



## Pupil Center Detection Using Radial Symmetry Transform to Measure Pupil Distance in the Eye

J. Utama\*, V. R. L. Fitriani

Faculty of Engineering and Computer Science, Universitas Komputer Indonesia, Bandung, Indonesia

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### ABSTRACT

In patients with refractive errors or impaired vision, light rays received by the pupil do not fall directly onto the retina. This can be corrected by wearing monocled glasses. The focal point of the eyeglass lens needs to be adjusted to the center of the user's pupil. This can be known through the measured pupil distance (PD) value information. The measurement of the PD is very important to determine the center distance of the pupils in both eyes, where the eye does not experience the prism effect. This study aims to apply the radial symmetry transformation (RST) method combined with self-quotient (SQI) imagery to detect the pupillary center and measure PD. This algorithm combines to get more optimal results in detecting the center of the pupil in dark conditions or those exposed to shadow illumination. The program created using the MATLAB software simulates PD measurements for pupillary center detection in bright and dark images conditions. The test was carried out ten times, and the results showed that the system was able to measure PD on low-resolution images of 300 x 300 pixels at 72 dpi in bright image conditions; with measurement uncertainty values in each image of 0.60 mm. As for testing on dark images, the uncertainty values are 0.80 mm. In this case, the standard deviation value is obtained from the effect of the different dimensions of the face object on the tested image.

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## 1. INTRODUCTION

Glasses can help improve the quality of vision because they use lenses to regulate the light rays entering the eye. Light passing through the lens will be focused on one point and directed to the eyes via the pupil [1]. The light that enters the eye will be focused first at a point called the optical center (OC), which is located on the eyeglass lens so that the light that can be received by the pupil is right on the retina. The OC needs to be adjusted to the pupil distance, also called pupil distance (PD), to be comfortable when using glasses [2]. It is important to measure PD to determine the distance between the pupillary locations in both eyes so as not to experience prism effects or irregular focus shifts [3]. However, the PD measurement is still often skipped for time effectiveness in the examination because the measurement is still done manually by the examiner using the PD ruler or with tools such as the pupilometer

[1]. The estimated time during the examination can be minimized by the presence of an automatic system for measuring pupil distance that utilizes image processing on photos or images of eye objects [4, 5].

Research conducted by Harto and Rahmani [6] utilized PD values for facial recognition systems. The system takes the face area to be converted from a color image into a grayscale image and then uses the Viola-Jones method, which focuses on cropping the eye area. The eye area image will go through several filter processes until the system can detect the pupil of the eye. The search for the centroid of the pupil of the eye is used to calculate the PD value by calculating the Euclidean distance, which will be displayed in pixels. This value will be used as a parameter by the system to recognize a person's face [6]. The pupil of the eye has the same shape as the iris, which is round. The existence of a characteristic pupil shape is used by some researchers as a way to detect it. Research conducted by Vázquez in

\*Corresponding Author Institutional Email:  
[jana.utama@email.unikom.ac.id](mailto:jana.utama@email.unikom.ac.id) (J. Utama)

detecting the center of the pupil was carried out using four methods, namely the Circular Hough Transform (CHT), Ellipse Fitting (EF), Integro-Differential Operator (IDO), and Radial Symmetry Transform (RST). The result of the experiment in detecting the pupillary center is that the RST method is better in terms of accuracy and robustness than other types of methods. RST has the advantage of being robust when detecting the center of the pupil under disturbance conditions such as the presence of glasses, eyelashes, or the reflection of light in the eye image. This is because the disturbance is not affected by the radial shape which is ignored when the system is running compared to three other methods such as CHT which is still easily disturbed by objects around the eye and results in inaccurate detection results. EF relies on the use of a threshold which results in the detection process being disturbed by other objects around the eye area, especially objects with dark colors, and IDO utilizes the contours of objects. The presence of other elements that appear around the eye area causes the contours of the pupil object to be disrupted and has an impact on the inaccuracy of the prediction of the detected pupil center [7]. In low-resolution images [8], such as those produced by webcam cameras, the center of the pupil of the eye can be detected properly using the RST method [9, 10]; because this method works well in detecting the center point of an object. Lyasheva et al. [11] proved it through a comparison of the Hough method with the Fast Radial Symmetry Transform (FRST) in counting the number of objects, with the FRST method being able to detect the number of objects through the proximity level of the detected pupillary center. Similarly to that study, Ram and Rodríguez [12] tested FRST in detecting the number of nuclear cells by detecting the object's center. The research conducted by Zafari et al. [13] is FRST testing on overlapping objects that give good segmentation results through the detection of the center of the object.

