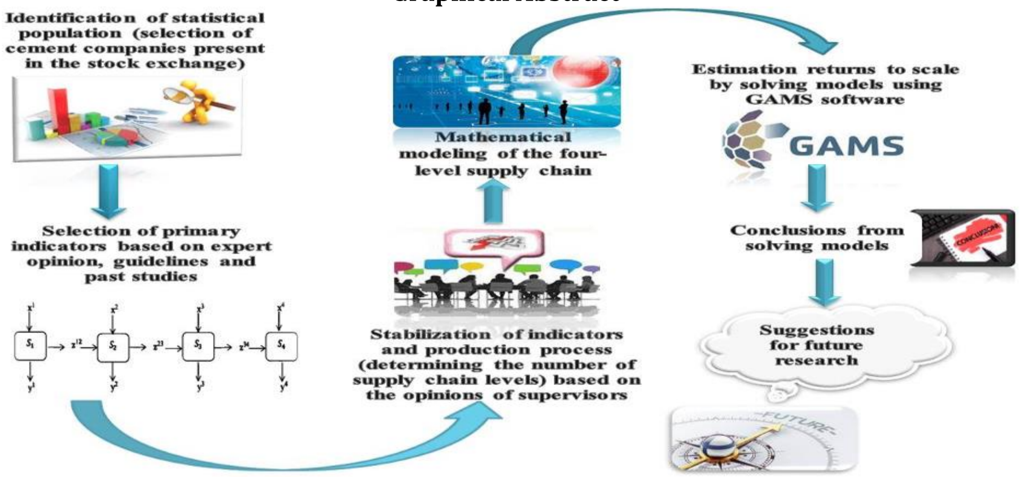
Graphical Abstract



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NOMENCLATURE

T_v	Production possibility set	X_{ij}^1	Primary inputs of supply chain
X_j	Inputs of decision-making unit j (DMU_j)	Y_{rj}^1	First level outputs
Y_j	Outputs of decision-making unit j (DMU_j)	Z_{lj}^{12}	Intermediate data (1st level output and 2nd level input)
λ_j	Decision variables of decision-making unit j (DMU_j)	X_{ij}^2	2nd level independent inputs
θ	Efficiency score of decision-making unit j (DMU_j)	Y_{rj}^2	2nd level outputs
Z_δ	Right-hand neighborhood of the unit under evaluation (DMU_o)	Z_{lj}^{23}	Intermediate data (2nd level output and 3rd level input)
$Z_\delta^{'}$	Left- hand neighborhood of the unit under evaluation (DMU_o)	X_{ij}^3	3rd level Independent inputs
v	Weights of the input data for decision-making unit j (DMU_j)	Y_{rj}^3	Third level output
u	Weights of the output data for decision-making unit j (DMU_j)	Z_{lj}^{34}	Intermediate data (3rd level output and 4th level input)
w	Weights of the intermediate data for decision-making unit j (DMU_j)	X_{ij}^4	4th level independent Inputs
E	Efficiency of supply chain levels	Y_{rj}^4	Final outputs
u_o^*	Optimal solution for return to scale in the supply chain		

1. INTRODUCTION

In the contemporary competitive market landscape, all enterprises, encompassing both manufacturing and service organizations, find it imperative to measure and evaluate the performance of their supply chain operations. This undertaking is essential for fostering productivity, ensuring survival, and achieving sustainable longevity.

The objective of each supply chain is to deliver a high-quality product to the end customer at minimum cost and within the shortest timeframe while concurrently adding value to all facets of the production process. In recent years, the concept of productivity has assumed a pivotal role in the ideologies and perspectives of various companies and organizations. Concurrently, researchers across diverse disciplines have increasingly employed the supply chain framework. A complete supply chain comprises four integral components: the supplier, manufacturer, distributor, and customer. It is crucial to note that the measurement and evaluation of optimal supply chain performance must be tailored to consider the unique characteristics of the chain networks and their interdependencies. Generally, assessing larger and more intricate supply chains becomes progressively more intricate and challenging.

Within the spectrum of evaluation methodologies, DEA emerges as a fitting approach for gauging the efficiency and performance of DMUs. Through the formulation of diverse models, this non-parametric method adeptly assesses DMUs that yield multiple outputs while consuming multiple inputs. DEA holds considerable significance within the literature on supply chain management.

The classical DEA methods, as delineated by Charnes et al. (1); lack of a theoretical framework for comprehending the internal operations of DMUs, treating them as a black box. These methods confine calculations to initial inputs and ultimate outputs, neglecting internal processes in their entirety.

Hence, to address this issue, several models, such as network DEA, have been introduced. The configuration of the supply chain stands out as a critical and pragmatic consideration within the context of network DEA. Methodical coordination of raw material procurement, the conception and manufacturing of suitable products, efficient distribution and transportation, and ultimately, the provision of services and customer satisfaction all hold significant importance in supply chain management. In the realm of network DEA, the emphasis extends to intermediate products and internal communications of DMUs, with performance evaluation incorporating a comprehensive examination of the internal components of a unit.

DEA serves as a tool for assessing the performance of DMUs. Within this framework, the evaluation of the return to scale for DMUs assumes paramount significance. The return to scale, in the context of DEA, holds economic importance as it signifies the maximum rate of output increase per unit increase in input. An essential consideration is that the diagnosis of the return to scale yields valuable insights into the developmental trajectory of DMUs. In economics, the concept of return to scale, whether ascending, descending, or constant, is defined as follows:

1. Ascending Return to scale occurs when, with an increase in inputs, outputs increase more than the input ratio, indicating that the expansion of DMUs is cost-effective.
2. Descending return to scale occurs when, with an increase in inputs, outputs grow proportionally less, indicating that limiting DMUs is cost-effective.
3. Constant return to scale occurs when, with an increase in inputs, the outputs also increase in the same proportion to the inputs. This implies that by expanding or limiting DMUs, we neither gain nor lose efficiency.

In this investigation, the researchers endeavor to ascertain the return to scale within a complete supply chain context, incorporating both independent and dependent inputs and outputs, utilizing the DEA method.

A seminal study addressing network structure and supply chain, dating back approximately 23 years, was introduced by Fare and Grosskopf (2, 3). Their approach to the overarching network structure involved initially establishing a Production Possibility Set (PPS) grounded in fundamental standard principles within a variable return-to-scale scenario. Subsequently, they constructed a production feasibility set within the supply chain by amalgamating the production feasibility sets of its internal components. Nevertheless, to gauge the efficacy of a supply chain, it is imperative to consider both the network properties inherent to the chain and the interrelationships among the "supplier of materials and components," "manufacturer," "distributor," and "end customer. This consideration has been prompted by Saranga and Moser (4), also by Chen and Yan (5), to present diverse models accommodating various supply chain structures. For instance, Chen and Yan (5) have delineated three network DEA models corresponding to centralized, decentralized, and hybrid organizational mechanisms for supply chain evaluation. Tavana et al. (6), introducing a network model founded on the Network Epsilon–Based Measure (NEBM), have investigated supply chain performance, simultaneously analyzing changes in inputs and outputs, both radially and non-radially within the network. In the realm of Green Supply Chain Management, Mirhedayatian et al. (7) have positioned it as a method to enhance environmental performance, asserting that companies, influenced by stakeholders, pressures, and regulations, must enhance the performance of Green Supply Chain Management (GSCM). Shafiee et al. (8), after an extensive examination of various tools for evaluating supply chain performance, have proposed a novel approach based on network DEA and the Balanced Score card (BSC) method. They have, in essence, furnished a comprehensive framework for supply chain performance evaluation through the amalgamation of BSC and DEA models. Grigoroudis et al. (9), in their work titled "Recursive Algorithm, a DEA-Based Recursive Algorithm for Optimal Design of Biomass Supply Chain Networks," have introduced an alternative method for designing supply chain networks. Khodakarami et al. (10) have conducted an evaluation of 27 Iranian companies in the sustainability realm of supply chain management, employing a two-level model. Tajbakhsh and Hassini (11) have proposed a methodology for assessing the sustainability of supply chain networks, with a sustainability approach aiming to harmonize economic, environmental, and social considerations. Tavana et al. (12) have suggested a 2-level DEA method to appraise the performance of a 3-level supply chain, encompassing the supplier, manufacturer, and distributor. This proposed model facilitates a comprehensive analysis of multi-level supply chains. Yousefi et al. (13), in their study, have introduced a combined model of DEA and Goal Programming within a network structure to offer improvement solutions and rank units within the supply chain. Fathi and Farzipoor Saen (14) have emphasized the complexity of evaluating sustainable supply chains and articulated the use of a realistic and practical model for this purpose. This paper proposes a novel directional network DEA model for the inaugural evaluation of the stability of distribution supply chains. In this scholarly work, Darvish Motevalli et al. (15) have introduced a novel model designed to assess the efficiency of extant cement companies employing a network structure. The findings demonstrate that this innovative model is proficient in evaluating the performance of factories characterized by a network structure incorporating diverse indicators. Beyond the conventional utilization of financial and technical indicators, the model accounts for undesirable outcomes and sustainability criteria within the supply network. Simultaneously, the authors have incorporated the decision-makers perspectives on the relative importance of specific indicators, acknowledging the weighted constraints as a means to align the efficiency assessment with actual values. The examination of the computation of efficiency within the context of a green supply chain is an ongoing process. Tavassoli and Farzipoor Saen (16), in their scholarly work, have underscored that, in numerous real-life scenarios, not all inputs or outputs can be precisely determined, with some potentially being contingent or accidental. Torabi et al. (17), in their publication, introduced a novel two-stage green supply chain network. Primarily, they present an innovative multi-objective model addressing a two-level green supply chain problem. Given the intricacy of this model, a novel multi-objective interior search algorithm (MOISA) is employed. The results illustrate that the proposed algorithm MOISA yields superior Pareto solutions, affirming the efficacy of the algorithm in a majority of cases. Meanwhile, the objective of Asadpour et al. (18) in this study was to formulate a green Blood Supply Chain (BSC) network, considering expiration dates and backup facilities. The proposed model takes the form of a bi-objective Mixed Integer Programming (MIP) structure. The two primary objectives are to minimize the overall cost and mitigate the adverse environmental impacts associated with shipping between facilities and waste generation within the network. Fathollahi-Fard et al. (19) have introduced two hybrid meta-heuristic algorithms to address a dual-channel closed-loop supply chain network design problem within the tire industry under conditions of uncertainty. This study represents a pioneering effort by proposing a dual-channel, multi-product, multi-period, multi-echelon closed-loop Supply Chain Network Design (SCND) tailored to the uncertainties prevalent in the tire industry. To contend with the uncertain parameters intrinsic to the problem, such as prices and demand, a fuzzy approach, specifically Jimenez's method, is employed. Another notable contribution of this work lies in the introduction of two innovative hybrid meta-heuristic algorithms featuring novel procedures. The integration of two contemporary nature-inspired algorithms, namely the red deer algorithm (RDA) and the whale optimization algorithm (WOA), with the genetic algorithm (GA) and simulated annealing (SA), serves to enhance the diversification and intensification phases respectively. Moosavi and Seifbarghy (20) presented a new mathematical model recognizing the significance of supply chain and environmental concerns. The model addresses a green closed-loop supply chain network with the primary objectives of maximizing profits, job creation, and reliability. The practical application of the model to a real case study within the Iranian engine oil industry demonstrates the efficacy of the derived solutions for this network. In this study, Sahraeian et al. (21) emphasize the critical significance of designing an efficient and reliable cold supply chain for the benefit of the company, suppliers, customers, and society. The research delves into all facets of the cost of quality within the design of a cold supply chain, encompassing considerations such as the cost of quality associated with suppliers and the cost of distribution service quality. Notably, the study takes a holistic approach by simultaneously evaluating the quality of suppliers, manufacturers, and distributors across the entirety of the supply chain. To address this, the problem is formally expressed as a mathematical model, accommodating multi-item and multi-period scenarios while considering two distinct objective functions. In this article, Gholizadeh et al. (22) highlight the significant impact of an electrical discharge machine (EDM) on production management. The study delves into the investigation of EDM machining parameters and their influence on the volumetric flow rate, electrode corrosion percentage, and surface roughness. These parameters play a crucial role in determining the quality of the final product, thereby enhancing customer satisfaction and increasing the company's market share. Given the dynamic nature of machine parameters and the variations in production environments, the use of an uncertain model becomes imperative. To address the machining data under uncertainty, the study introduces a mathematical modeling approach based on the fuzzy possibility regression integrated (FPRI) model. Considering the uncertainty and unclear distribution of results and numbers obtained from the neural network, a robust data envelopment analysis approach (RDEA) is applied to identify the optimal tuning level of the parameters. The findings substantiate the accuracy and reliability of the proposed method for predicting and optimizing EDM parameters. In this study, Moosavi et al. (23) present a comprehensive set of contemporary bibliometric, network, and thematic analyses aimed at discerning influential contributors, principal research streams, and strategies for disruption management concerning supply chain (SC) performance within the context of the COVID-19 pandemic. The analyses conducted unveil resilience and sustainability as the predominant SC topics. Additionally, the primary research themes identified are centered around food, health-related supply chains, and technology-aided tools such as artificial intelligence (AI), the Internet of Things (IoT), and blockchains.

