IJE TRANSACTIONS C: Aspects Vol. 36, No. 03, (March 2023) 585-593

International Journal of Engineering

Journal Homepage: www.ije.ir

An Enhanced McCormick Envelopes to Represent Kron's Loss Formula

H. Sharifzadeh*

Department of Electrical and Computer Engineering, Hakim Sabzevari University, Sabzevar, Iran

PAPER INFO

Paper history:
Received 28 January 2022
Received in revised form 18 January 2023
Accepted 02 February 2023

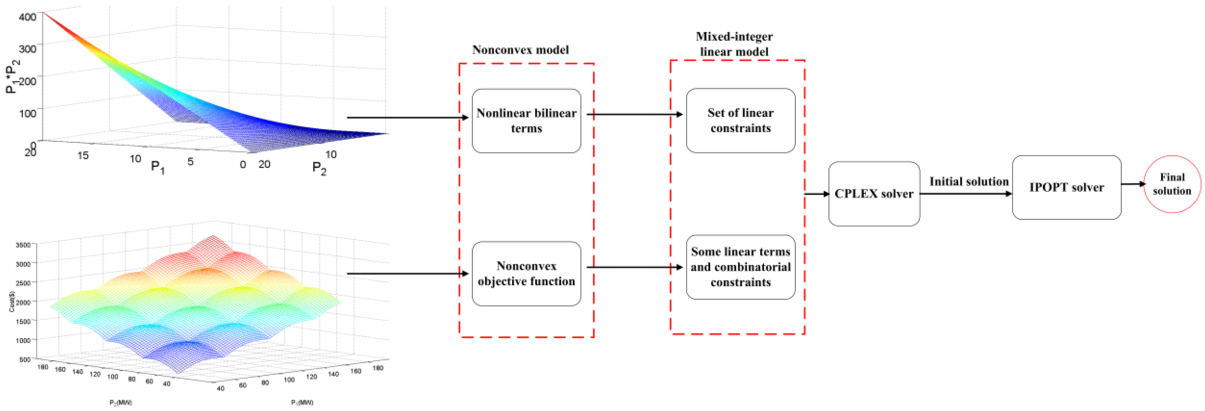
Keywords:
Kron's Loss Formula
McCormick Envelopes
Tight Relaxations

ABSTRACT

Recently, some researchers have employed the McCormick envelopes method to convexify some NP-hard optimization problems with bilinear terms. However, few publications concentrate on its variants to derive a more tight convex relaxation for practical applications. This paper proposes a new viewpoint on Kron's loss formula, also known as the B-matrix formula, as an equation having bilinear terms. Relying on the perspective, we transform the loss equation to some linear constraints using an enhanced McCormick relaxation. In the technique, the domain of bilinear variables is divided into some smaller parts to improve the relaxation tightness. Some case studies with different nonconvex terms are considered to verify the effectiveness of the enhanced envelopes for capturing Kron's loss formula. The findings from the numerical simulations suggest that the proposed approach can represent Kron's loss equation precisely. Moreover, the method performs more effectively than the other methods available in the literature as it usually converges to more optimal solutions.

***doi**: 10.5829/ije.2023.36.03c.19*

Graphical Abstract



NOMENCLATURE

Indices		D	Demand
h	Breakpoint indices of cost functions	$e_{Max}^{k,j}$	The maximum distance in McCormick envelopes
k, j	Generating unit indices	$M^{k,j,n}$	Big-M for constraint relaxation
n	Subinterval indices of Partitioned McCormick	P_k^L, P_k^U	Lower and upper bounds of generation in unit k
x	POZ indices	$P_k^{pozd^x}, P_k^{pozu^x}$	Lower and upper bounds of x^{th} POZ in unit k
Sets		P_k^0	Initial generation levels obtained from previous hour ED solution

*Corresponding Author Institutional Email: *h.sharifzade@hsu.ac.ir*
(H. Sharifzadeh)