However, the RST performance is less than optimal in dark-state images. The use of the self-quotient image (SQI) has the advantage of reducing the illumination or shadow in the image. SQI can remove shadows in dark areas of the image [14-16]. In addition, there are also Multi-Scale Retinex and Histogram Equalization [17] which are used for face illumination. Multi-Scale Retinex has an advantage for image improvement in terms of SNR (Signal to Noise Ratio), but the histogram distribution is not evenly distributed in improving images. Histogram Equalization can improve the illumination effect on dark images by equalizing the spread of the histogram to reduce the illumination effect on the image. SQI was used in the experiment because it does not destroy important information in low quality images such as edges. Once the pupil has been detected by the system, the next most important part is measuring the distance between the two pupils. This distance can be

measured with Euclidean measurement, which is a calculation method for measuring the distance between two points in Euclidean space [18, 19].

## 2. METHOD

RST is one of the methods used to detect the center point of an object, especially objects that have circular characteristics. RST can work optimally on this object. The advantages of RST, which can optimally detect the center point of a circular object, are utilized in finding the center point of the pupil of the eye because of the characteristic circular shape of the pupil. The search area for the center of the pupil will focus on the negative area (p-ve), or the area with a low level of gray because the pupil has the darkest color of the rest of the eye.

At first, the pupillary area boundary is sought to determine the edge of the area around the pupil. This edge will follow the shape of the circle around the pupil, the iris. The known edge of the circle is then used as a place to consider the number of points that can be formed into a line of radius  $n$ . The magnitude of the radii value given to the system influences the radius  $n$ . The radii then meet and intersect at the center of the circle ( $p$ ). The point of intersection at the center of the circle resulting from the radius  $n$  becomes a marker of the position of the center of the pupil that has been detected by RST, as illustrated in Figure 1. The range of  $n$  to be used is a minimum of 7 and a maximum of 11. The point of marking the center of the pupil can be confirmed by the large alpha value given to the system. The clarity of the point as a marker can be set by its thickness at the given standard factor value.

In dark images, the eye area on the face is often affected by the illumination of shadows. Information obtained in the pupillary area of the eye becomes difficult to obtain. This has an impact on the RST method when searching for a disturbed pupillary center due to the interference factor. SQI is a solution so that the detection of pupillary centers by RST can be carried out optimally, especially in dark images. SQI was used in the

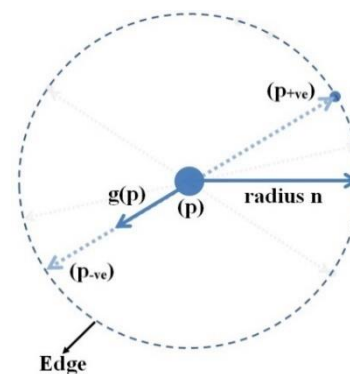
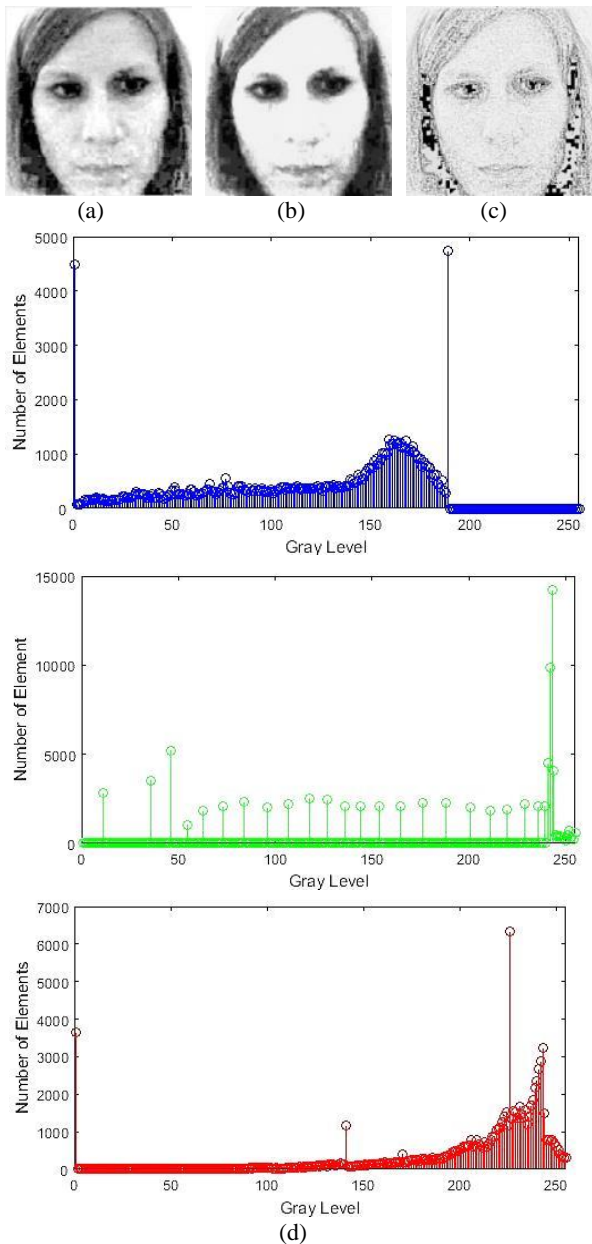