In this article authored by Berlin et al. (24), the discussion revolves around the configuration of closed-loop supply chains (CLSCs) for original equipment manufacturers (OEMs) to recover and remarket products, representing a pivotal avenue in the transition towards the circular economy (CE). Through a systematic literature review spanning from 2007 to 2021, this paper contributes to the literature by delineating the characteristics of open-loop supply chains (OLSC), providing empirical illustrations, and constructing a conceptual framework for the open- and closed-loop supply chain continuum. In this scholarly article, Haghshenas et al. (25) assert that the primary aim of supply chain design is the enhancement of profitability. Consequently, they introduced a cutting-edge model for a three-echelon closed-loop supply chain comprising the manufacturer, retailer, and collection centers. Notably, this model pioneers a distinct and autonomous channel dedicated to the sale of Reman products strategically aimed at augmenting manufacturer profitability. The model also incorporates considerations for the location, inventory, and pricing of the product, contributing to a comprehensive approach.

In this research, Asghari et al. (26) focused on pricing and advertising decisions within a closed-loop supply chain network. While pricing decisions have been extensively studied in this context, advertising decisions have received comparatively little attention. It is widely acknowledged that advertising plays a significant role in influencing customer behavior regarding the return of end-of-life products in a closed-loop supply chain. The primary novelty of this paper lies in the development of a new optimization model that incorporates both pricing and advertising decisions within a direct-sales closed-loop supply chain.

The study conducted by Simonetto et al. (27) directs its attention to the analysis of the benefits that Industry 4.0 technologies can offer in terms of mitigating risks in Closed Loop Supply Chains (CLSCs), specifically focusing on operational risks. Through two systematic literature reviews, the paper identifies the primary operational risks associated with CLSC activities and elucidates the impact of Industry 4.0 technologies on mitigating these identified risks. To summarize the reviews and support future managerial initiatives in the CLSC domain, the paper proposes a conceptual framework and a new cross-sectional matrix. The conclusion of the paper outlines identified open research opportunities.

Technical analysis indicators serve as widely used tools in financial markets, aiding investors in discerning buy and sell signals with a margin of error. The primary objective of this study is to develop novel and practical methods for identifying false signals emanating from technical analysis indicators within the precious metals market. As articulated by Fathollahi-Fard and Soleimani (28) in this article, the key innovation of this research lies in proposing hybrid neural network-based metaheuristic algorithms for accurate analysis while enhancing the performance of signals derived from technical analysis indicators. The ultimate finding underscores that the suggested neural network-based metaheuristics can function as valuable decision-support tools for investors, addressing and managing the significant uncertainties inherent in financial and precious metals markets.

In their paper, Ali et al. (29) assert that closed-loop supply chain (CLSC) networks offer a viable solution for effective waste management by facilitating the recycling, reassembly, and reuse of waste products. They tackle these challenges by presenting a comprehensive CLSC network that optimizes environmental, economic, and social footprints through a multi-objective optimization approach. To address the proposed model's scenario-based stochastic nature, they transform the multi-objective formulation into a single-objective model using a weighted sum method, incorporating a set of problem-specific heuristics. Additionally, they leverage Lagrangian relaxation theory to formulate various problem reformulations, aiming to achieve an optimal lower bound for the CLSC problem. This is complemented by a neighborhood-based algorithm designed to identify a feasible upper bound.

As highlighted earlier, the assessment of green supply chain efficiency has been explored in various scenarios; however, there has been a relatively limited emphasis on estimating the return to scale within green supply chains.

In this context, a seminal work by Banker et al. (30) has introduced a significant approach, emphasizing that extending the measurement of return to scale from a single number to an interval could broaden its applicability to Data Envelopment Analysis (DEA) domains with multiple inputs and outputs. They delineate an optimal boundary consisting of three segments representing increasing, constant, and decreasing returns to scale, respectively. The paper establishes a precise framework facilitating the identification of several optimal solutions.

Gholam Abri (31) has contributed a paper addressing the determination of "stability radius." A noteworthy aspect of this work is the introduction of a novel method for calculating the 'stability radius,' wherein variations in data neither alter the class of efficiency units nor the class of return to scale. The method's value lies in its capacity to discuss input and output changes through various strategies, subsequently fostering the enhancement of evaluated Decision Making Unit (DMU) performance via diverse options.

Zhang and Yang (32) have presented a paper on the calculation of network return to scale in two stages. They expound on network DEA as a non-parametric method for determining the return to scale of Decision-Making Units (DMUs) with multi-stage structures, examining a two-stage production process using network DEA techniques.

Alirezaee et al. (33), in their study, address a critical issue in developing a DEA model, specifically the identification of relevant returns to scale for the data. They introduced a purposeful, non-statistical method named the Angles method to identify technological return to scale in the data.

In a comprehensive investigation, Wang et al. (34) delve into the performance of energy in terms of CO_2 pollution and emission, employing variable return to scale and a non-radial model. Three main conclusions emerge:

1) Performance varies across the economic levels of the members,

2) Development of the members holds potential for further reduction in energy consumption and CO_2 emissions and

3) Reduction in energy and CO_2 emission intensity, coupled with the contribution of industrial-added value to total GDP, can effectively enhance energy performance and reduce CO_2 emissions.

This paper examines the use of DEA to determine the return to scale in the structure of green supply chains and considers the relationship between intermediate products. A new model is presented to estimate the return to scale at each stage and in the entire chain, thus estimating the return to scale for cement companies.

In continuation of this article, we will review basic concepts. Then, we will introduce suitable models for estimating return to scale in the structure of a four-level supply chain network. In this regard, we provide a practical example in the cement industry to illustrate the proposed models, and finally, we discuss and analyze the results obtained from model execution in the conclusion.

2. BASIC CONCEPTS

A set of production possibilities is defined as follows:
{T = (X, Y) output Y can be generated by input X}

It is observed that the set of production possibilities T is determined when the production function is known. Suppose n is an existing DMU that $Y_j = (y_{1j},\ldots,y_{sj})^t$, $X_j = (x_{1j},\ldots,x_{mj})^t$ are the input and output vectors of DMU_j, respectively and $Y_j \neq 0$, $Y_j \geq 0$, $X_j \neq 0$, $X_j \geq 0$.

In general, the production function is not available. Therefore, by accepting the principles of inclusion of observations, convexity, feasibility and the minimum of interpolation, a set of production possibilities will be defined as follows:

$$T_v=\left\{\binom{X}{Y} \middle| X \geq \sum_{j=1}^{n} \lambda_j X_j \ , \ Y \leq \sum_{j=1}^{n} \lambda_j Y_j \ , \ \sum_{j=1}^{n} \lambda_j = 1 \ , \ \lambda_j \geq 0 \ , \ j = 1,2,...,n \right\} \quad (1)$$

Now, suppose the unit under evaluation, DMU_o, is in T_v. For evaluating DMU_o, the following model, by an input nature, must be solved.

$$\text{Min } \theta, \quad \text{s.t:} \quad \binom{\theta X_o}{Y_o} \in T_v.$$

$$\begin{aligned} &\text{Min } \theta, \\ &\text{s.t:} \\ &\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, && i = 1,2,\ldots,m \\ &\sum_{j=1}^{n} \lambda_j y_{ij} \geq y_{ro}, && r = 1,2,\ldots,s \\ &\sum_{j=1}^{n} \lambda_j = 1, \\ &\lambda_j \geq 0 && j = 1,2,\ldots,n \end{aligned} \quad (2)$$

Model 2, identified as BCC in terms of input, was introduced by Banker et al. (30). It is evident that DMU_o achieves "strong efficiency" or "Pareto-Koopmans Efficiency" if and only if $\theta^* = 1$ and in every optimal solution of Model 2, the value of all auxiliary variables is zero.

Definition1. Return to scale in the Black Box. To define the return to scale in Black Box, suppose, $(x_o, y_o) \in T_v$.

a. If $\delta_1^* > 0$, so that, $0 \leq \delta < \delta_1^*$:

$$\begin{aligned} &Z_\delta = ((1+\delta)\, x_o, (1+\delta)\, y_o) \in T_v, \\ &Z'_\delta = ((1-\delta)\, x_o, (1-\delta)\, y_o) \notin T_v \end{aligned} \quad (3)$$

Then, "return to scale" will be increasing at this point.

b. If $\delta_2^* > 0$, so that, $0 \leq \delta < \delta_2^*$:

$$\begin{aligned} &Z_\delta = ((1+\delta)\, x_o, (1+\delta)\, y_o) \notin T_v, \\ &Z'_\delta = ((1-\delta)\, x_o, (1-\delta)\, y_o) \in T_v \end{aligned} \quad (4)$$

Then, "return to scale" will be descending at this point.

c. whereas DMU_o does not apply in **definition (a)** increasing and **(b)** descending return to scale does not apply, then, "return to scale" will be constant at this point.