Please cite this article as: H. Sharifzadeh, An Enhanced McCormick Envelopes to Represent Kron's Loss Formula, ***International Journal of Engineering, Transactions C: Aspects***, Vol. 36, No. 03, (2023) 585-593

H_k	Set of breakpoint indices of cost functions	$P_k^{L,n}, P_k^{U,n}$	New lower and upper bounds of generation for unit k in subinterval n for partitioned McCormick
K	Set of generating units	$\tilde{p}_{h,k}$	Generation in breakpoint h for unit k
N	Set of Subinterval indices of Partitioned McCormick	UR_k/DR_k	Ramp up/down limit for unit k
$X = \{1,2,\ldots,q\}$	Set of POZ indices	**Variables**	
Parameters		P_k	Generation level of unit k
c_k, b_k, a_k, e_k, f_k	Coefficients of cost function. characteristics	$\tilde{p}_{h,k}$	Generation level of unit k in segment h for piecewise linear approximation of cost. function
B_{00}, B_{k0}, B_{kj}	Loss coefficients in Kron's formula	P_{losses}	Transmission losses
$v_{k,j}$	Auxiliary continuous variables in the McCormick relaxation	**Functions**	
$v_{k,j}^n$	Auxiliary continuous variables in the partitioned McCormick relaxation	$O(P)$	The total cost of generating units
$z_R^{k,n}$	Binary variables in the McCormick relaxation	$\tilde{O}(P)$	The approximated total cost of generating units
$z_{h,k}$	Binary variables in the partitioned McCormick relaxation	PC_k	Generation cost of unit k

1. INTRODUCTION

The increasing pressure on enhancing power systems' economic and environmental performance requires more efficient tools for electrical network management. However, most current tools, such as market-clearing models, usually ignore the transmission losses due to emerging complex optimization problems [1-4]. Nevertheless, this simplification leads to inefficient and imprecise modeling.

One can directly incorporate the physic of the problem to model the losses as accurately as possible using the AC power flow equations. Nonetheless, the accurate model creates highly nonlinear nonconvex equations constituting an NP-hard problem. On the other hand, one can use the DC power flow model as an alternative approach, which is the current practice of some electricity markets [5]. Although the DC power flow equations build a linear model, they do not consider the losses and, as a result, can not capture the network behavior accurately.

An intermediate technique can include one or some equations solely to approximate the transmission losses as the network effect model. The most well-known technique for approximating the losses is Kron's equation employing a nonlinear equation to represent the network losses [6].

Kron's formula yields a more straightforward loss computation approach than the complex nonlinear AC power equations. Kron's formula yields a more simple loss computation approach than the complex nonlinear AC power equations. However, it also questions the efficacy of the conventional optimization algorithms to solve the constructed model even in a relatively simple economic dispatch (ED) problem, especially when a model includes some other practical constraints such as wire drawing effects.

The ED problem involving the loss formula as a constraint can be solved using traditional nonlinear programming techniques such as the interior-point method or sequential quadratic programming (SQP) [7]. The techniques exhibit reliable behavior to solve nonlinear problems in general. Nevertheless, as a weakness, these solution algorithms naturally converge to local solutions rather than the global ones, which is problematic in multimodal problems.

In the past decade, some researchers have used newly emerged artificial intelligence (AI) algorithms to solve the complex problem [8]. In the area of the nonconvex ED, to name a few, differential evolution [9], teaching-learning algorithm [10], hybrid particle swarm optimization [11], chameleon swarm [12], artificial bee colony [13], peafowl optimization [14], hybridization of ETLBO and IPSO [15], Hybrid Multi-Verse Optimizer [16], ray optimization algorithm [17], particle swarm [18-20], GA–API [21], shuffled differential evolution [22], quasi-oppositional teaching learning [23], oppositional real coded chemical reaction [24], and krill herd algorithm [25] have been employed to solve the ED.