Figure 1. Center Point Detection on RST

experiment because it does not destroy important information in low quality images such as edges. The SQI method works to separate intrinsic elements as identity in images that are mixed with extrinsic elements such as shadow illumination in images. In accordance with system requirements because it tests cropping images of the eye with low image quality. As shown in Figure 2 for image enhancement using SQI, shadows in the area around the eyes are reduced and the image is created in a bright state. The histogram on SQI shows that the image tends towards a graylevel value of 255 (white in color)

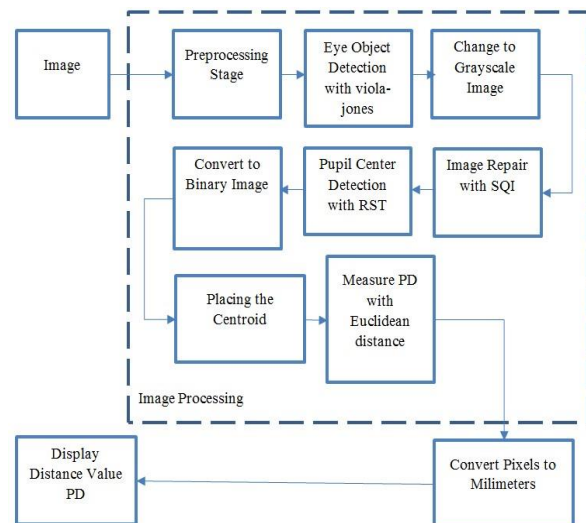


**Figure 2.** Comparison of (a) MSR, (b) HE, and (c) SQI in correcting dark images. Also presented are (d) histograms of each method; blue for MSR, green for HE, and red for SQI

with brighter images compared to MSR and HE. The shadow effect on the image can be reduced through an image separation process based on extrinsic elements, namely the effect of shadow illumination, and an intrinsic element, namely the object in the image itself. The results of image improvement by SQI help RST detect the center of the pupil more accurately in dark images.

**2. 1. PD Measurement System Design** The system created is a simulation that implements the RST method combined with SQI in detecting the center of the pupil to determine the size of the pupil distance (PD). Images with bright or dark conditions are used in the test. The image processing process can be seen in the block diagram in Figure 3.

Detects pupillary center distance, measured as the PD value, in images using datasets from MPIIGaze shown in Figure 4 and Real Self-Portraits in Figure 5. MPII Gaze are datasets containing face image data on 15 different models of people using their laptops daily for three months with a total of 213,659 images with an image resolution of 1280 x 720 pixels at 96 dpi. Real Self-Portrait datasets are a collection of real face images taken using a mobile phone camera on 2 different models of people with a total of 40 images. The use of different cameras makes the resolution of the image different for each model. The first model has a resolution of 2448 x 2448 pixels at 72 dpi and the second model has a resolution of 2576 x 1932 pixels at 72 dpi. The resolution for each dataset image will be changed and uniformed to a size of 300 x 300 pixels at 72 dpi to suit the system so it can work optimally. Done by processing the image to get information about the distance to the center of the pupil. The image used is an image that shows the face area with a straight head facing the camera. This system was built using Matlab 2015a software.



**Figure 3.** Block Diagram of the PD Value Search Process



Figure 4. Image of MPII Gaze Dataset



Figure 5. Image of Real Self-Photo Dataset

**2. 1. 1. Eye Detection with Viola-Jones** The object area in the image will be focused on the eye area. As shown in Figure 6, the system will automatically crop the eye area using the Viola-Jones method. The cropped image is used as an object that is focused on detecting the center of the pupil.