Banker and Thrall (35) have articulated and substantiated the following two propositions regarding the estimation of return to scale in the Black Box using the BCC multiplicative model when the solutions are distinct and singular:

Theorem 1: Suppose (x_o, y_o) is an efficient DMU in the BCC model (on BCC border) and (v^*, u^*, u_o^*), the optimal answer is the BCC multiplicative model in the evaluation of this DMU. In addition, suppose the optimal answer u_o^* is unique, in which:

* Return to scale is increasing at the point (x_o, y_o), if and only if $u_o^* > 0$.

* Return to scale is constant at the point (x_o, y_o), if and only if $u_o^* = 0$.

* Return to scale is descending at the point (x_o, y_o), if and only if $u_o^* < 0$.

Proof: To prove above the theorem, refer to Banker and Thrall (35).

Theorem 2: Suppose, (x_o, y_o) is an efficient DMU in the BCC model (on the BCC border), and (v^*, u^*, u_o^*) is the optimal answer to the BCC multiplicative model in the evaluation of this DMU. In addition, suppose u_o^* is not unique; that is, we have multiple answers. Now, to calculate the return to scale, we consider models 5 and 6:

$$\begin{aligned} &u_o^{*+} = \max u_o, \\ &\text{s.t:} \\ &v^t x_o = 1, \\ &u^t y_j - v^t x_j + u_o \leq 0, && j = 1,2,\ldots,n \\ &u^t y_o - v^t x_o + u_o = 0, \\ &u \geq 0, \ v \geq 0, \ u_o \ \text{free} \end{aligned} \quad (5)$$

$$\begin{aligned} &u_o^{*-} = \min u_o, \\ &\text{s.t:} \\ &v^t x_o = 1, \\ &u^t y_j - v^t x_j + u_o \leq 0, && j = 1,2,\ldots,n \\ &u^t y_o - v^t x_o + u_o = 0, \\ &u \geq 0, \ v \geq 0, \ u_o \ \text{free} \end{aligned} \quad (6)$$

where in, $u_o^{*-} < u_o^{*+}$ and $u_o^* \in [u_o^{*-}, u_o^{*+}]$.

Whereas,

- If $u_o^{*+} > 0$ and $u_o^{*-} > 0$, the return to scale will be increasing at the point (x_o, y_o).
- If $u_o^{*+} \geq 0$ and $u_o^{*-} \leq 0$, the return to scale will be constant at the point (x_o, y_o).
- If $u_o^{*+} < 0$ and $u_o^{*-} < 0$, the return to scale will be descending at the point (x_o, y_o).

Proof: To prove the above theorem, refer to Banker and Thrall (35).

Definition2. Return to scale in the network: to define return to scale efficiency in the supply chain network, suppose $(x_o^1, y_o^1, z_o^{12}) \in T_v$ is the vector of the first level of the supply chain in DMU_o.

a. If $\delta_1^* > 0$, so that, $0 \le \delta < \delta_1^*$:

$$Z_\delta = \left((1+\delta)\, x_o^1 ,\, (1+\delta)\, y_o^1 ,\, (1+\delta)\, z_o^{12}\right) \in T_v ,$$
$$Z'_\delta = \left((1-\delta)\, x_o^1 ,\, (1-\delta)\, y_o^1 ,\, (1-\delta)\, z_o^{12}\right) \notin T_v \tag{7}$$

In that case, the "return to scale of the first level" will increase at the points $(x_o^1, y_o^1$, and $z_o^{12})$.

b. If $\delta_2^* > 0$, so that, $0 \le \delta < \delta_2^*$:

$$Z_\delta = \left((1+\delta)\, x_o^1 ,\, (1+\delta)\, y_o^1 ,\, (1+\delta)\, z_o^{12}\right) \notin T_v ,$$
$$Z'_\delta = \left((1-\delta)\, x_o^1 ,\, (1-\delta)\, y_o^1 ,\, (1-\delta)\, z_o^{12}\right) \in T_v \tag{8}$$

Where the "return to scale of the first level" will be descending at the points (x_o^1, y_o^1, z_o^{12}).

c. If DMU_o does not apply in the **definition (a)** increasing return to scale and **(b)** descending return to scale, then, "return to scale" is constant at the point (x_o^1, y_o^1, z_o^{12}). The definition of return to scale in levels 2, 3 and 4, as well as the return to scale of the whole supply chain, is done similarly. Further explanation is avoided to summarize the issue.

Subsequently, the proposed method will be examined utilizing the aforementioned definitions and theorems pertaining to return to scale in the Black Box and supply chain network.

3. PROPOSED METHOD

In traditional DEA models, the consideration of intermediate products and the interactions among different components within the system is often neglected. Addressing this limitation, network DEA models are employed. In these instances, the output of one level may be computed as the input of the subsequent level, and conversely, each level may possess independent inputs. The return to scale is an economic concept within DEA, denoting the maximum increase in output per unit increase in input. The identification of return to scale holds particular significance in developing DMU_o.

In order to draw the figure and to construct the framework, relevant supply chain literature, including the work of Darvish Motevalli et al. (15), has preferred from the available sources. This paper aims to estimate the return to scale of a green supply chain under various conditions within the cement industry using DEA methodology. The exploration follows the outlined structure:

In the above complete supply chain:

- L_1, L_2, L_3 and L_4 represent the supplier, manufacturer, distributor, and customer respectively.
- $X^f = (x_{ij}^f ,\ i = 1,2,\ldots,m)$ for $f = 1,2,3,4$ of input vector DMU_j, including independent inputs to the level L_f.
- $Z^{k_1k_2} = (z_{lj}^{k_1k_2} ,\ l=1,2,\ldots,p)$ for $k_1 = 1,2,3$ and $k_2 = 2,3,4$ intermediate data from the level L_{k_1} to L_{k_2} is the unit j, or, in other words, the output vector of the level L_{k_1} in the unit j, is also the input vector of level $L_{k_{1+1}}$.
- $Y^{k_3} = (y_{rj}^{k_3} ,\ r=1,2,\ldots,s)$ for $k_3 = 1,2,3,4$ is the output vector of the level L_{k_3}.

Consider a set of n identical supply chains similar to the configuration depicted in Figure 1, denoted as n Decision-Making Units (DMUs) as $DMU_1, DMU_2, \ldots, DMU_n$ in the DEA literature. The development of a novel network DEA model is imperative to assess the return to scale of the supply chain under varying conditions. Notably, the BCC model proves insufficient in identifying the corresponding black box returns to scale of the supply chain illustrated in Figure 1. This inadequacy arises due to its exclusive consideration of the inputs and outputs of the supply chain, disregarding the intermediate products generated at different levels within the supply chain. Hence, this study introduces a network BCC model for the complete supply chain, structured according to the nature of input. This model aims to calculate the return to scale at each level and, ultimately, for the entire supply chain. The model is articulated as follows:

$$\begin{aligned}
&\text{Min } \theta, \quad \text{s.t:}\\
&\textstyle\sum_{j=1}^{n} \lambda_j^1\ x_{ij}^1 \le \theta\, x_{io}^1 , && i = 1,2,\ldots,m_1\\
&\textstyle\sum_{j=1}^{n} \lambda_j^1\ z_{lj}^{12} \ge \sum_{j=1}^{n} \lambda_j^2\, z_{lj}^{12}, && l = 1,2,\ldots,p_1\\
&\textstyle\sum_{j=1}^{n} \lambda_j^1\ y_{rj}^1 \ge y_{ro}^1 , && r = 1,2,\ldots,s_1\\
&\textstyle\sum_{j=1}^{n} \lambda_j^2\ x_{ij}^2 \le \theta\, x_{io}^2, && i = 1,2,\ldots,m_2\\
&\textstyle\sum_{j=1}^{n} \lambda_j^2\ z_{lj}^{23} \ge \sum_{j=1}^{n} \lambda_j^3\ z_{lj}^{23}, && l = 1,2,\ldots,p_2\\
&\textstyle\sum_{j=1}^{n} \lambda_j^2\ y_{rj}^2 \ge y_{ro}^2, && r = 1,2,\ldots,s_2\\
&\textstyle\sum_{j=1}^{n} \lambda_j^3\ x_{ij}^3 \le \theta\, x_{io}^3, && i = 1,2,\ldots,m_3\\
&\textstyle\sum_{j=1}^{n} \lambda_j^3\ z_{lj}^{34} \ge \sum_{j=1}^{n} \lambda_j^4\ z_{lj}^{34}, && l = 1,2,\ldots,p_3\\
&\textstyle\sum_{j=1}^{n} \lambda_j^3\ y_{rj}^3 \ge y_{ro}^3, && r = 1,2,\ldots,s_3\\
&\textstyle\sum_{j=1}^{n} \lambda_j^4\ x_{ij}^4 \le \theta\, x_{io}^4, && i = 1,2,\ldots,m_4\\
&\textstyle\sum_{j=1}^{n} \lambda_j^4\ y_{rj}^4 \ge y_{ro}^4, && r = 1,2,\ldots,s_4\\
&\textstyle\sum_{j=1}^{n} \lambda_j^1 = 1 ,\quad \sum_{j=1}^{n} \lambda_j^2 = 1,\quad \sum_{j=1}^{n} \lambda_j^3 = 1,\quad \sum_{j=1}^{n} \lambda_j^4 = 1,\\
&\lambda_j^1 ,\ \lambda_j^2 ,\ \lambda_j^3 ,\ \lambda_j^4 \ge 0 , \quad j = 1,2,\ldots,n ,\ \theta \text{ free}
\end{aligned} \tag{9}$$

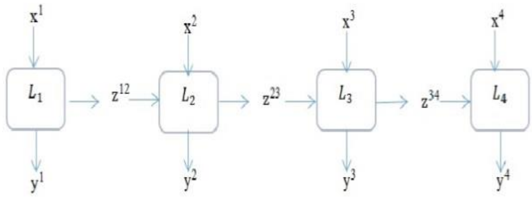

Figure 1. Four-level supply chain

Where in:

1. For $f = 1,2,3,4$ the constraints $\sum_{j=1}^{n} \lambda_j^f \, x_{ij}^f \le \theta \, x_{io}^f$, $i = 1,2,...,m$ are written corresponding to the independent inputs of the level S_f.

