



## A Baseline Free Method for Multiple Damage Identification and Localization using the Roving Mode Shape Response

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### ABSTRACT

The identification of structural systems with unknown vibration signature response is still a challenging issue which has been addressed by many reviewers. The current sensor technology states that the sensor position should be very close to the damaged element in order to identify and localize the damage. The primary goal of this research is to present a baseline-free method using the roving mode shape response based, multiple damage localization in a cantilever beam. Consequently, the damage location indicator is based on the roving mode shape approach (DLRA). The theoretical development is carried out on a cantilever beam, a finite element model. The different cases for multiple damages i.e. 2 elements damage, 3 elements damage and 5 elements to be damage, at a time, have been modelled on the structural member. The system response, for the healthy and damaged structural systems, has been determined using the roving mode shape approach. Further, the algorithm has been developed for multiple damage identification and localization using MATLAB software. The combined mass and stiffness damage, as well as only the mass change damage, both cases were considered. From the results, it was found that the proposed method can reliably identify the damage and its position. The method will also be helpful while keeping the sensor's position very close to the damage. The novelty of this method is that it uses the response which is basically a field output and no prior assumptions have been made at the damaged element's location.

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### NOMENCLATURE

NOMENCLATURE		Greek Symbols	
$M$	Distributed mass matrix (N)	$\omega$	Modal frequency (Hz)
$C$	Damping coefficient matrix	$\Delta\phi$	Modal amplitude perturbation matrix
$K$	Stiffness matrix (N/mm)	$\phi_d$	Modal amplitude of the damaged structure
$t$	Time period (s)	$\phi_u$	Modal amplitude of the undamaged structure
$u(t)$	Structural response matrix in terms of displacement (mm)	$\alpha$	Coefficient of proportionality
$\dot{u}(t)$	Structural response matrix in terms of velocity (mm/s)	<b>Subscripts</b>	
$\ddot{u}(t)$	Structural response matrix in terms of acceleration (mm/s <sup>2</sup> )	$d$	Damage case
$A$	Column matrix of modal amplitude (mm)	$u$	Undamage case
$N$	Total number of elements in the Finite Element model	$i$	Natural number

## 1. INTRODUCTION

Vibration is a motion that repeatedly revolves about its mean position while being constrained between two clearly defined boundaries (known as extreme positions) on each side of the mean position. The vibration shows the build-up of energy in the system. The various

techniques for free vibration analysis of beams on elastic foundations has been given by Ozturk and Coskun [1-3]. Various researchers have found that damage alters the system's dynamic response such as Natural Frequency and Mode shape [4]. Also, these responses can be evaluated by vibration analysis, hence, this property has been used by researchers for structural system

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identification [5-7]. Various structural system identification techniques particularly in the beam have been developed to date. Still, complete system identification using in-field data is a challenging issue [8]. The damage-tolerant and fail-safe design of civil structures necessitates extensive inspection and defect monitoring at regular intervals, and due to this, a lot of structural health monitoring research is going on [9]. Intelligent real-time monitoring is essential, to provide safe and affordable buildings since the danger of failure and the expense of scheduled but unnecessary maintenance are both rising. The sensor-based technologies are frequently used for this. The actuators and sensors are integrated into the structures or can be used instantly. With the advancement in sensor technology, the researchers found that the appropriate positioning of the sensor is very important in order to find out the damaged element [10]. The same algorithm may provide misleading data when the sensors are away from damaged members. The sensor position should be very close to the damage location in order to find out the damage in the structural systems [11-14]. Patel and Dewangan [15] have also found that the sensor if placed in the purview of the damage will only be able to detect the damage accurately.