**2. 1. 2. Converting RGB Image to Grayscale and Image Improvement with SQI** Figure 7 shows the cropped area of the eye that is converted from an RGB image to a grayscale image. In the eye area that is disturbed by shadow illumination, the image is then corrected by SQI. This process is done to reduce the shadow effect that interferes with the system's ability to detect the center of the pupil.

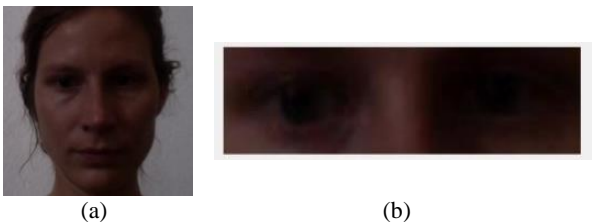


Figure 6. Eye Area Cropping Process on (a) Original Image and (b) Cropping Results

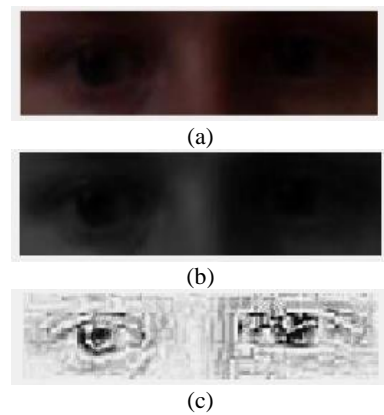


Figure 7. The Process of Changing Color and Image Quality with (a) RGB Image, (b) Grayscale Image, (c) Image Improvement Results with SQI

**2. 1. 3. Detection of the Pupil Center with RST**

The corrected image generated by SQI is then used for the pupillary center detection process using RST, which is shown in Figure 8. The pupil center can be detected at the point of intersection of the n-radius line formed around the pupil. The magnitude of the given radii value determines the number of radii n.

**2. 1. 4. Placement of Centroids to Measure PD Value**

The detected pupil center point in the form of a binary image is then marked using a centroid. The PD measurement results are obtained from the calculation of the distance between the first centroid (Point 1) and the second centroid (Point 2) as shown in Figure 9 using Equation (1) from the Euclidean method.

$$d(x, y) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2} \tag{1}$$

The measurement results are then divided by a reference value of 1.325 pixels per millimeter so that the resulting PD value is in millimeter units.

**2. 2. System Design with Simulation**

The system was tested on the image dataset that has been provided, namely Real Self-Portraits taken using a mobile phone camera and images from the MPII Gaze dataset. The PD

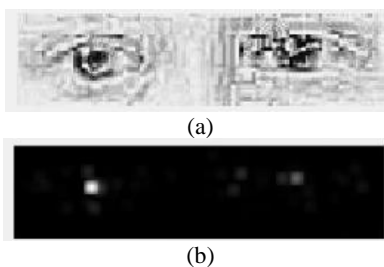
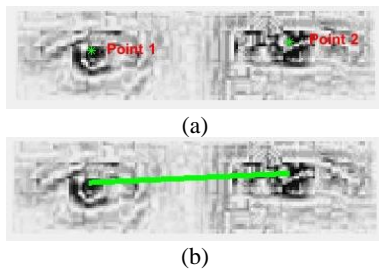


Figure 8. Pupil Center Detection Process with (a) SQI Corrected Image and (b) Pupil Center Detection Results by RST in Binary Image