The algorithm utilizing stochastic parallel search mechanisms can solve the problem more effectively than the nonlinear programming methods and find the global solution. Moreover, they do not rely on objective function/constraint gradients to search the feasible space. However, they lack compelling evidence for convergence. Moreover, one generally can find some discrepancies in their identified solutions in different algorithm runs in practice. Although a deterministic technique has been previously presented [26], it does not consider trnamission losses in its ED model.

On the other side, relatively new deterministic global optimization methods usually utilize the convex relaxation of the nonconvex region to solve the nonconvex problems. McCormick envelopes have been introduced to relax generally nonconvex bilinear terms to one convex region [27]. A bilinear term is defined as the product of two different variables, i.e., the 'x×y' term. The nonlinear parts of Kron's equation are bilinear terms in $P_k \times P_j$ form, where P_k and P_j represent the generation level of generating units k and j, respectively.

In the literature, the transmission losses usually are ignored or managed heuristically, which usually leads to infeasibility. To the best of our knowledge, this work, for the first paper, calculates the losses using a robust deterministic technique known as the McCormick relaxation. Moreover, we utilize an enhanced version of the relaxation to improve the solution optimality. As the distinct advantage, the proposed method reliably converges to the optimal solution while the convergence is guaranteed.

We propose an enhanced McCormick technique to relax the unit generation product expressions in this work. The enhanced envelopes leverage deterministic approaches rather than stochastic searches used in AI algorithms, thereby presenting a robust and stable convergence behavior.

In summary, the contribution of this paper include the following:

1) We proposed a new viewpoint on Kron's formula as an equation having bilinear terms. Relying on the viewpoint, we transform the loss equation to some linear constraints that can be solved efficiently using available optimization software.

2) The presented enhanced version of the McCormick formulation provides a tight linear problem. Moreover, we linearized the nonconvexity terms due to wire drawing effects, and thereby we transformed the nonlinear nonconvex ED model to a fully tight linear model.

3) We also proposed a new mixed-integer technique to enforce prohibited operating zones (POZs) of the generating units having nonconvex space due to disjoint feasible space rather than nonconvex functions.

The rest of the paper organizes as follows: Firstly, in section 2, we formulate an ED model with the transmission losses. In the next section, we formulate the enhanced McCormick relaxation to recast Kron's formula as linear constraints. To verify the effectiveness of the proposed approach, we use two case studies having multimodal objective functions. The simulation results obtained from applying the solution method in the case studies are reported in section 4. Section 5 concludes the paper.

2. PROBLEM STATEMENT

The ED problem includes the sum of the generation costs as the objective function and a set of equality and inequality constraints describing the physical and technical limits of the power system [18]. The details of the considered ED formulation are provided below.

Objective function: Traditionally, the production costs of the generating units are shown by quadratic expressions. The objective function of the ED problem usually is defined as the sum of the generating unit production cost. Mathematically, the ED objective function can be computed as follows:

$$Min \sum_{k\in K} PC_k(P_k) = \sum_{k\in K}\left(c_k P_k^2 + b_k P_k + a_k\right) \quad (1a)$$

The representation of the cost functions in Equation (1a) makes the implicit assumption that a thermal unit has only one steam valve. However, the current modern units with multiple steam valves have more complex cost functions. For these modern units, the cost functions usually include sinusoidal terms, modeling the wire drawing effects, in addition to the quadratic expressions. Therefore, a more general objective function with the complex cost function can be expressed as follows:

$$MinO(P) = \sum_{k\in K} PC_k(P_k) = \sum_{k\in K}\begin{pmatrix} c_k P_k^2 + b_k P_k + a_k \\ +e_k\left|sin(f_k \times (P_k^L - P_k))\right| \end{pmatrix} \quad (1b)$$

The surface of the objective function, considering only two generating units, is shown in Figure 1. As can be seen, it forms a nonconvex space with many local minimal challenging optimization algorithms.