Aydin et al. [16-17] provided the information on support optimization. The different beam vibrations will cause different distributions of elastic supports. Internal forces can occasionally exceed yield limits, which is significant for damage. This could be a guess for the damage existence. The mode shapes are more vulnerable to local damage since they carry local information [18]. Also, they are less sensitive to the temperature effect than the natural frequency [19]. Huang et al. [20] developed a baseline-free method for system identification based on the node displacement of structural mode shapes. It has been found that the natural frequency is significantly influenced by both temperature and damage. Consequently, using a frequency-based technique to identify the damage separately is challenging. The monitored region may be split into larger and smaller areas to improve the damage location resolution. With additional sensors, this technique might provide more accurate detection. Malekinejad and Rahgozar [21] presented a mathematical model of the cantilever beam for each tube through the structure's height. The free vibration analysis case has been considered. The mode shape has been calculated by simplifying the mathematical equation using Hamilton's principle and the assumptions. Various mode shape based techniques have been used by the researchers for damage identification [22-31]. Nahvi and Jabbari [32] used the experiment modal data and the natural frequency to identify the damage in the form of a crack in a cantilever beam. It can be inferred that the variations in the frequencies of higher modes rely on the distance between

the crack position and the appropriate mode shape's nodes. As a result, for a crack located at the nodal points of the corresponding mode shape, the natural frequency of the cracked beam remains unchanged. Since early 1979 the introduction of the digital FFT spectrum analyzer and various other equipment has grown in popularity for determining the mode shape. However, since these types of equipment are highly expensive, they cannot be easily affordable. An effort has been made by Chandra and Samal [33] to use experimental roving impact tests to identify a beam's mode shapes without using these expensive types of equipment. The experimental modal analysis has been performed in the cantilever beam by Prashant et al. [34] and Zhang et al. [35].

Until now, there has been no reported literature on baseline free structural damage identification and localization by DLRA for multiple damages using the infield data. Hence, the primary goal of this research is to present a baseline free method using the roving mode shape response based, multiple damage identification and localization in a cantilever beam. The combined mass and stiffness damage, as well as only the mass change damage, both cases were considered. Finding the ideal position for the sensor during on-site structure health monitoring is another goal of this paper. Further, for the single damage case, many research works have been published. In order to identify the structural system with multiple damages, the sensor can be located in advance using the DLRA.

## 2. THEORETICAL DEVELOPMENT

The general form of the equation of motion of the linear structural system in structural dynamics is given by Equation (1):

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = 0 \tag{1}$$

Hence, the distributed mass and stiffness matrix is given by Equations (2) and (3), respectively.

$$M = \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & 3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & 3l^2 & -22l & 4l^2 \end{bmatrix} \tag{2}$$

$$K = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & \dots & & & \\ & & & & \\ & & & k_{i-1} + k_i & -k_i \\ & & & & k_i \end{bmatrix} \tag{3}$$

For the multiple degree of freedom system, the solution to the problem is reduced to the solution of Equation (4), which is given below:

$$([K] - \omega^2[M])\{A\} = 0 \tag{4}$$

Equation (4) represents the general form of the modal characteristic equation. The above equation is rank deficient since the rank of the matrix is less than the number of rows. Hence the mode shape and the natural frequency of the system can only be determined using the above equation. The modal amplitude perturbation matrix should be introduced as the difference between the modal amplitude of the damage structure and the undamage structure, for the  $i^{\text{th}}$  mode and  $(i + 1)^{\text{th}}$  mode, it is given by Equations (5) and (6), respectively.

$$(\Delta\phi)_i = (\phi_d)_i - (\phi_u)_i \tag{5}$$

$$(\Delta\phi)_{(i+1)} = (\phi_d)_{(i+1)} - (\phi_u)_{(i+1)} \tag{6}$$

Damage to the structure leads to a change in the modal amplitude. At the position of damage, it is assumed that the damage modal amplitude will change by say  $\alpha$  times the undamage modal amplitude, hence it can be given by the below expression:

$$\phi_d = \alpha \times \phi_u \tag{7}$$

On subtracting Equation (6) from Equation (5) and substituting the value from Equation (7) into the resulted equation will be given as follows:

$$(\Delta\phi)_i - (\Delta\phi)_{(i+1)} = (1 - \alpha)((\phi_d)_i - (\phi_d)_{(i+1)}) \tag{8}$$

It can be noted that the expression on the right-hand side of Equation (8) contains only the damage response along with the constant term. This damage response only, will be useful for obtaining the baseline free equation. But, the expression is not sufficient, hence further pre and post multiply Equation (8) by  $(\sum_{i=1}^N((\phi_d)_{(i)} - (\phi_d)_{(i+1)}))/N$  on both sides and further simplifying, it can be rewritten as follows:

$$\frac{1}{1-\alpha} = X \times \frac{((\phi_d)_{(i)} - (\phi_d)_{(i+1)})/N}{(\Delta\phi)_i - (\Delta\phi)_{(i+1)}} \tag{9}$$

The value of  $X$  is represented by Equation (10). It has been observed that the expression contains only the damage structure responses hence the expression could be used as the damage detection and localization indicator. The proposed equation is formulated by the small amount of available structural information.