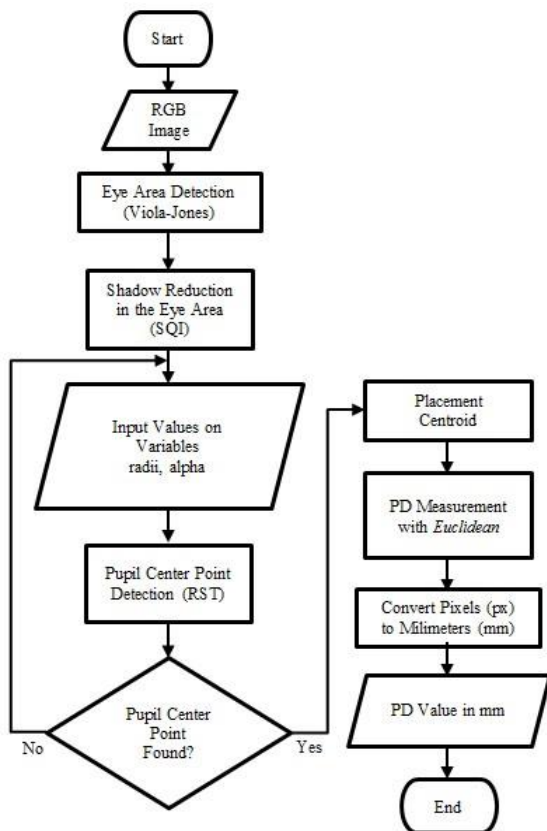


**Figure 9.** Measurement of PD by the system by (a) Marking the Pupil Center Point with Centroids and (b) Giving Illustration with Lines for PD Distance

measurement process is carried out by displaying the simulation through the MATLAB GUI as the interface. The system process in detecting the pupillary center using RST combined with SQI to measure the PD value which is shown Figure 10 by the flowchart.

**3. RESULT AND DISCUSSION**

The test was carried out using two datasets, namely the MPII gaze dataset and the Real Self-Photo dataset. The MPII Gaze dataset is used to test the system's ability to



**Figure 10.** Flowchart of Automatic PD Measurement System

detect the center of the pupil in the image in bright or dark conditions as well as the success of the system in measuring PD distance. While the Real Self-Photo dataset is used for the comparison of PD values to know the accuracy of PD measurements made by the system.

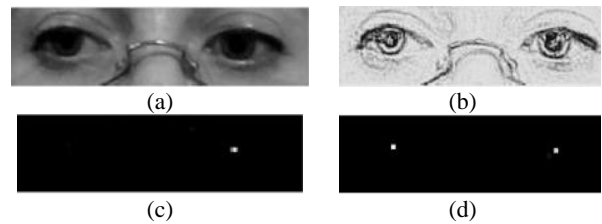
**3. 1. Testing PD Measurements**

Tests in the form of simulations are carried out through the MATLAB GUI intermediary in displaying the pupil center detection process to measure the pupil distance (PD) in the eye. The test was carried out Real Self-Potraits and MPII Gaze dataset with various light conditions on objects of two different people. The test is done by using the image from the MPII Gaze dataset to detect the center of the pupil. There are two lighting conditions in the image, namely a bright state in Figure 11 and a dark state in Figure 12.

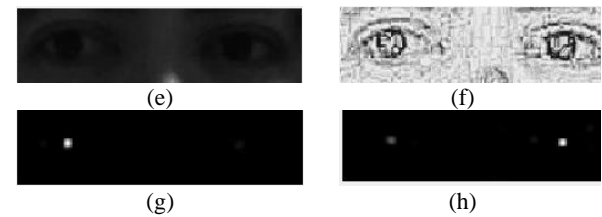
Figure 13 shows the PD measurements made by the system with size information in pixels (px) as well as millimeters (mm) using a low-resolution image of 300 x 300 pixels with 72 dpi, this resolution is the minimum limit for the system to work optimally and make the image at the lowest resolution. The test was carried out ten times on the same object model but there were various positions of the eye glances in each lighting state of the image shown in Figure 13. The experimental results were generated from 60 images from MPIIGaze and Real Self-Portraits like the example shown in Figure 14, consisting of 18 images with glasses and 42 without glasses.

**3. 2. Evaluation**

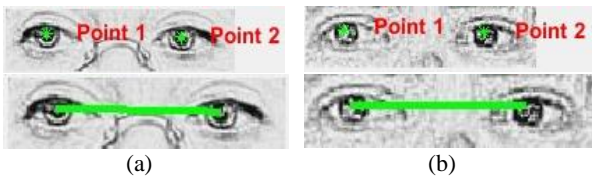
Previously on Real Self-Potraits manual measurements had been carried out using the PD ruler as shown in Figure 15 with a PD range in general is