Equality constraints: Here, we represent the transmission network losses using well-known Kron's formula. Accordingly, the generating units should meet the total system load as well as the transmission losses. The requirement usually is called the power balance equation and can be written as follows:

$$\sum_{k\in K} P_k = D + P_{losses} \quad (2)$$

The P_{losses} denotes the transmission losses and can be computed using Kron's formula by the following equation:

$$P_{losses} = B_{00} + \sum_{k\in K} P_k B_{k0} + \sum_{k\in K} \sum_{j\in K} P_k B_{kj} P_j \quad (3)$$

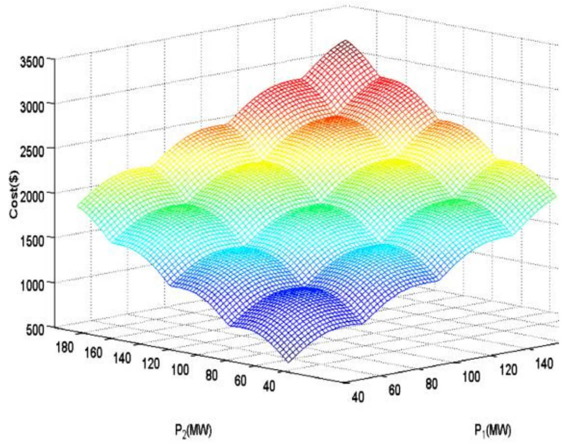


Figure 1. Nonconvex space of the objective function considering two generating units

$P_k B_{kj} P_j$, shown in red in Equation (3), are bilinear terms that are highly nonlinear and nonconvex in general.

Figure 2 illustrates the complex structure of the bilinear term. Next, we attempt to relax the bilinear terms tightly using the enhanced McCormick envelopes.

Inequality constraints: The technical limits of the generating units require that the units should be operated within the feasible range of generation:

$$P_k^L \leq P_k \leq P_k^U \forall k \in K \tag{4}$$

Furthermore, some units have prohibited operating zone (POZ), namely $P_k \notin \left[P_k^{pozd^x}, P_k^{pozu^x}\right]$. The generation limits considering the POZ can be described as follows:

$$P_k^L \leq P_k \leq P_k^{pozd^1} \forall k \in K \; P_k^{pozu^x} \leq P_k \leq P_k^{pozd^{x+1}} \forall k \in K, \forall x \in X = \{1,2,...,q\} \; P_k^{pozu^q} \leq P_k \leq P_k^U \forall k \in K \tag{5}$$

Finally, the ramp rate limits of the generation units for a single period ED problem can be modeled by the following constraints:

$$P_k \leq P_k^0 + UR_k \forall k \in K \tag{6a}$$

$$P_k \geq P_k^0 - DR_k \forall k \in K \tag{6b}$$

In the next section, we reformulate the bilinear terms ($P_k P_j$) of Kron's equation in Equation (3) as well as the rectified sinusoidal terms ($|sin(f_k \times (P_k^L - P_k))|$) in Equation (1b) to achieve a (mixed-integer) linear model.

3. PROPOSED METHOD

The McCormick relaxation forming the convex hull of the bilinear term $P_k P_j$ using two underestimators and two overestimators can be computed as follows [27]:

$$v_{k,j} \geq P_k^L P_j + P_k P_j^L - P_k^L P_j^L \tag{7a}$$

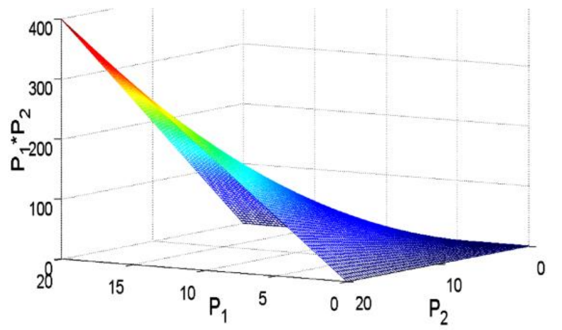


Figure 2. The complex surface of the bilinear expression

$$v_{k,j} \geq P_k^U P_j + P_k P_j^U - P_k^U P_j^U \tag{7b}$$

$$v_{k,j} \leq P_k^U P_j + P_k P_j^L - P_k^U P_j^L \tag{7c}$$

$$v_{k,j} \leq P_k P_k^U + P_k^L P_j - P_k^L P_j^U \tag{7d}$$

The new variable v_{kj}, together with the four additional linear constraints (7a)-(7d), replaces the bilinear nonconvex space with a convex one. As an advantage, the constructed constraints are linear, and as a result, one can solve the new problem using matured linear programming (LP) solvers.

As can be seen, the built constraints depend on the variable bounds, namely $[P_k^L, P_k^U]$ and $[P_j^L, P_j^U]$. It can be shown that the maximum distance of the relaxed space from the bilinear surfaces can be computed by the following equation [27]:

$$e_{Max}^{k,j} = \frac{(P_k^U - P_k^L)(P_j^U, P_j^L)}{4} \tag{8}$$

The expression in Equation (8) demonstrates that the maximum distance is proportional to the variable ranges: as the variable ranges widen, the relaxation performance weakens. Thus, one can divide the ranges into smaller parts and formulate the relaxation based on the new bounds to improve the relaxation tightness [28]. In this way, the McCormick relaxation is constructed for each sub-interval separately. Figure 3 illustrates the idea behind the partitioning mechanism more clearly [29]. As the number of partitions (N) increases, the envelopes build a set of tighter relaxations.

In this paper, we divide the generation bounds to N smaller subintervals and apply the McCormick envelopes for each of the subintervals separately. To this end, consider a unit whose generation level is denoted by P_k^n. The new generation variable in smaller subinterval has the following bounds:

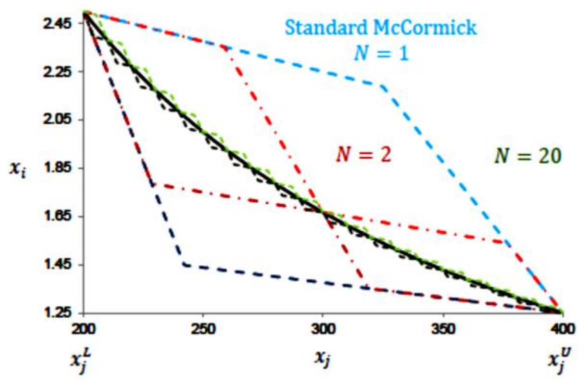


Figure 3. Tightening the McCormick relaxation by increasing the number of partitions [29]

$$P_k^{L,n} \leq P_k^n \leq P_k^{U,n} \tag{9}$$

Therefore, the original generation level variables and subinterval generation variables have the following relationships:

$$P_k = \sum_{\forall n \in N} P_k^n \tag{10}$$

$$P_k^n \leq z_R^{k,n} P_k^{U,n} \forall n \in N, \forall k \in K \tag{11}$$

$$\sum_{\forall n \in N} z_R^{k,n} = 1 \forall k \in K \tag{12}$$

Constraint (12) requires only one subinterval can be selected at the same time, and other subinterval generations are enforced to be zero using constraint (11). To avoid introducing too many partitions and binary variables, we build the McCormick relaxation using one variable partitioning rather than two variables partitioning. Thus, the enhanced McCormick for the new generation variables with the novel bounds can be expressed as follows:

$$v_{k,j}^n \geq P_k^{L,n} P_j + P_k^n P_j^L - P_k^{L,n} P_j^L - M^{k,j,n} \times (1 - z_R^{k,n}) \tag{13a}$$