Although the above equation provides only, some general information regarding the existence of the damage, it cannot precisely detect the position of the damage. It has also been observed that the damage in its early stages could not be directly detected using the damage index given by Equation (10).

$$X = \frac{((\phi_d)_{(i)} - (\phi_d)_{(i+1)})}{(\sum_{i=1}^N((\phi_d)_{(i)} - (\phi_d)_{(i+1)}))/N} \tag{10}$$

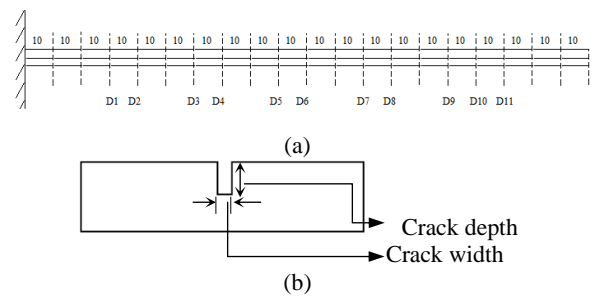
This is due to the fact that differences are directly averaged over all measurement points. Further, when these data are directly plotted, the damage can not be clearly identified and located. Hence Equation (10) is further modified and given by the expression below:

$$DLRA = \frac{Abs((\phi_d)_{(i)}) - Abs((\phi_d)_{(i+1)})}{(\sum_{i=1}^N(Abs(\phi_d)_i - Abs(\phi_d)_{i+1}))/N} \tag{11}$$

In the above expression, the response from the roving mode shape technique will be employed to determine the value of DLRA. Consequently, Equation (11) will be used as the damage position indicator based on the roving mode shape approach. The combined mass and stiffness damage, as well as only the mass change damage, both cases could be applicable for determining the DLRA using the above expression.

### 3. EXPERIMENTAL ROVING TEST APPROACH

In this section, the experimental roving test approach described by Chandra and Samal [33] has been applied to estimate the modal amplitude of the cantilever beam. Aluminum was used as the beam specimen, and its material characteristics and dimensions are listed in Table 1. As shown in Figure 1(a), the 200 mm long beam was discretized into 20 elements. As a result, 10 mm was found to be the distance between the two nodes. Various damages, with varying degrees and locations of damage, were introduced. For this purpose, eleven locations were randomly chosen on the cantilever beam at a distance of 30 mm, 40 mm, 60 mm, 70 mm, 90 mm, 100 mm, 120 mm, 130 mm, 150 mm, 160 mm and 170 mm. Further, the details of the damage considered are shown in Figure 1. D1, D2, D3, D4, D5, D6, D7, D8, D9, D10 and D11



**Figure 1.** (a) Damage location in cantilever beam (b) Vertical section of the crack damage region

**TABLE 1.** Specimen details

Details	Value
Young's modulus (E)	69.1 N/mm <sup>2</sup>
Poisson's ratio (t)	0.334
Density (q)	2668.32 x 10 <sup>-10</sup> N/mm <sup>3</sup>
Dimension	(200 x 9 x 50) mm

represent the crack position, the depth of crack was taken in terms of percentage reduction in depth as 30 %, 50 % and 60 % reduction and the width of the crack is considered negligible. Table 2 provides a summary of these cases' specifics.