2. For $k_1 = 1,2,3$ and $k_2 = 2,3,4$ the constraints $\sum_{j=1}^{n} \lambda_j^{k_1} \, z_{lj}^{k_1k_2} \ge \sum_{j=1}^{n} \lambda_j^{k_2} \, z_{lj}^{k_1k_2}$, $l = 1,2,...,p$ corresponding to the output of the level S_{k_1} is written in unit jth and indicates that the convex combination of these outputs as the inputs of the level S_{k_2} must be less or equal to the production of the level S_{k_1}.

3. For $k_3 = 1,2,3,4$ the constraints $\sum_{j=1}^{n} y_j^{k_3} \, y_{rj}^{k_3} \ge y_{ro}^{k_3}$, $r = 1,2,...,s$ corresponding to the output of the level S_{k_3} is written.

Employing model 9, the dual formulation of the BCC model within a complete supply chain (represented in multiplicative form) is articulated as follows. This formulation serves as a tool for estimating the efficiency of a complete supply chain:

$$\text{Max } Z = \sum_{r=1}^{s_1} u_{r1} \, y_{ro}^1 + \sum_{r=1}^{s_2} u_{r2} \, y_{ro}^2 + \sum_{r=1}^{s_3} u_{r3} \, y_{ro}^3 + \sum_{r=1}^{s_4} u_{r4} \, y_{ro}^4 + u_{o1} + u_{o2} + u_{o3} + u_{o4},$$

s.t:

$$-\sum_{i=1}^{m_1} v_{i1} \, x_{ij}^1 + \sum_{l=1}^{p_1} w_{l1} \, z_{lj}^{12} + \sum_{r=1}^{s_1} u_{r1} \, y_{rj}^1 + u_{o1} \le 0, \quad j = 1,2,\dots,n$$

$$-\sum_{l=1}^{p_1} w_{l1} \, z_{lj}^{12} - \sum_{i=1}^{m_2} v_{i2} \, x_{ij}^2 + \sum_{l=1}^{p_2} w_{l2} \, z_{lj}^{23} + \sum_{r=1}^{s_2} u_{r2} \, y_{rj}^2 + u_{o2} \le 0, \quad j = 1,2,\dots,n$$

$$-\sum_{l=1}^{p_2} w_{l2} \, z_{lj}^{23} - \sum_{i=1}^{m_3} v_{i3} \, x_{ij}^3 + \sum_{l=1}^{p_3} w_{l3} \, z_{lj}^{34} + \sum_{r=1}^{s_3} u_{r3} \, y_{rj}^3 + u_{o3} \le 0, \quad j = 1,2,\dots,n \qquad (10)$$

$$-\sum_{l=1}^{p_3} w_{l3} \, z_{lj}^{34} - \sum_{i=1}^{m_4} v_{i4} \, x_{ij}^4 + \sum_{r=1}^{s_4} u_{r4} \, y_{rj}^4 + u_{o4} \le 0, \quad j = 1,2,\dots,n$$

$$\sum_{i=1}^{m_1} v_{i1} \, x_{io}^1 + \sum_{i=1}^{m_2} v_{i2} \, x_{io}^2 + \sum_{i=1}^{m_3} v_{i3} \, x_{io}^3 + \sum_{i=1}^{m_4} v_{i4} \, x_{io}^4 = 1,$$

$$v_{i1}, \ v_{i2}, \ v_{i3}, \ v_{i4} \ge 0,$$

$$u_{r1}, \ u_{r2}, \ u_{r3}, \ u_{r4} \ge 0,$$

$$w_{l1}, \ w_{l2}, \ w_{l3} \ge 0,$$

$$u_{o1}, \ u_{o2}, \ u_{o3}, \ u_{o4} \ \text{free}$$

To estimate the return to scale of levels 1, 2, 3, and 4 under distinct or identical conditions utilizing the BCC multiplicative model, let DMU_o represent an arbitrary complete supply chain in accordance with the structure depicted in Figure 1.

Initially, the return to scale of the first level is estimated, and subsequently, the estimations for levels 2, 3, and 4 are derived, mirroring the procedure applied to the first level.

3. 1. Estimation of the Return to Scale of the First Level (supplier)

For estimation of the return to scale of the first level, model 10 is first solved.

To do this, the target function of model 10 is considered in the optimization of Z^*. On Z^*, the efficiency of the first level (E_1^*) or (θ_1^*) will be obtained from the following model.

In order to determine the efficiency of the first level, the following model is solved:

$$\text{Max} \quad E_1 = \sum_{l=1}^{p_1} w_{l1} \, z_{lo}^{12} + \sum_{r=1}^{s_1} u_{r1} \, y_{ro}^1 + u_{o1},$$

s.t:

$$\sum_{i=1}^{m_1} v_{i1} \, x_{io}^1 = 1,$$

$$-\sum_{i=1}^{m_1} v_{i1} \, x_{ij}^1 + \sum_{l=1}^{p_1} w_{l1} \, z_{lj}^{12} + \sum_{r=1}^{s_1} u_{r1} \, y_{rj}^1 + u_{o1} \le 0, \quad j = 1,2,\dots,n$$

$$-\sum_{l=1}^{p_1} w_{l1} \, z_{lj}^{12} - \sum_{i=1}^{m_2} v_{i2} \, x_{ij}^2 + \sum_{l=1}^{p_2} w_{l2} \, z_{lj}^{23} + \sum_{r=1}^{s_2} u_{r2} \, y_{rj}^2 + u_{o2} \le 0 \quad j = 1,2,\dots,n$$

$$-\sum_{l=1}^{p_2} w_{l2} \, z_{lj}^{23} - \sum_{i=1}^{m_3} v_{i3} \, x_{ij}^3 + \sum_{l=1}^{p_3} w_{l3} \, z_{lj}^{34} + \sum_{r=1}^{s_3} u_{r3} \, y_{rj}^3 + u_{o3} \le 0, \quad j = 1,2,\dots,n \qquad (11)$$

$$-\sum_{l=1}^{p_3} w_{l3} \, z_{lj}^{34} - \sum_{i=1}^{m_4} v_{i4} \, x_{ij}^4 + \sum_{r=1}^{s_4} u_{r4} \, y_{rj}^4 + u_{o4} \le 0, \quad j = 1,2,\dots,n$$

$$Z^* = \frac{\sum_{r=1}^{s_1} u_{r1} \, y_{ro}^1 + \sum_{r=1}^{s_2} u_{r2} \, y_{ro}^2 + \sum_{r=1}^{s_3} u_{r3} \, y_{ro}^3 + \sum_{r=1}^{s_4} u_{r4} \, y_{ro}^4 + u_{o1} + u_{o2} + u_{o3} + u_{o4}}{\sum_{i=1}^{m_1} v_{i1} \, x_{io}^1 + \sum_{i=1}^{m_2} v_{i2} \, x_{io}^2 + \sum_{i=1}^{m_3} v_{i3} \, x_{io}^3 + \sum_{i=1}^{m_4} v_{i4} \, x_{io}^4},$$

$$v_{i1}, \ v_{i2}, \ v_{i3}, \ v_{i4} \ge 0,$$

$$u_{r1}, \ u_{r2}, \ u_{r3}, \ u_{r4} \ge 0,$$

$$w_{l1}, \ w_{l2}, \ w_{l3} \ge 0,$$

$$u_{o1}, \ u_{o2}, \ u_{o3}, \ u_{o4} \ \text{free}$$

In these conditions, two cases will be considered:

1. Where u_{o1} is unique;

2. Where u_{o1} has multiple answers.

To identify the uniqueness of u_{o1}, models 12 and 13 are considered.

In fact, to estimate the return to scale in u_{o1}, the following two models by the same achievable area and two target functions are used to estimate u_{o1}^- and u_{o1}^+.

$$u_{o1}^+ = \max u_{o1},$$

s.t: (12)

$$\sum_{i=1}^{m_1} v_{i1} \, x_{io}^1 = 1,$$

$-\sum_{i=1}^{m_1} v_{i1}\ x_{ij}^1\ +\sum_{l=1}^{p_1} w_{l1}\ \ z_{lj}^{12}+\sum_{r=1}^{s_1} u_{r1}$
$y_{rj}^1\ + u_{o1} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_1} w_{l1}\ \ z_{lj}^{12}\ -\sum_{i=1}^{m_2} v_{i2}\ x_{ij}^2 +\sum_{l=1}^{p_2} w_{l2}\ \ z_{lj}^{23}\ +\sum_{r=1}^{s_2} u_{r2}$
$y_{rj}^2\ + u_{o2} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_2} w_{l2}\ \ z_{lj}^{23}\ -\sum_{i=1}^{m_3} v_{i3}\ x_{ij}^3 +\sum_{l=1}^{p_3} w_{l3}\ \ z_{lj}^{34}\ +\sum_{r=1}^{s_3} u_{r3}$
$y_{rj}^3\ + u_{o3} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_3} w_{l3}\ \ z_{lj}^{34}\ -\sum_{i=1}^{m_4} v_{i4}\ x_{ij}^4 +\sum_{r=1}^{s_4} u_{r4}\ y_{rj}^4\ +$
$u_{o4}\ \leq 0, \qquad j = 1,2,\ldots,n$

$$Z^* = \frac{\sum_{r=1}^{s_1} u_{r1}\ y_{ro}^1 + \sum_{r=1}^{s_2} u_{r2}\ y_{ro}^2 +\sum_{r=1}^{s_3} u_{r3}\ y_{ro}^3 +\sum_{r=1}^{s_4} u_{r4}\ y_{ro}^4\ +\ u_{o1} +\ u_{o2} +\ u_{o3} +\ u_{o4}}{\sum_{i=1}^{m_1} v_{i1}\ x_{io}^1 +\sum_{i=1}^{m_2} v_{i2}\ x_{io}^2 +\sum_{i=1}^{m_3} v_{i3}\ x_{io}^3 +\sum_{i=1}^{m_4} v_{i4}\ x_{io}^4},$$

$E_1^* = \sum_{l=1}^{p_1} w_{l1}\ \ z_{lo}^{12}\ +\ \sum_{r=1}^{s_1} u_{r1}\ y_{ro}^1\ +\ u_{o1}\ ,$

$v_{i1}\ ,\ v_{i2}\ ,\ v_{i3}\ ,\ v_{i4}\ \geq\ 0,$

$u_{r1}\ ,\ u_{r2}\ ,\ u_{r3}\ ,\ u_{r4} \geq 0,$

$w_{l1}\ , w_{l2}\ , w_{l3} \geq 0,$

$u_{o1}\ ,\ u_{o2}\ \ ,\ u_{o3}\ \ ,\ u_{o4}$ free.

Also, model 13 is estimated with the same achievable area and different target function as model 12 to estimate u_{o1}^-.