$$v_{k,j}^n \geq P_k^{U,n} P_j + P_k^n P_j^U - P_k^{U,n} P_j^U - M^{k,j,n} \times (1 - z_R^{k,n}) \tag{13b}$$

$$v_{k,j}^n \leq P_k^{U,n} P_j + P_k^n P_j^L - P_k^{U,n} P_j^L - M^{k,j,n} \times (1 - z_R^{k,n}) \tag{13c}$$

$$v_{k,j}^n \leq P_j P_k^{U,n} + P_j^L P_k^n - P_k^{L,n} P_j^U - M^{k,j,n} \times (1 - z_R^{k,n}) \tag{13d}$$

(13a)-(13d): $\forall n \in N, \forall k, j \in K$

$$v_{k,j}^n \leq P_k^{U,n} P_j^U z_R^{k,n} \forall n \in N, \forall k, j \in K \tag{14}$$

$$P_{losses} = B_{00} + \sum_{k \in K} P_k B_{k0} + \sum_{n \in N} \sum_{j \in K} \sum_{k \in K} B_{kj} \times v_{k,j}^n \tag{15}$$

The Nonconvex cost functions also pose a challenge for the solution of the ED problem. Based on a technique that has been proposed by sharifzadeh [4]. We represent the nonconvex cost functions Equation (1b) through piecewise linear approximation rendering mixed-integer linear programming (MILP):

$$Min\tilde{O}(P) = \sum_{k \in K} \sum_{h \in H_k} (\alpha_{h,k} \tilde{p}_{h,k} + \beta_{h,k} z_{h,k}) \tag{16}$$

$$P_k = \sum_{h \in H_k} \tilde{p}_{h,k} \; \forall k \in K \tag{17}$$

$$\bar{p}_{h-1,k} z_{h,k} \leq \tilde{p}_{h,k} \leq \bar{p}_{h,k} z_{h,k} \forall k \in K, \forall h \in H_k \tag{18}$$

$$\sum_{h \in H_k} z_{h,k} = 1 \; \forall k \in K \tag{19}$$

Moreover, relying on the MILP representation, we propose a new technique to handle POZ restrictions. We can include the POZ segment bounds as the pairs of breakpoints in the piecewise linear approximation and, thereby, POZ segments can be avoided by imposing the pertained integer variables to become zero, namely:

$$\tilde{p}_{h,k} \in \left[P_k^{pozd^x}, P_k^{pozu^x}\right] \rightarrow z_{x,k} = 0 \forall k \in K, \forall x \in X \tag{20}$$

To summarize, the Mixed Integer-McCormick envelopes (MI-ME) model can be expressed as follows:
Objective function: Equation (16)
Constraints: (2), (5), (6a), (6b), (9)-(15) and (17) - (20)

As the solution obtained by the MI-ME model may contain error because of the approximations, we use its optimal solution as the initial solution of the original NLP model. The goal is to obtain a more accurate solution without approximation error. Clearly, the NLP model objective function is Equation (1b), and its constraints include Equations (2), (3), (5), (6a), and (6b). It is noted as the NLP model is a nonconvex problem, the pertained NLP solver converges to the nearest point to MI-ME optimal solution. Later in the numerical result section, we analyze the MI-ME and NLP roles to enhance the quality of obtained solutions.

4. NUMERICAL RESULTS

To show the effectiveness of the proposed solution method, we adopt two case studies, including 6-unit and 40-unit test systems modeling the transmission losses using Kron's formula. We draw required data for the 6-unit and 40-unit case studies from [18]. The case studies also include the sinusoidal cost functions stemmed from the wire drawing effects. The generation ranges of the 6-unit case study also contain POZs complicating the solution space. Therefore, the ED feasible spaces of both case studies make a complex nonconvex problem. As a result, they need particular solution techniques with global search ability rather than locally based solvers .