**TABLE 2.** Damage induced in the cantilever beam

Damage case	Crack location	Damage induced	
		Depth reduction (%)	Mass reduction (%)
<b>Two elements damage cases</b>			
Case 1	D2, D6	0.3	-
Case 2	D2, D7	0.3	-
Case 3	D4, D8	0.3	-
Case 4	D4, D9	0.3	-
Case 5	D5, D9	0.3	-
Case 6	D5, D10	0.3	-
Case 7	D2, D6	0.6	0.6
Case 8	D2, D7	0.6	0.6
Case 9	D4, D8	0.6	0.6
Case 10	D4, D9	0.6	0.6
Case 11	D5, D9	0.6	0.6
Case 12	D5, D10	0.6	0.6
<b>Three elements damage cases</b>			
Case 13	D2, D6, D9	0.5	-
Case 14	D2, D7, D11	0.5	-
Case 15	D4, D6, D9	0.5	-
Case 16	D4, D7, D11	0.5	-
Case 17	D5, D7, D9	0.5	-
Case 18	D5, D8, D11	0.5	-
Case 19	D2, D6, D9	0.6	0.6
Case 20	D2, D7, D11	0.6	0.6
Case 21	D4, D6, D9	0.6	0.6
Case 22	D4, D7, D11	0.6	0.6
Case 23	D5, D7, D9	0.6	0.6
Case 24	D5, D8, D11	0.6	0.6
<b>Five elements damage cases</b>			
Case 25	D1, D3, D5, D7, D9	0.3	-
Case 26	D1, D3, D5, D7, D11	0.3	-
Case 27	D2, D3, D5, D7, D9	0.3	-
Case 28	D2, D3, D5, D7, D11	0.3	-
Case 29	D2, D4, D6, D8, D10	0.3	-
Case 30	D2, D4, D6, D8, D11	0.3	-

Case 31	D1, D3, D5, D7, D9	0.6	0.6
Case 32	D1, D3, D5, D7, D11	0.6	0.6
Case 33	D2, D3, D5, D7, D9	0.6	0.6
Case 34	D2, D3, D5, D7, D11	0.6	0.6
Case 35	D2, D4, D6, D8, D10	0.6	0.6
Case 36	D2, D4, D6, D8, D11	0.6	0.6

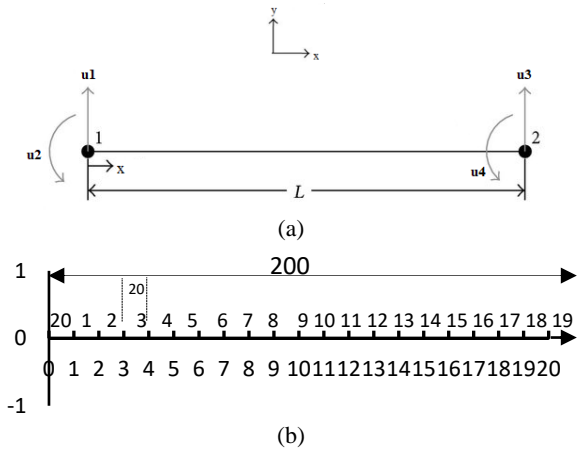
Free vibration has been introduced by applying initial displacement at the free end of the beam. Signals from the accelerometers were analyzed in order to determine the natural frequencies and the amplitudes correspondingly. It has been observed that the system response for the healthy and damaged structural system has been determined using the roving mode shape approach. Damage in its early stages could not be detected using these data directly. This is due to the fact that differences are averaged over all measurement points when determining the mode shape.

Further, when these data are directly plotted, the damage could not be clearly identified and located. Hence further analysis is required, which has been addressed in section 4. The algorithm has been developed for multiple damage identification and localization using MATLAB software. It should also be noted that the proposed damage detection method inevitably and essentially depends on determining the displacement modal amplitude. It is extremely important to note that inaccurate assessments of the structure's original physical characteristics result in inaccurate damage detection.

#### 4. STRUCTURAL MODELLING AND IDENTIFICATION

The theoretical development is carried out on a cantilever beam, using the finite element method. The two noded linear elements are used for meshing, as shown in Figure 2(a). The degree of freedom considered is vertical deflection  $u_1$  and  $u_3$  and rotation about the z-axis is  $u_2$  and  $u_4$ . The finite element model is shown in Figure 2(b).

The physical properties and dimensions of the beam are mentioned in Table 1. In order to determine the effectiveness of suggested damage localization technique, several damage scenarios are taken into consideration. Table 2 provides a summary of these cases' specifics. The damages result in a sudden fluctuation in amplitude as well as adverse vibrational performances. These generated damages cause the structure to behave dynamically inadequate. The suggested DLRA using the data gathered, such as the modal parameters of the structure after the occurrence of damage, is used to identify the existence and then the position of induced damage.