$u_{o1}^- = \min u_{o1},$

s.t:

$\sum_{i=1}^{m_1} v_{i1}\ x_{io}^1\ =1,$

$-\sum_{i=1}^{m_1} v_{i1}\ x_{ij}^1\ +\sum_{l=1}^{p_1} w_{l1}\ \ z_{lj}^{12}+\sum_{r=1}^{s_1} u_{r1}$
$y_{rj}^1\ + u_{o1} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_1} w_{l1}\ \ z_{lj}^{12}\ -\sum_{i=1}^{m_2} v_{i2}\ x_{ij}^2 +\sum_{l=1}^{p_2} w_{l2}\ \ z_{lj}^{23}\ +\sum_{r=1}^{s_2} u_{r2}$
$y_{rj}^2\ + u_{o2} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_2} w_{l2}\ \ z_{lj}^{23}\ -\sum_{i=1}^{m_3} v_{i3}\ x_{ij}^3 +\sum_{l=1}^{p_3} w_{l3}\ \ z_{lj}^{34}\ +\sum_{r=1}^{s_3} u_{r3}$
$y_{rj}^3\ + u_{o3} \leq 0, \qquad j = 1,2,\ldots,n$

$-\sum_{l=1}^{p_3} w_{l3}\ \ z_{lj}^{34}\ -\sum_{i=1}^{m_4} v_{i4}\ x_{ij}^4 +\sum_{r=1}^{s_4} u_{r4}\ y_{rj}^4\ + \qquad (13)$
$u_{o4}\ \leq 0, \qquad j = 1,2,\ldots,n$

$$Z^* = \frac{\sum_{r=1}^{s_1} u_{r1}\ y_{ro}^1 + \sum_{r=1}^{s_2} u_{r2}\ y_{ro}^2 +\sum_{r=1}^{s_3} u_{r3}\ y_{ro}^3 +\sum_{r=1}^{s_4} u_{r4}\ y_{ro}^4\ +\ u_{o1} +\ u_{o2} +\ u_{o3} +\ u_{o4}}{\sum_{i=1}^{m_1} v_{i1}\ x_{io}^1 +\sum_{i=1}^{m_2} v_{i2}\ x_{io}^2 +\sum_{i=1}^{m_3} v_{i3}\ x_{io}^3 +\sum_{i=1}^{m_4} v_{i4}\ x_{io}^4},$$

$E_1^* = \sum_{l=1}^{p_1} w_{l1}\ \ z_{lo}^{12}\ +\ \sum_{r=1}^{s_1} u_{r1}\ y_{ro}^1\ +\ u_{o1}\ ,$

$v_{i1}\ ,\ v_{i2}\ ,\ v_{i3}\ ,\ v_{i4}\ \geq\ 0,$

$u_{r1}\ ,\ u_{r2}\ ,\ u_{r3}\ ,\ u_{r4} \geq 0,$

$w_{l1}\ , w_{l2}\ , w_{l3} \geq 0,$

$u_{o1}\ ,\ u_{o2}\ \ ,\ u_{o3}\ \ ,\ u_{o4}$ free.

1. If $u_{o1}^+ = u_{o1}^-$ that is, u_{o1}^* is unique and **theorem 1** will be correct.
2. If $u_{o1}^- < u_{o1}^+$ then $u_{o1}^* \in [u_{o1}^-\ , u_{o1}^+\ ]$ and **theorem 2** will be.
Tip 1: To get the returns to scale of the level 2, 3 and 4, the same procedure is followed.

3. 2. Examining the Conformity of the Calculation of the Return to Scale to its Definition in a Green Supply Chain

Suppose DMU_o is a supply chain with an intended structure as shown in Figure 1. This chain can be considered as follows:

$$\left(\underbrace{(x_o^1\ , y_o^1\ ,z_o^{12})}_{\text{Input and output vector in level (1)}}\ ,\ \underbrace{(x_o^2\ , z_o^{12}, y_o^2\ ,z_o^{23})}_{\text{Input and output vector in level (2)}}\ ,\ \underbrace{(x_o^3\ , z_o^{23}, y_o^3\ ,z_o^{34})}_{\text{Input and output vector in level (3)}}\ ,\ \underbrace{(x_o^4\ , z_o^{34}, y_o^4)}_{\text{Input and output vector in level (4)}} \right)$$

In this section, it is necessary to examine the conformity of the definition of return to scale in all four levels and finally, the return to scale of the whole chain and the way of estimation. In other words, the relationship between definition 1 and theorems 1 and 2 in Black Box will be studied in each level of the chain and finally the whole supply chain.

Theorem 3: The return to scale of the first level is increasing at the point $(x_o^1\ , y_o^1\ ,z_o^{12})$, if and only if $u_{o1}^* > 0$.

The return to scale of the first level is constant at the point $(x_o^1\ , y_o^1\ ,z_o^{12})$, If and only if $u_{o1}^* = 0$.

The return to scale of the first level is descending at the point $(x_o^1\ , y_o^1\ ,z_o^{12})$, If and only if $u_{o1}^* < 0$.

Proof: Suppose $(x_o^1\ , y_o^1\ ,z_o^{12})$ is the vector of the first level in the supply chain of DMU_o. In order to represent that the return to scale is increasing at this level, it is sufficient, by definition 1, to show that $\delta_1^* > 0$, that for every $0 \leq \delta < \delta_1^*$
We have:

a) $Z_\delta = \Big((1+\delta)\ x_o^1\ , (1+\delta)\ y_o^1\ , (1+\delta)\ z_o^{12}\ \Big) \in T_v\ ,$

b) $Z_\delta' = \Big((1-\delta)\ x_o^1\ , (1-\delta)\ y_o^1\ , (1-\delta)\ z_o^{12}\ \Big) \notin T_v$

To show the relation (a) by using the second constraint of model 12, we have:

$-v^*(1+\delta)\ x_o^1 + w^*(1+\delta)\ z_o^{12} + u^*(1+\delta)\ y_o^1 + u_{o1}^* =$

$(1+\delta)\ [-v^*x_o^1 + w^*z_o^{12} + u^*y_o^1 + u_{o1}^*\ ] - \delta u_{o1}^*$

But, in the above equation: $1+\delta > 0$ and $[-v^*x_o^1 + w^*z_o^{12} + u^*y_o^1 + u_{o1}^*\ ] \leq 0$. Because, $(x_o^1\ , y_o^1\ ,z_o^{12})$ is in the set of product possibilities. Therefore, $Z_\delta \in T_v$, if and only if $-\delta u_{o1}^* \leq 0$. So $-\delta \leq 0$, it should be $u_{o1}^* \geq 0$.
On the other hand, to show the relation (b) by using the second constraint of model 12, we have:

$-v^*(1-\delta)\, x_o^1 + w^*(1-\delta)\, z_o^{12} + u^*(1-\delta)\, y_o^1 + u_{o1}^* =$

$(1-\delta)\,[-v^* x_o^1 + w^* z_o^{12} + u^* y_o^1 + u_{o1}^*\,] + \delta u_{o1}^*$

But, in the above equation: $1\text{-}\delta > 0$ and $[-v^* x_o^1 + w^* z_o^{12} + u^* y_o^1 + u_{o1}^*\,] \leq 0$. Because, (x_o^1, y_o^1, z_o^{12}) is in the set of product possibilities. Therefore, $Z'_\delta \notin T_v$ if and only if $\delta u_{o1}^* > 0$. So $\delta > 0$, it should be $u_{o1}^* \geq 0$.

As a result, by the relation $u_{o1}^* \geq 0$ and $u_{o1}^* > 0$ proved previously, we conclude $u_{o1}^* > 0$. Exactly by this argument and simply, it can be possible to prove the return to scale in a fixed and descending state.

Theorem 4: The returns to scale in the second level increase at the point $(x_o^2, z_o^{12}, y_o^2, z_o^{23})$, if and only if $u_{o2}^* > 0$.

The returns to scale in 2nd level are constant at the point $(x_o^2, z_o^{12}, y_o^2, z_o^{23})$, if and only if $u_{o2}^* = 0$.

The returns to scale in 2nd level is descending at the point $(x_o^2, z_o^{12}, y_o^2, z_o^{23})$, if and only if $u_{o2}^* < 0$.

Proof: It is exactly the same as proving theorem 3.

Theorem 5: The returns to scale of the 3rd level are increasing at the point $(x_o^3, z_o^{23}, y_o^3, z_o^{34})$, if and only if $u_{o3}^* > 0$.

The return to scale of 3rd level is constant at the point $(x_o^3, z_o^{23}, y_o^3, z_o^{34})$, if and only if $u_{o3}^* = 0$.

The return to scale of 3rd level is descending at the point $(x_o^3, z_o^{23}, y_o^3, z_o^{34})$, if and only if $u_{o3}^* < 0$.

Proof: It is exactly the same as proving theorem 3.

Theorem 6: The return to scale in the 4th level is increasing at the point (x_o^4, z_o^{34}, y_o^4), if and only if $u_{o4}^* > 0$.

The returns to scale of the 4th level scale are constant at the point (x_o^4, z_o^{34}, y_o^4), if and only if $u_{o4}^* = 0$.

The returns to scale of the 4th level are descending at the point (x_o^4, z_o^{34}, y_o^4), if and only if $u_{o4}^* < 0$.

Proof: It is exactly the same as proving Theorem 3.

Note 2: Theorems 3, 4, 5 and 6, regarding the method of estimating four-level return to scale, were proved in the case that the optimal answer is unique. It can be easily proved that these 4 theorems are correct in the case that the optimal answer is not unique. To prove the case, for example, for the level 1, that is, u_{o1}^* is an optimal answer of models 12 and 13, that is, u_{o1}^{*+} and $u_{o1}^{*-}$and similar to the process of proving theorem 3. The proof of the theorem in levels 2, 3 and 4 is also similar.it is avoided to be written for the sake of summary.

Now, we will study the way of estimating the return to scale of the whole chain(u_o^*).

Note 3: As we know, we no longer solve the model to estimate the return to scale of the whole supply chain.

But, we estimate the return to scale of the whole supply chain(u_o^*) through the following relation:

$$u_o^* = u_{o1}^* + u_{o2}^* + u_{o3}^* + u_{o4}^* \tag{14}$$

1. If $u_{o1}^*, u_{o2}^*, u_{o3}^*, u_{o4}^*$ all are unique, u_o^* is unique and **theorem 1** will be correct.

2. If at least one of $u_{o1}^*, u_{o2}^*, u_{o3}^*, u_{o4}^*$ is not unique, so u_o^* (return to scale of the whole chain) will certainly not be unique and due to interval operations, u_o^* will be calculated.

Alefeld and Herzberger (36), in the first part (the calculation of real distances) of the book Preliminary Distance Estimations, have talked about the estimations of interval operations (distances):

The operations can be explicitly estimated in the ranges $A = [a_1, a_2]$ and $B = [b_1, b_2]$, so that:

$A + B = [a_1 + b_1\ ,\ a_2 + b_2]$.