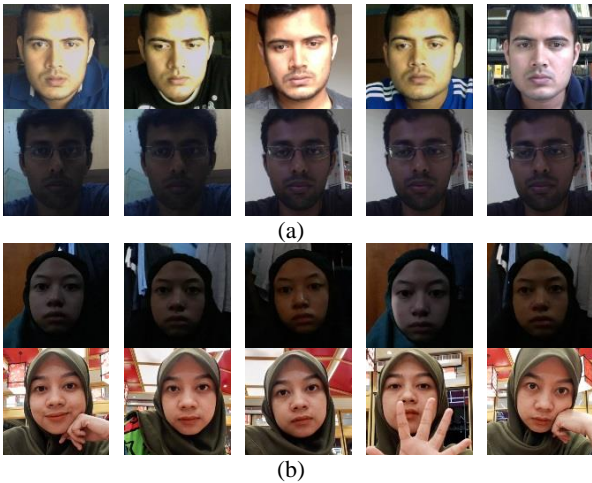
**Figure 11.** Detection of the Pupil Center on a Bright Image use radii  $n = 8$  with (a) Grayscale Image, (b) Image Quality Improvement with SQI, (c) Pupil Center Detection with RST, and (d) Pupil Center Detection with RST Improve SQI



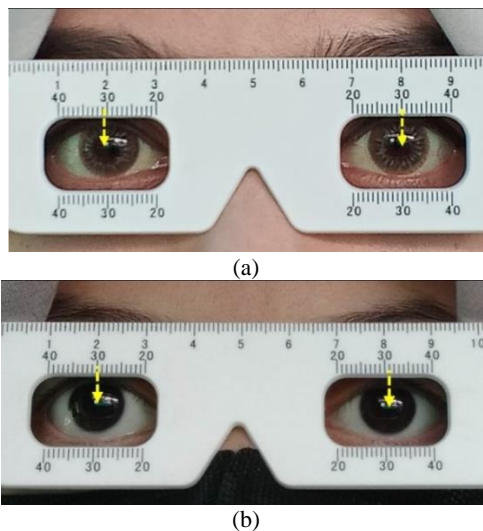
**Figure 12.** Detection of the Pupil Center in a Dark Image use radii  $n = 11$  with (e) Grayscale Image, (f) Image Quality Improvement with SQI, (g) Pupil Center Detection with RST, and (h) Pupil Center Detection with RST Improve SQI



**Figure 13.** Measurement of PD on Bright Image with Value (a) 63.1 mm and PD in Dark Image (b) 63.0 mm



**Figure 14.** Bright State and Dark State Image from (a) MPIIGaze Dataset and (b) Real Self-Photograph Dataset



**Figure 15.** Manual Measurement Using PD Ruler with Image PD Value (a) 61 mm and (b) 61 mm

58-70 mm. The PD value data in Tables 1 and 2 will then be compared with the manual PD value to determine the percentage of system accuracy in measuring PD. Then, Tables 3 and 4 presents pure PD value data because MPIIGaze does not have a manual PD value. This data will be processed with a standard deviation to find out the value of the uncertainty in the PD measurement carried out by the system.

**TABLE 1.** Test Results on Real Self-Potraits with Bright Conditions

Image	PD (px)	PD (mm)	
		$x_1$	$x_1^2$
1	81.35	61.4	3769.96
2	80.67	60.9	3708.81
3	80.74	60.9	3708.81
4	79.59	60.1	3612.01
5	81.63	61.6	3794.56
6	81.53	61.5	3782.25
7	80.62	60.8	3696.64
8	82.06	61.9	3831.61
9	79.61	60.1	3612.01
10	80.66	60.9	3708.81
n = 10		$\sum x_1 = 610.1$	$\sum x_1^2 = 37225.47$

**TABLE 2.** Test Results on Real Self-Potraits with Dark Conditions

Image	PD (px)	PD (mm)	
		$x_2$	$x_2^2$
1	79.64	60.1	3612.01
2	77.09	58.2	3387.24
3	79.68	60.1	3612.01
4	79.55	60.0	3600.00
5	81.43	61.5	3782.25
6	79.55	60.0	3600.00
7	78.98	59.6	3552.16
8	79.61	60.1	3612.01
9	80.80	61.0	3721.00
10	80.33	60.6	3672.36
n = 10		$\sum x_2 = 601.2$	$\sum x_2^2 = 36151.04$

**TABLE 3.** Test Results on MPIIGaze with Bright Conditions

Image	PD (px)	PD (mm)	
		$x_3$	$x_3^2$
1	88.51	66.8	4462.24
2	89.06	67.2	4515.84
3	90.66	68.4	4678.56
4	88.51	66.8	4462.24
5	88.50	66.8	4462.24
6	88.29	66.6	4435.56
7	88.59	66.9	4475.61
8	90.27	68.1	4637.61
9	90.71	68.5	4692.25
10	88.91	67.1	4502.41
n = 10		$\sum x_3 = 673.2$	$\sum x_3^2 = 45324.56$