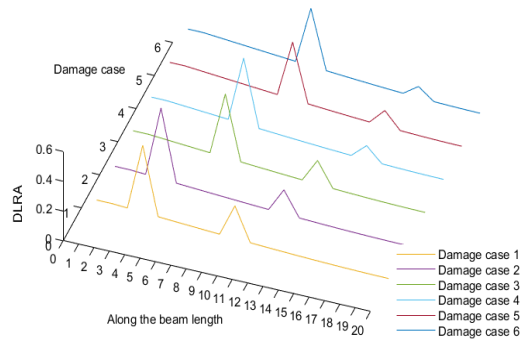


**Figure 2.** (a) Beam element with 2 degrees of freedom at each node. (b) Finite element model of a cantilever beam

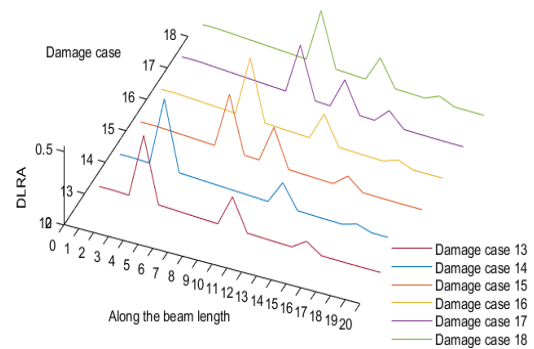
**5. RESULTS AND DISCUSSIONS**

The damage localization by ‘DLRA’ has been plotted for various damage cases. It could predict the damage existence and locations for that particular structure very clearly. Figures 3 to 5 illustrate all the cases for, the damage without considering the mass variation. Figures 6 to 8 illustrate all the cases for the damage, by considering the mass variation. The technique provides a clear indication of DLRA for that element with multiple damages at a time, with a significance change in the numerical value, as well as it is very clear form Tables 3 to 8. For each of the damage cases, it has been observed that there are variations in the DLRA value of damage and undamaged condition of the structure at particular locations, which denotes the presence and location of the damage. This variation is approximately between 11% to 16% as stated in Table 3, the sudden fluctuation in the graph, at the damaged location, is for this reason.

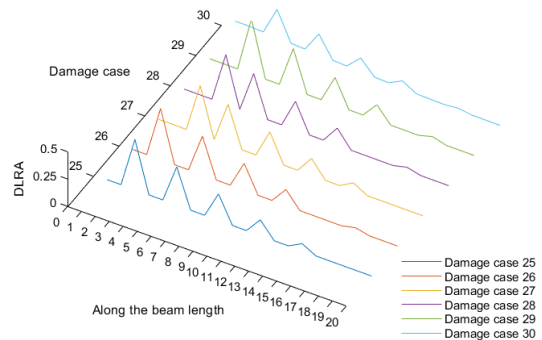
The drastic shift in the amplitude of the DLRA points out the damage location. Hence, in Figure 3, that the damage position is the 4<sup>th</sup> and 10<sup>th</sup> element for damage Case 1. The DLRA is easier to calculate than the other methods due to the mass matrix's simplicity and associated damage index. Along with this, the multiple damages have been taken for up to 5 damages at a time in the present study, and the proposed algorithm is able to identify and localize the damage. These results can be clearly observed in Figures 3 to 8. Further, this algorithm will be effective for even more than 5 damages at a time. This result will also be helpful in finding the position of the sensor placement in advance. The proposed method is based on the roving mode shape response. Hence, the algorithm is useful for computer automation, which provides the self-generated technique by element automation, which could predict the damage location in terms of DLRA value element by element.



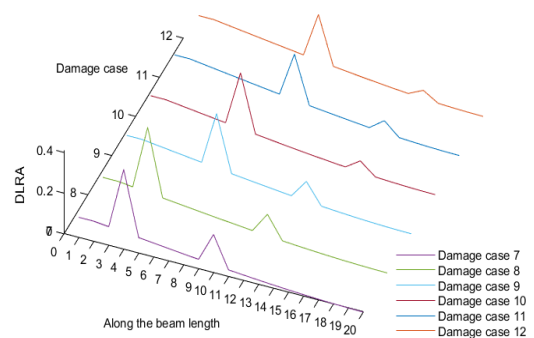
**Figure 3.** Plot for DLRA, for 2 elements damage