So, to calculate u_o^*, we have:

$$\begin{aligned} &u_o^* = u_{o1}^* + u_{o2}^* + u_{o3}^* + u_{o4}^*, \\ &[u_o^-, u_o^+] = [u_{o1}^-, u_{o1}^+] + [u_{o2}^-, u_{o2}^+] + [u_{o3}^-, u_{o3}^+] + [u_{o4}^-, u_{o4}^+], \\ &\Rightarrow [u_o^-, u_o^+] = [u_{o1}^- + u_{o2}^- + u_{o3}^- + u_{o4}^-\ ,\ u_{o1}^+ + u_{o2}^+ + u_{o3}^+ + u_{o4}^+] \end{aligned} \tag{15}$$

Theorem 7: Suppose $u_o^* = u_{o1}^* + u_{o2}^* + u_{o3}^* + u_{o4}^*$, in that case:

- The return to scale of the whole supply chain is increasing, if and only if $u_o^* > 0$.
- The return to scale of the whole supply chain is constant if and only if $u_o^* = 0$.
- The return to scale of the whole supply chain is descending, if and only if $u_o^* < 0$.

Proof: Suppose DMU_o is a supply chain with an assumptive structure. In order to represent that the return to scale of the whole chain is increasing, it is enough, by definition 1, to show that if $\delta_1^* > 0$ for each $0 \leq \delta < \delta_1^*$, we have:

a) $Z_\delta \in T_v$,

b) $Z'_\delta \notin T_v$

To show the relation (a), we, from the sum of 4 constraints including the second to fifth, will have model 12:

$$-v_1^* x_j^1 + w_1^* z_j^{12} + u_1^* y_j^1 + u_{o1}^* - w_1^* z_j^{12} - v_2^* x_j^2 + w_2^* z_j^{23} + u_3^* y_j^3 + u_{o2}^* - w_2^* z_j^{23} - v_3^* x_j^3 + w_3^* z_j^{34} + u_3^* y_j^3 + u_{o3}^* - w_3^* z_j^{34} - v_4^* x_j^4 + u_4^* y_j^4 + u_{o4}^* \leq 0.$$

Now, we will construct Z_δ as follows:

$$\left[-v_1^*(1+\delta)\, x_o^1 + w_1^*(1+\delta)\, z_o^{12} + u_1^*(1+\delta)\, y_o^1 + u_{o1}^*\right] + \left[-w_1^*(1+\delta)\, z_o^{12} - v_2^*(1+\delta)\, x_o^2 + w_2^*(1+\delta)\, z_o^{23} + u_2^*(1+\delta)\, y_o^2 + u_{o2}^*\right] + \left[-w_2^*(1+\delta)\, z_o^{23} - v_3^*(1+\delta)\, x_o^3 + w_3^*(1+\delta)\, z_o^{34} + u_3^*(1+\delta)\, y_o^3 + u_{o3}^*\right] + \left[-w_3^*(1+\delta)\, z_o^{34} - v_4^*(1+\delta)\, x_o^4 + u_4^*(1+\delta)\, y_o^4 + u_{o4}^*\right] = (1+\delta)\left[\overbrace{-v_1^*\, x_o^1 + w_1^*\, z_o^{12} + u_1^*\, y_o^1 + u_{o1}^*}^{\text{The first sentence}}\right] + (1+\delta)\left[\overbrace{-w_1^*\, z_o^{12} - v_2^*\, x_o^2 + w_2^*\, z_o^{23} + u_2^*\, y_o^2 + u_{o2}^*}^{\text{The second sentence}}\right] + (1+\delta)\left[\overbrace{-w_2^*\, z_o^{23} - v_3^*\, x_o^3 + w_3^*\, z_o^{34} + u_3^*\, y_o^3 + u_{o3}^*}^{\text{The third sentence}}\right] + (1+\delta)\left[\overbrace{-w_3^*\, z_o^{34} - v_4^*\, x_o^4 + u_4^*\, y_o^4 + u_{o4}^*}^{\text{The fourth sentence}}\right] - \delta\left(\overbrace{u_{o1}^* + u_{o2}^* + u_{o3}^* + u_{o4}^*}^{\text{The fifth sentence}}\right) \tag{16}$$

But, since $(1+\delta) > 0$ and DMU_o is a supply chain, in the last sentence of the above 5 sentences, the first 4 sentences are less than or equal to zero. Therefore, $Z_\delta \in T_v$, if and only if $-\delta\, u_o^* \leq 0$, and because $-\delta < 0$, so, it must be $u_o^* \geq 0$.

On the other hand, to represent the relation (b) in a similar way to the above process and to prove theorem (3), we will obtain $u_o^* > 0$. As a result, we conclude $u_o^* > 0$ by the relations $u_o^* \geq 0$ and $u_o^* > 0$. The return to scale can be easily proved in the case of constant and increasing.

4. CASE STUDY

The effective management of a green supply chain stands as a critical concern for organizations, necessitating managerial efforts to formulate suitable models for performance estimation.

This study examines 42 companies operating within the cement industry to ascertain the overall return to scale across various levels. The comprehensive costs encompass annual expenditures associated with green training, eco-friendly design (pertaining to economic sustainability), personnel costs (relating to social sustainability), and environmental costs (reflecting environmental sustainability).

These factors manifest at different stages of the supply chain. The impetus for undertaking this research lies in the need to estimate the return to scale for cement factories, treating them as complete supply chains while accounting for these environmental considerations, where changes lie either entirely or partially beyond managerial control.

Validation of the proposed model is executed using real data from the cement industry in 2021. The model adeptly estimates the return to scale for the supply chain within this industry.

Additionally, the supporting source used to determine input, output, and intermediate data indicators, are determined based on authoritative sources and expert opinions, prominently featuring the work of Darvish Motevalli et al. (15), which is duly referenced.

Subsequently, the indicators outlined in Table 1 encompass financial, economic, and production metrics.

TABLE 1. Introduction of indicators and their definitions for J th DMU in independent inputs, intermediate data and outputs

Symbols		Classification of indicators	Title of Indicators
X_{ij}^1	x_{1j}^1	Primary inputs of supply chain	Quality of suppliers in terms of stability in supplying raw materials and consumables
	x_{2j}^1		Cost of green training and sustainability for addressing related issues along the supply chain
	x_{3j}^1		Total initial investment in mining and factory operation
	x_{4j}^1		Total cost of purchasing minerals, chemicals and other consumables
	x_{5j}^1		Total cost paid to contractors for mining extraction
	x_{6j}^1		Total transportation costs paid
	x_{7j}^1		Total financial costs
	x_{8j}^1		Total cost of salaries and wages paid
Y_{rj}^1	y_{1j}^1	First level outputs	Factory performance impact on creating adverse environmental effects in mining
	y_{2j}^1		Total mineral reserves available
Z_{lj}^{12}	z_{1j}^{12}	Intermediate data (1st level output and 2nd level input)	Total tonnage of chemical and mineral additives in the production process
	z_{2j}^{12}		Total mineral raw materials stored to use in cold season
	z_{3j}^{12}		Quality of training programs for suppliers and employees for sustainable production and TQM

Symbols		Classification of indicators	Title of Indicators
	z_{4j}^{12}		Total research and development costs
	z_{5j}^{12}		Industry's actual capacity
X_{ij}^{2}	x_{1j}^{2}	2nd level independent inputs	Suppliers flexibility
	x_{2j}^{2}		Improvement of relationships throughout the supply chain
	x_{3j}^{2}		Total cost to increase reliability in the supply chain
	x_{4j}^{2}		Paying attention to principles of legal standards and government regulations along the chain
	x_{5j}^{2}		Total energy payment costs
	x_{6j}^{2}		Electricity consumption per year in kilowatt-hours (kw/h)
	x_{7j}^{2}		Gas consumption per year in cubic meters per ton (m3/ton)
	x_{8j}^{2}		Mazut fuel energy consumption per year in liters per ton (liters/ton)
Y_{rj}^{2}	y_{1j}^{2}	2nd level outputs	Total dust particles produced in milligrams per cubic meter (mg / m3)
	y_{2j}^{2}		Annual average of emitted NOX greenhouse gases (mg/m3)
	y_{3j}^{2}		Annual average of emitted CO greenhouse gases (mg/m3)
	y_{4j}^{2}		Annual average of emitted SO2 greenhouse gases (mg/m3)
	y_{5j}^{2}		Impact of total infiltrated water consumption and wastewater on groundwater
Z_{lj}^{23}	z_{1j}^{23}	Intermediate data (2nd level output and 3rd level input)	Total tonnage of the clinker production by the factory
	z_{2j}^{23}		Total tonnage of the cement production by the factory
X_{ij}^{3}	x_{1j}^{3}	3rd level Independent inputs	Reverse logistics
	x_{2j}^{3}		Efforts to use advanced technologies and alternative raw materials
	x_{3j}^{3}		Total marketing costs
	x_{4j}^{3}		Cost of environmentally compatible design
	x_{5j}^{3}		Number of consumed cement bags per year of type pp
Y_{rj}^{3}	y_{1j}^{3}	Third level output	Total value of assets and inventory held ready for sale in Rials
Z_{lj}^{34}	z_{1j}^{34}	Intermediate data (3rd level output and 4th level input)	Total tonnage of bagged and bulk cement sales in the domestic and export markets
	z_{2j}^{34}		Total tonnage of clinker sold
	z_{3j}^{34}		Total finished product cost
X_{ij}^{4}	x_{1j}^{4}	4th level independent Inputs	Implementation of quality of life principles and social welfare for personnel
	x_{2j}^{4}		Factory's impact in the area of activity
	x_{3j}^{4}		Social responsiveness
Y_{rj}^{4}	y_{1j}^{4}	Final outputs	Total assets
	y_{2j}^{4}		Competitiveness and globalization of the factory brand
	y_{3j}^{4}		Cultural attitude towards creating green spaces
	y_{4j}^{4}		Total revenue from products sales
	y_{5j}^{4}		Total profit earned
	y_{6j}^{4}		Annual growth rate based on performance
	y_{7j}^{4}		Returns on assets (ROA)
	y_{8j}^{4}		Return on equity owners' accounts
	y_{9j}^{4}		Customer satisfaction

To exemplify the application of the proposed method, authentic data from 42 cement companies in the year 2021 is meticulously scrutinized, encompassing inputs, outputs, and intermediate data.

Subsequently, the implementation of models 10 and 11 is effectuated through GAMS software. Following the execution of models 10 and 11 using the GAMS software, the overall efficiency of the supply chain and the efficiency of levels one through four were estimated. According to the basic DEA definitions and principles mentioned by Banker et al. (30), a value of “1.00000” in Table 2 indicates "strong efficiency" or "Pareto-Koopmans Efficiency" for DMUs. The results are presented in Table 2.