**TABLE 4.** Test Results on MPIIGaze with Dark Conditions





Image	PD (px)	PD (mm)	$x_4^2$
		$x_4$	
1	87.25	65.8	4329.64
2	90.02	67.9	4610.41
3	88.39	66.7	4448.89
4	88.77	67.0	4489.00
5	89.59	67.6	4569.76
6	90,10	68.0	4624.00
7	88,81	67.0	4489.00
8	88,63	66.9	4475.61
9	88,79	67.0	4489.00
10	88,94	65.6	4303.36
n = 10		$\sum x_4 = 669.5$	$\sum x_4^2 = 44828.67$



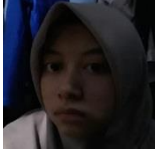

In Real Self-Potraits was found that there are differences in the dimensions of the image object that can affect the value of the PD measurement processed by the system. The size of the object in question is the position of the face that is larger or smaller in the image.

A smaller face dimension will result in a smaller PD measurement value than the original value. This is influenced by the image quality of the eye object experiencing a decrease, which causes a burst effect and results in inaccurate pupil center detection. The results of the PD measurement are not the manual PD measurement values, which can be seen in Table 5. Therefore, it is necessary to pay attention to the dimensions of the image so that the measurement of the PD value on the system can match the PD value that has been measured manually.

From the test results, it can be seen that the value of the standard deviation (S) of measurement uncertainty is calculated using Equation (2).

**TABLE 5.** Differences in Face Object Dimensions Affecting PD Values on Real Self-Potraits

Correct Face Dimensions	PD (mm)	Incorrect Face Dimensions	PD (mm)
	60.1		24.7
	61.6		42.4

	60.1		53.4
	60.6		44.2

$$S = \sqrt{\frac{n \sum x_i^2 - (\sum x_i)^2}{n(n-1)}} \tag{2}$$

The uncertainty value for PD measurement on Real Self-Potraits is as follows:

$$S_{\text{bright}} = \sqrt{\frac{10(37225.47) - (610.1)^2}{10(10-1)}} = \sqrt{0.36} = 0.60 \text{ mm}$$

$$S_{\text{dark}} = \sqrt{\frac{10(36151.04) - (601.2)^2}{10(10-1)}} = \sqrt{0.77} = 0.88 \text{ mm}$$

The uncertainty value for PD measurement on MPIIGaze is as follows:

$$S_{\text{bright}} = \sqrt{\frac{10(45324.56) - (673.2)^2}{10(10-1)}} = \sqrt{0.53} = 0.73 \text{ mm}$$

$$S_{\text{dark}} = \sqrt{\frac{10(44828.67) - (669.5)^2}{10(10-1)}} = \sqrt{0.63} = 0.80 \text{ mm}$$

PD measurements made by the system have an average value of uncertainty less than 1 mm. From the known uncertainty values, it can be determined that the PD value of each object from Real Self-Potraits Dataset:

$$\begin{aligned} PD_{\text{bright}} &= \left( \frac{\sum x_1}{10} \right) \pm S_{\text{Table 1}} \\ &= 61.01 \pm 0.60 \text{ mm} \end{aligned}$$

$$\begin{aligned} PD_{\text{dark}} &= \left( \frac{\sum x_4}{10} \right) \pm S_{\text{Table 2}} \\ &= 60.12 \pm 0.88 \text{ mm} \end{aligned}$$

Meanwhile, PD value from MPIIGaze Dataset:

$$\begin{aligned} PD_{\text{bright}} &= \left( \frac{\sum x_3}{10} \right) \pm S_{\text{Table 3}} \\ &= 67.32 \pm 0.73 \text{ mm} \end{aligned}$$

$$\begin{aligned} PD_{\text{dark}} &= \left( \frac{\sum x_4}{10} \right) \pm S_{\text{Table 4}} \\ &= 66.95 \pm 0.80 \text{ mm} \end{aligned}$$

In the Real Self-Potraits dataset, it can be seen that the accuracy of the system in measuring PD by calculating the Mean Absolute Percentage Error (MAPE) using Equation (3).