**Figure 4.** Plot for DLRA, for 3 elements damage



**Figure 5.** Plot for DLRA, for 5 elements damage



**Figure 6.** Plot for DLRA, for 2 elements damage

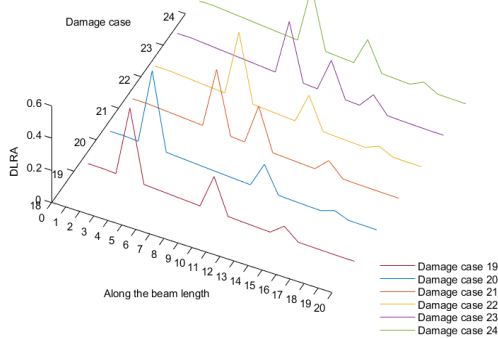


Figure 7. Plot for DLRA, for 3 elements damage

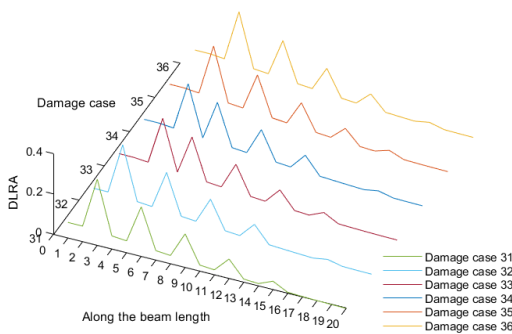


Figure 8. Plot for DLRA, for 5 elements damage

TABLE 3. The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 1	D2	0.06	0.51	11.76
	D6	0.03	0.25	12.00
Case 3	D4	0.06	0.49	12.24
	D8	0.03	0.18	11.11
Case 5	D5	0.05	0.44	11.36
	D9	0.02	0.12	16.67

TABLE 4. The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 13	D2	0.06	0.50	12.00
	D6	0.03	0.25	12.00
	D9	0.01	0.08	12.50
Case 15	D4	0.05	0.43	11.62
	D6	0.03	0.29	10.34
	D9	0.01	0.09	11.11
Case 17	D5	0.05	0.40	12.50
	D7	0.03	0.24	12.50
	D9	0.02	0.11	18.18

TABLE 5. The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 25	D1	0.012	0.475	2.53
	D3	0.010	0.360	2.78
	D5	0.007	0.249	2.81
	D7	0.004	0.149	2.68
	D9	0.002	0.068	2.94
Case 27	D2	0.013	0.460	2.82
	D3	0.011	0.381	2.89
	D5	0.007	0.265	2.64
	D7	0.004	0.159	2.52
	D9	0.002	0.073	2.74
Case 29	D2	0.015	0.515	2.91
	D4	0.011	0.382	2.88
	D6	0.007	0.256	2.73
	D8	0.004	0.143	2.80
	D10	0.001	0.057	1.75

TABLE 6. The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 7	D2	0.08	0.39	20.51
	D6	0.04	0.19	21.05
Case 9	D4	0.07	0.34	20.59
	D8	0.03	0.12	25.00
Case 11	D5	0.06	0.29	20.69
	D9	0.02	0.08	25.00

TABLE 7. The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 19	D2	0.08	0.39	20.51
	D6	0.04	0.19	21.05
	D9	0.01	0.06	16.67
Case 21	D4	0.06	0.32	18.75
	D6	0.04	0.21	19.05
	D9	0.01	0.06	16.67
Case 23	D5	0.07	0.27	25.93
	D7	0.04	0.16	25
	D9	0.02	0.07	28.57

**TABLE 8.** The numerical result of values of DLRA

	Crack Location	Undamage DLRA	Damage DLRA	% Variation in DLRA
Case 31	D1	0.077	0.328	23.48
	D3	0.059	0.246	23.98
	D5	0.040	0.168	23.81
	D7	0.023	0.098	23.47
	D9	0.010	0.043	23.26
Case 33	D2	0.073	0.310	23.55
	D3	0.060	0.254	23.62
	D5	0.040	0.173	23.12
	D7	0.024	0.101	23.76
	D9	0.011	0.045	24.44
Case 35	D2	0.077	0.329	23.40
	D4	0.057	0.241	23.65
	D6	0.037	0.158	23.42
	D8	0.020	0.086	23.26
	D10	0.007	0.032	21.88