By implementing models 12 and 13 using the GAMS software and considering theorem 2 proposed by Banker and Thrall (35), the return to scale of levels one to four was estimated. Finally, based on the relationship presented by Alfield and Herzberger (36), the return to scale of complete supply chain was estimated. The results are illustrated in Table 3.

TABLE 2. The efficiency of the whole supply chain and the efficiency of the first to fourth level

Company Name		**Z**	**E1**	**E2**	**E3**	**E4**
Sabad	DMU1	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sabik	DMU2	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sarab	DMU3	0.9981847	0.9993864	1.0000000	1.0000000	0.9817792
Sarbil	DMU4	0.9986577	1.0000000	1.0000000	0.9342312	1.0000000
Sarum	DMU5	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Saveh	DMU6	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sebagher	DMU7	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sebajnu	DMU8	0.9978771	1.0000000	1.0000000	0.9835436	1.0000000
Sabzeva	DMU9	0.9993072	1.0000000	1.0000000	0.9485462	1.0000000
Sebahan	DMU10	0.9985984	0.9985984	1.0000000	1.0000000	1.0000000
Sepaha	DMU11	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Satran	DMU12	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sajam	DMU13	0.9966413	1.0000000	0.9989980	0.9198612	1.0000000
Sakhash	DMU14	0.9996298	1.0000000	1.0000000	1.0000000	0.9900012
Sakhrom	DMU15	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sekhazar	DMU16	0.9999267	1.0000000	1.0000000	1.0000000	1.0000000
Sakhvaf	DMU17	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sakhuz	DMU18	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sedasht	DMU19	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sadur	DMU20	0.9986574	1.0000000	1.0000000	0.9541445	1.0000000
Sarud	DMU21	0.9996711	1.0000000	1.0000000	0.9720452	1.0000000
Seshargh	DMU22	0.9985093	1.0000000	1.0000000	0.9765528	1.0000000
Seshomal	DMU23	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sesafha	DMU24	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sesufi	DMU25	0.9981047	1.0000000	1.0000000	0.9686542	1.0000000
Saghrab	DMU26	0.9948798	1.0000000	1.0000000	0.9525511	1.0000000
Sefar	DMU27	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sefars	DMU28	0.9987711	1.0000000	1.0000000	0.9965400	1.0000000
Sefarum	DMU29	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sefanu	DMU30	0.9989667	0.9994331	1.0000000	0.9912634	1.0000000
Sefiruz	DMU31	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000

Company Name		Z	E1	E2	E3	E4
Seghain	DMU32	0.9990562	1.0000000	1.0000000	0.9853776	0.9505419
Sekarun	DMU33	0.9990175	1.0000000	1.0000000	0.9998777	1.0000000
Sekard	DMU34	0.9976153	1.0000000	0.9991167	0.9204183	1.0000000
Sekarma	DMU35	0.9990327	1.0000000	1.0000000	0.9139310	1.0000000
Selaar	DMU36	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Semazen	DMU37	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Samtaz	DMU38	0.9987039	0.9989622	1.0000000	1.0000000	1.0000000
Senir	DMU39	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sehormoz	DMU40	1.0000000	1.0000000	1.0000000	1.0000000	1.0000000
Sehegmat	DMU41	0.9995315	0.9995318	1.0000000	1.0000000	1.0000000
Silam	DMU42	0.9990813	0.9990813	1.0000000	1.0000000	1.0000000

TABLE 3. Estimating the returns to scale of the first to fourth levels and estimating the return to scale of the whole supply chain

Company Name		u_{o1}^+	u_{o1}^-	1st level return to scale	u_{o2}^+	u_{o2}^-	2nd level return to scale	u_{o3}^+	u_{o3}^-	3rd level return to scale	u_{o4}^+	u_{o4}^-	4th level return to scale	u_o^+	u_o^-	Total return to scale
Sabad	DMU1	+	-	constant	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Sabik	DMU2	+	+	increasing	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sarab	DMU3	-	-	decreasing	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Sarbil	DMU4	+	-	constant	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Sarum	DMU5	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Saveh	DMU6	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sebagher	DMU7	+	-	constant	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Sebajnu	DMU8	+	-	constant	+	-	constant	-	-	decreasing	+	-	constant	+	-	constant
Sabzeva	DMU9	-	-	decreasing	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Sebahan	DMU10	-	-	decreasing	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sepaha	DMU11	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Satran	DMU12	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sajam	DMU13	+	-	constant	+	+	increasing	+	+	increasing	+	-	constant	+	-	constant
Sakhash	DMU14	+	-	constant	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Sakhrom	DMU15	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sekhazar	DMU16	+	-	constant	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Sakhvaf	DMU17	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sakhuz	DMU18	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sedasht	DMU19	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sadur	DMU20	+	-	constant	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Sarud	DMU21	+	-	constant	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Seshargh	DMU22	+	-	constant	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Seshomal	DMU23	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sesafha	DMU24	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sesufi	DMU25	-	-	decreasing	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Saghrab	DMU26	+	-	constant	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Sefar	DMU27	+	-	constant	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Sefars	DMU28	-	-	decreasing	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Sefarum	DMU29	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant

Company Name		u_{o1}^{+}	u_{o1}^{-}	1st level return to scale	u_{o2}^{+}	u_{o2}^{-}	2nd level return to scale	u_{o3}^{+}	u_{o3}^{-}	3rd level return to scale	u_{o4}^{+}	u_{o4}^{-}	4th level return to scale	u_{o}^{+}	u_{o}^{-}	Total return to scale
Sefanu	DMU30	-	-	decreasing	+	-	constant	-	-	decreasing	-	-	decreasing	-	-	decreasing
Sefiruz	DMU31	+	-	constant	+	-	constant	+	-	constant	-	-	decreasing	+	-	constant
Seghain	DMU32	-	-	decreasing	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Sekarun	DMU33	+	-	constant	+	-	constant	+	+	increasing	-	-	decreasing	-	-	decreasing
Sekard	DMU34	+	-	constant	-	-	decreasing	+	+	increasing	-	-	decreasing	-	-	decreasing
Sekarma	DMU35	+	-	constant	+	-	constant	+	+	increasing	+	-	constant	+	-	constant
Selaar	DMU36	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Semazen	DMU37	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Samtaz	DMU38	-	-	decreasing	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Senir	DMU39	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sehormoz	DMU40	+	-	constant	+	-	constant	+	-	constant	+	-	constant	+	-	constant
Sehegmat	DMU41	-	-	decreasing	+	-	constant	+	-	constant	-	-	decreasing	-	-	decreasing
Silam	DMU42	+	+	increasing	+	-	constant	+	-	constant	+	-	constant	+	-	constant

5. CONCLUSION

In a broader context, the estimation of returns to scale furnishes valuable insights into the progression or constraints within Decision-Making Units (DMUs). An escalating return to scale implies that when the input is doubled, the output surpasses a twofold increase. This signifies that the expansion of DMUs is cost-effective. Conversely, a diminishing return to scale indicates that when the input is doubled, the output falls short of doubling. In this scenario, the constraints on DMUs are deemed reasonable.

In the contemporary business landscape, the enhancement of supply chain performance stands as the sole avenue to attain a competitive advantage in the global market. Thus, leveraging the Multiple BCC model within a complete supply chain, the initial step involves estimating the overall efficiency model and the efficiency at each level within the supply chain. Subsequently, the return-to-scale model of each level was estimated. Finally the return to scale model for complete supply chain was estimated. Given the pivotal role of the cement industry in national development, a thorough examination of total efficiency, efficiency at various levels, and the return to scale at each level and for the entire cement industry is conducted. This examination is based on the data from 2021 and employs the models and theorems expounded in this paper. The return to scale for 42 cement companies listed on the Stock Exchange, each comprising four levels (supplier, producer, distributor, and customer), is meticulously estimated.

The results of model execution indicate that, in the entire supply chain, 28 Companies exhibit constant return to scale. This implies that if inputs increase, the outputs also increase in the same proportion to the inputs. Therefore, economically, expanding or limiting DMUs results in neither profit nor loss. However, based on the defined input indices in Table 1, various decision-making approaches can be considered. For instance:

1) Considering the eighth input index, "Total cost of salaries and wages paid" if the development of these 28 Companies leads to increased employment, it may not be financially beneficial for the company. Still, it could be considered a socially and culturally beneficial activity, contributing to job creation and preventing social harm overall.

2) Regarding the second input index, "Cost of green training and sustainability for addressing related issues along the supply chain", if the development of these 28 companies results in a healthier environment and a reduction in harmful effects from factory activities, it may not yield financial benefits for the company. However, it can be considered a useful activity for health and environmental sustainability, serving future generations.

Based on the above examples, the decision regarding whether a DMU should expand or not depends on the decision-maker.

The results of the model execution indicate that, in the entire supply chain, 14 remaining companies exhibit decreasing return to scale. This implies that if inputs increase, outputs increase less than the input ratio, economically justifying the limitation of DMUs.

Furthermore, executing the model using GAMS software revealed that 2 cement companies at the supplying level, 1 company at the production level, and 14 companies at the distributing level have ascending return to scale. It means that while the overall return to scale of these 42 companies is either constant or decreasing, some companies, according to the results in Table 3, exhibit ascending return to scale at various levels. In economic terms, this indicates that developing these DMUs at specific levels within the supply chain is economically viable.

The analysis suggests that if a company's return to scale is upward at a specific level, it needs to be investigated whether the overall return to scale of that company is constant or decreasing. If the overall return to scale of the company throughout the supply chain is decreasing, and the return to scale at one of the levels is increasing, the development of this DMU at the intended level is not economically feasible. This means that by expanding a specific level of the company's chain where its return to scale has increased, the overall return to scale of the chain in the company remains reduced.

However, if the overall return to scale of the company throughout the supply chain is constant and the return to scale at one of the levels is increasing, the development of the company can be considered, depending on the decision-maker's perspective. It is recommended to consider relevant indicators regarding employment generation, reduction of social damages, health, environmental sustainability, etc.

The following topics are proposed for consideration as future research needs:

- Estimating return to scale in a four-level supply chain in the presence of uncontrollable and undesirable factors;
- Determining units with the Most Productive Scale Size (MPSS) in the green supply chain using data envelopment analysis;
- Presenting non-radial models for evaluating the performance of the complete supply chain with dependent and independent inputs and outputs;
- Determining left and right scale efficiencies in the 4-level green supply chain using data envelopment analysis;
- Furthermore, in future research, it would be possible to investigate the waste of intermediate outputs in the supply chain, resulting from the imbalance between supply and demand in internal sectors. A comprehensive study on resource efficiency for the supply chain could also be presented.