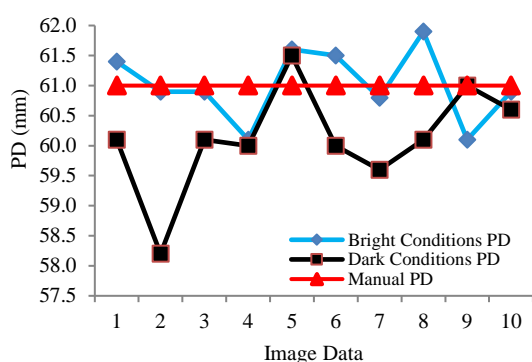
$$MAPE = \frac{1}{n} \sum \left| \frac{\text{Manual PD} - \text{Result PD Detection}}{\text{Manual PD}} \right| \times 100\% \tag{3}$$

$$MAPE_{\text{bright}} = \frac{1}{10} \times |-0.16\%| = 0.02\%$$

$$MAPE_{\text{dark}} = \frac{1}{10} \times |14.42\%| = 1.44\%$$

From the results of the MAPE calculation, it is obtained that a percentage value  $\leq 10$  proves that the PD measurement carried out by the system has good accuracy in bright or dark images. This statement is in accordance with the interpretation of the MAPE value when the percentage value is less than 10, indicating that the prediction of the existing measurement value matches or approaches the actual measurement value.

In Figure 16, the measured PD in bright conditions is closer to the manual PD value than the PD in dark conditions from Real Self-Portraits datasets. Evidenced by the difference in the uncertainty value for bright conditions of 0.60 mm which is small compared to 0.88 mm for dark conditions.



**Figure 16.** Graph of PD Value Proximity Measured by the System with Manual PD

#### 4. CONCLUSION

PD measurements on the system were successfully carried out on test images from the MPIIGaze and Real Self-Portraits datasets with two varying lighting conditions. Test images that have a resolution of 300 x 300 pixels with 72 dpi provide optimal work for the system in measuring PD. The lighting in the image is divided into two conditions, namely bright conditions and dark conditions. The PD results measured on the system have an uncertainty value of less than 1 mm as in Real Self-Portrait which has an uncertainty value of 0.60 mm for bright conditions and 0.88 mm for dark conditions. MPIIGaze has an uncertainty value of 0.73 mm for bright conditions and 0.80 for dark conditions. The existence of this uncertainty value is influenced by the difference in the dimensions of the face object in the image being tested. The system for measuring PD has good accuracy as evidenced by the smallest MAPE value of 0.02% in bright images. However, the system is still constrained by dark image conditions with high shadow illumination when detecting the center of the pupil and the difference in image dimensions is also an obstacle to produce accurate PD measurements. For further research

in measuring pupil distance (PD) through images, it is hoped that the system can detect the center of the pupil accurately and precisely in various image dimensions so that dimension changes are not done manually.

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### Persian Abstract

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#### چکیده

در بیماران مبتلا به عیوب انکساری یا اختلال بینایی، پرتوهای نور دریافتی مردمک به طور مستقیم روی شبکیه نمی‌افتد. این مشکل را می‌توان با استفاده از عینک های مونوکلد اصلاح کرد. نقطه کانونی عدسی عینک باید با مرکز مردمک چشم کاربر تنظیم شود. این را می‌توان از طریق اطلاعات مقدار فاصله مردمک اندازه گیری شده (PD) دانست. اندازه گیری PD برای تعیین فاصله مرکزی مردمک ها در هر دو چشم بسیار مهم است. جایی که چشم اثر منشور را تجربه نمی‌کند. این مطالعه با هدف استفاده از روش تبدیل تقارن شعاعی (RST) همراه با تصاویر خود بهره (SQI) برای تشخیص مرکز مردمک و اندازه گیری PD است. این الگوریتم ترکیب می‌شود تا نتایج بهینه تری در تشخیص مرکز مردمک در شرایط تاریک یا در معرض نور سایه بدست آورد. برنامه ایجاد شده با استفاده از نرم افزار متلب، اندازه گیری PD را برای تشخیص مرکز مردمک در شرایط تصاویر روشن و تاریک شبیه سازی می‌کند. آزمایش ده بار انجام شد و نتایج نشان داد که سیستم قادر به اندازه گیری PD بر روی تصاویر با وضوح پایین 300 x 300 پیکسل در 72 dpi در شرایط تصویر روشن است. با مقادیر عدم قطعیت اندازه گیری در هر تصویر 0.6 میلی متر. در مورد آزمایش روی تصاویر تاریک، مقادیر عدم قطعیت 0.8 میلی متر است. در این حالت، مقدار انحراف استاندارد از تأثیر ابعاد مختلف جسم صورت بر روی تصویر آزمایش شده به دست می‌آید.

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