**TABLE 9.** Comparative study of DLRA with similar methods, for the Damage Case 31

S. No.	Method	Multiple Damage location (5 damages at a time)		Percentage variance (%)	Average Percentage variance (%)
		Actual damage location (mm)	Identified damage location (mm)		
1.	Approximate curvature method [36]	30	33	10	9.4
		60	65	8.3	
		90	99	10	
		120	131	9.2	
		150	164	9.3	
2.	Mode shape damage index method (MSDI) [37]	30	31	3.3	3.3
		60	62	3.3	
		90	92	2.2	
		120	125	4.2	
		150	155	3.3	
3.	Mode shape based damage detection (MBDD) [38]	30	31	3.3	1.5
		60	61	1.7	
		90	89	-1.1	
		120	122	1.7	
		150	153	2.0	
4.	DLRA	30	30	0	0.1
		60	60	0	
		90	90	0	
		120	120	0	
		150	151	0.7	

## 6. FINAL REMARKS

In this study, a new method for damage identification has been proposed for assessing the structural multiple damages at a time, in the cantilever beam. Further for the single damage case, many research works have been published, and the multiple damages have been taken for up to 4 damages at a time. This algorithm will be effective for even more than 5 damages at a time. In the first stage of this method, the existence and position of damaged elements are identified by considering the damage with no mass variation. Subsequently, the effect of mass variation is considered. In order to assess the proposed method, the experimental roving test approach described has been applied to estimate the modal amplitude of the cantilever beam. The extracted modal data from experimental modal testing are usually complex values. For identifying the damage positions, further analysis is required, which has been addressed in section 2, where the algorithm is described to form a damage location indicator. The location indicator is defined by the DLRA. It should be noted that the results of DLRA depend on the correct determination of the structural model's initial information. It has been found that the DLRA was able to capture and localize the damage accurately which is clear from Figures 3 to 8. The proposed methodology will also be helpful while keeping the sensor positions very close to the damaged location. The algorithm is useful for computer automation, which provides the self-generated technique by element automation, which could predict the damage location in terms of DLRA value element by element.

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### Persian Abstract

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#### چکیده

شناسایی سیستم‌های سازه‌ای با پاسخ امضای ارتعاش ناشناخته هنوز یک موضوع چالش برانگیز است که توسط بسیاری از بازمین‌ها مورد توجه قرار گرفته است. فناوری حسگر فعلی بیان می‌کند که موقعیت سنسور باید بسیار نزدیک به عنصر آسیب‌دیده باشد تا آسیب را شناسایی و محلی‌سازی کند. هدف اصلی این تحقیق ارائه یک روش بدون خط مینا با استفاده از پاسخ شکل حالت چرخشی مبتنی بر مکان‌یابی آسیب چندگانه در یک تیر کنسول است. در نتیجه، نشانگر مکان آسیب بر اساس رویکرد شکل حالت چرخشی (DLRA) است. توسعه نظری بر روی یک تیر کنسول، یک مدل المان محدود انجام شده است. موارد مختلف برای آسیب‌های متعدد یعنی آسیب ۲ المان، آسیب ۳ عنصر و ۵ المان آسیب در یک زمان بر روی عضو سازه مدل شده است. پاسخ سیستم، برای سیستم‌های ساختاری سالم و آسیب‌دیده، با استفاده از رویکرد شکل حالت چرخشی تعیین شده است. علاوه بر این، الگوریتم برای شناسایی آسیب‌های متعدد و محلی‌سازی با استفاده از نرم افزار MATLAB توسعه یافته است. آسیب ترکیبی جرم و سفتی، و همچنین تنها آسیب تغییر جرم، هر دو مورد در نظر گرفته شد. از نتایج، مشخص شد که روش پیشنهادی می‌تواند آسیب و موقعیت آن را با اطمینان شناسایی کند. این روش همچنین در حالی که موقعیت سنسور را بسیار نزدیک به آسیب نگه می‌دارد مفید خواهد بود. تازگی این روش این است که از پاسخی استفاده می‌کند که اساساً یک خروجی میدانی است و هیچ فرض قبلی در محل عنصر آسیب‌دیده انجام نشده است.

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