6. REFERENCES

1. Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European journal of operational research. 1978;2(6):429-44. https://doi.org/10.1016/0377-2217(78)90138-8
2. Färe R, Grosskopf S. Intertemporal production frontiers: with dynamic DEA. Journal of the operational research society. 1997;48(6):656-. https://doi.org/10.1057/palgrave.jors.2600779
3. Färe R, Grosskopf S, Whittaker G. Network Dea II. Data Envelopment Analysis: A Handbook on the Modeling of Internal Structures and Networks. 2014:307-27. https://doi.org/10.1016/S0038-0121(99)00012-9
4. Saranga H, Moser R. Performance evaluation of purchasing and supply management using value chain DEA approach. European journal of operational research. 2010;207(1):197-205. https://doi.org/10.1016/j.ejor.2010.04.023
5. Chen C, Yan H. Network DEA model for supply chain performance evaluation. European journal of operational research. 2011;213(1):147-55. https://doi.org/10.1016/j.ejor.2011.03.010
6. Tavana M, Mirzagoltabar H, Mirhedayatian SM, Saen RF, Azadi M. A new network epsilon-based DEA model for supply chain performance evaluation. Computers & Industrial Engineering. 2013;66(2):501-13. https://doi.org/10.1016/j.cie.2013.07.016
7. Mirhedayatian SM, Azadi M, Saen RF. A novel network data envelopment analysis model for evaluating green supply chain management. International Journal of Production Economics. 2014;147:544-54. https://doi.org/10.1016/j.ijpe.2013.02.009
8. Shafiee M, Lotfi FH, Saleh H. Supply chain performance evaluation with data envelopment analysis and balanced scorecard approach. Applied mathematical modelling. 2014;38(21-22):5092-112. https://doi.org/10.1016/j.apm.2014.03.023
9. Grigoroudis E, Petridis K, Arabatzis G. RDEA: A recursive DEA based algorithm for the optimal design of biomass supply chain networks. Renewable Energy. 2014;71:113-22. https://doi.org/10.1016/j.renene.2014.05.001
10. Khodakarami M, Shabani A, Saen RF, Azadi M. Developing distinctive two-stage data envelopment analysis models: An application in evaluating the sustainability of supply chain management. Measurement. 2015;70:62-74. https://doi.org/10.1016/j.measurement.2015.03.024
11. Tajbakhsh A, Hassini E. A data envelopment analysis approach to evaluate sustainability in supply chain networks. Journal of Cleaner Production. 2015;105:74-85. https://doi.org/10.1016/j.jclepro.2014.07.054
12. Tavana M, Kaviani MA, Di Caprio D, Rahpeyma B. A two-stage data envelopment analysis model for measuring performance in three-level supply chains. Measurement. 2016;78:322-33. https://doi.org/10.1016/j.measurement.2015.10.023
13. Yousefi S, Soltani R, Saen RF, Pishvaee MS. A robust fuzzy possibilistic programming for a new network GP-DEA model to evaluate sustainable supply chains. Journal of Cleaner Production. 2017;166:537-49. https://doi.org/10.1016/j.jclepro.2017.08.054
14. Fathi A, Saen RF. A novel bidirectional network data envelopment analysis model for evaluating sustainability of distributive supply chains of transport companies. Journal of Cleaner Production. 2018;184:696-708. https://doi.org/10.1016/j.jclepro.2018.02.256
15. Darvish Motevalli MH, Hosseinzadeh Lotfi F, Shoja N, Gholamabri A. A new model for calculating the efficiency of existing cement companies with a network structure (an application of data envelopment analysis). Financial engineering and portfolio management. 2019;10(38):65-93. 20.1001.1.22519165.1398.10.38.4.0
16. Tavassoli M, Saen RF. Predicting group membership of sustainable suppliers via data envelopment analysis and discriminant analysis. Sustainable Production and Consumption. 2019;18:41-52. https://doi.org/10.1016/j.spc.2018.12.004
17. Torabi N, Tavakkoli-Moghaddam R, Najafi E, Hosseinzadeh-Lotfi F. A two-stage green supply chain network with a carbon emission price by a multi-objective interior search algorithm. International Journal of Engineering, Transactions C: Aspects. 2019;32(6):828-34. 10.5829/ije.2019.32.06c.05
18. Asadpour M, Boyer O, Tavakkoli-Moghaddam R. A blood supply chain network with backup facilities considering blood groups and expiration date: a real-world application. International

Journal of Engineering, Transactions B: Applications. 2021;34(2):470-9. 10.5829/ije.2021.34.02b.19

19. Fathollahi-Fard AM, Dulebenets MA, Hajiaghaei–Keshteli M, Tavakkoli-Moghaddam R, Safaeian M, Mirzahosseinian H. Two hybrid meta-heuristic algorithms for a dual-channel closed-loop supply chain network design problem in the tire industry under uncertainty. Advanced engineering informatics. 2021;50:101418. https://doi.org/10.1016/j.aei.2021.101418
20. Moosavi S, Seifbarghy M. A robust multi-objective fuzzy model for a green closed-loop supply chain network under uncertain demand and reliability (a case study in engine oil industry). International Journal of Engineering, Transactions C: Aspects. 2021;34(12):2585-603. 10.5829/ije.2021.34.12c.03
21. Sahraeian R, Afshar MA, Hosseini SMH. A bi-objective cold supply chain for perishable products considering quality aspects: a case study in Iran dairy sector. International Journal of Engineering Transactions B: Applications. 2022;35(2). 10.5829/ije.2022.35.02b.22
22. Gholizadeh H, Fathollahi-Fard AM, Fazlollahtabar H, Charles V. Fuzzy data-driven scenario-based robust data envelopment analysis for prediction and optimisation of an electrical discharge machine's parameters. Expert Systems with Applications. 2022;193:116419. https://doi.org/10.1016/j.eswa.2021.116419
23. Moosavi J, Fathollahi-Fard AM, Dulebenets MA. Supply chain disruption during the COVID-19 pandemic: Recognizing potential disruption management strategies. International Journal of Disaster Risk Reduction. 2022;75:102983. https://doi.org/10.1016/j.ijdrr.2022.102983
24. Berlin D, Feldmann A, Nuur C. The relatedness of open-and closed-loop supply chains in the context of the circular economy; framing a continuum. Cleaner Logistics and Supply Chain. 2022;4:100048. https://doi.org/10.1016/j.ijdrr.2022.102983
25. Haghshenas P, Sahraeian R, Golmohammadi A-M. A state-of-the-art model of location, inventory, and pricing problem in the closed-loop supply chain network. International Journal of Engineering, Transactions B: Applications,. 2022;35(8):1558-70. 10.5829/ije.2022.35.08b.12
26. Asghari M, Afshari H, Mirzapour Al-e-hashem S, Fathollahi-Fard AM, Dulebenets MA. Pricing and advertising decisions in a direct-sales closed-loop supply chain. Computers & Industrial Engineering. 2022;171:108439. https://doi.org/10.1016/j.cie.2022.108439
27. Simonetto M, Sgarbossa F, Battini D, Govindan K. Closed loop supply chains 4.0: From risks to benefits through advanced technologies. A literature review and research agenda. International Journal of Production Economics. 2022:108582. https://doi.org/10.1016/j.ijpe.2022.108582
28. Mousapour Mamoudan M, Ostadi A, Pourkhodabakhsh N, Fathollahi-Fard AM, Soleimani F. Hybrid neural network-based metaheuristics for prediction of financial markets: a case study on global gold market. Journal of Computational Design and Engineering. 2023;10(3):1110-25. https://doi.org/10.1093/jcde/qwad039
29. Ali SM, Fathollahi-Fard AM, Ahnaf R, Wong KY. A multi-objective closed-loop supply chain under uncertainty: An efficient Lagrangian relaxation reformulation using a neighborhood-based algorithm. Journal of Cleaner Production. 2023;423:138702. https://doi.org/10.1016/j.jclepro.2023.138702
30. Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management science. 1984;30(9):1078-92. https://doi.org/10.1287/mnsc.30.9.1078
31. Abri AG. An investigation on the sensitivity and stability radius of returns to scale and efficiency in data envelopment analysis. Applied mathematical modelling. 2013;37(4):1872-83. https://doi.org/10.1016/j.apm.2012.04.047
32. Zhang Q, Yang Z. Returns to scale of two-stage production process. Computers & industrial engineering. 2015;90:259-68. https://doi.org/10.1016/j.cie.2015.09.009
33. Alirezaee M, Hajinezhad E, Paradi JC. Objective identification of technological returns to scale for data envelopment analysis models. European journal of operational research. 2018;266(2):678-88. https://doi.org/10.1016/j.ejor.2017.10.016
34. Wang Z, He W, Wang B. Performance and reduction potential of energy and CO2 emissions among the APEC's members with considering the return to scale. Energy. 2017;138:552-62. https://doi.org/10.1016/j.energy.2017.07.059
35. Banker RD, Thrall RM. Estimation of returns to scale using data envelopment analysis. European Journal of operational research. 1992;62(1):74-84. https://doi.org/10.1016/0377-2217(92)90178-C
36. Alefeld G, Herzberger J. Introduction to interval computation: Academic press; 2012.

Persian Abstract

چکیده

امروزه، تمرکز بر کسب مزیت رقابتی در بازار جهانی کسب و کار در بهبود عملکرد زنجیره تامین نهفته است. در این مقاله سعی شده است نحوه بدست آوردن بازده به مقیاس در ساختار زنجیرۀ تأمین سبز چهار سطحی با استفاده از تحلیل پوششی داده‌ها مورد بررسی قرار گیرد. برای دستیابی به این هدف ، با استفاده از مدل مضربی زنجیرۀ تأمین کامل، ابتدا بازده به مقیاس هر مرحله از زنجیره تخمین زده می شـــود و در نهایت منجر به تخمین بازده به مقیاس کل زنجیرۀ تأمین می گردد. جامعه آماری این پژوهش کاربردی، همسو با اهداف آن، ۴۲ شرکت سیمان می‌باشد. بازده به مقیاس این شرکتها که زنجیرۀ متناظر هر یک از آنها دارای چهار سطح تأمین‌کننده، تولیدکننده، توزیع‌کننده و مشتری می‌باشد، مورد ارزیابی قرار گرفتند. نتایج اجرای مدل نشان داد که در کل زنجیره، بازده به مقیاس ۲۸ شرکت ثابت و ۱۴ شرکت کاهشی می باشد و از طرفی تعداد ۲ شرکت در بخش تأمین‌کننده، ۱ شرکت در بخش تولیدکننده و ۱۴ شرکت در بخش توزیع‌کننده بازده به مقیاس صعودی دارند. یافته‌ها تأکید می‌کنند که از نظر اقتصادی افزایش بازده به مقیاس، گسترش واحدهای تصمیم‌گیری را مقرون‌به‌صرفه می‌کند و برعکس، کاهش بازده به مقیاس، محدودیت منطقی DMU ها را مقرون‌به‌صرفه می‌کند.