We use GAMS 24.1.3 to implement our solution approach and CPLEX 12 to solve the constructed ED model[1]. The absolute optimality gap is set to 0. Moreover, IPOPT as the NLP solver is employed to refine the final solution. We perform all simulations in a laptop with 8 GB RAM and 2.7 GHz Core i7 processors .

Table 1 displays the solutions of the ED problem reported in earlier works as well as the proposed method results for 6-unit case study. The first column in the tables shows the different ED solution methods that can be found in earlier works as well as the solution obtained by the

[1] http://www.gams.com

proposed method. The next three columns (columns two, three, and four) exhibit the reported optimal costs, i.e., objective function in Equation (1b). As the AI-based algorithms use random search mechanisms, they usually converge to different solutions in each algorithm application. The algorithms generally run several times, for example, 50 times. Then, the best and worst identified solutions, as well as the average of the identified solutions among all trial runs, are recorded. However, note that the proposed approach always converges to a unique solution in different runs as it relies on deterministic mechanisms to find the optimal solution. In other words, we solely write down the obtained solution of the proposed method three times in the pertained three columns of Tables 1 and 3.

Consider the second column of Table 1, which shows the best cost, among some conducted experiments as reported in the corresponding study, of the solution methods in case study I. As can be seen, the MI-ME identifies a more optimal solution compared with the other techniques. A couple of the meta-heuristic algorithms randomly obtain some optimal points close the MI-ME solution as shown in the second column of the table; however, the poor performance of their average and especially the worst solutions, in columns three and four of Table 1, respectively, suggests they exhibit unacceptable behavior in different trial runs, as they statistically converge to a weaker solution compared with their own best solution.

Table 2 shows the optimal scheduling of the generating units in case I, based on the proposed method, leading to $15449.89, as the considered objective function in Equation (1b).

Table 3 shows the solutions of earlier studies and the proposed method result for case II, namely 40-unit case study. Case study II with a larger and more complex feasible space reasonably reveals the efficacy of the solution methods more clearly. From the viewpoint of the best solutions, rarely the previous studies have found the solutions close to the proposed Mi-ME solution, as can be seen in the second column of Table 3. More importantly, the result reported in columns three and four of Table 3 shows that the earlier works have low success rates. Namely, a large discrepancy can be seen between their identified solutions in different trial runs.

Apart from the discrepancy drawback, the AI techniques heavily rely on their parameters to search the solution space. In other words, their obtained solutions change depending on their parameter values. For example, the different versions of Particle swarm optimization (PSO) in Table 1, such as PSO, NPSO, NPSO-LRS, RDPSO, and IRDPSO, have parameters such as cognitive coefficient and social coefficient, number of iterations, and number of particles that need to be tuned beforehand. In other words, the results of the AI algorithms are sensitive to their parameter setup. The sensitivity challenges the application of these algorithms in practice, as a new difficult problem arises on how to tune the parameters effectively, which is an unsolved problem in general.

TABLE 1. Comparison of the solution method results in the 6-unit case study

Method	Cost($)		
	Best	**Average**	**Worst**
SA [17]	15545.5	15488.98	15461.1
GA [17]	15524.69	15477.71	15457.96
TS [17]	15498.05	15472.56	15454.89
PSO [17]	15491.71	15465.83	15450.14
GA [18]	15459	15469	15524
MTS [17]	15453.64	15451.17	15450.06
NPSO [19]	15450	15452	15454
NPSO-LRS [19]	15450	15450.5	15452
PSO [18]	15450	15454	15492
RDPSO [19]	15449.89	15458.01	NA[a]
IRDPSO [19]	15449.89	15456.55	NA
Proposed	15449.89	15449.89	15449.89

a: Not available

TABLE 2. The optimal generation scheduling of the proposed method in case I

Units	Generation (MW)
P1	447.5038
P2	173.3182
P3	263.4628
P4	139.0653
P5	165.4734
P6	87.1347
The objective function($)	**15449.89**

TABLE 3. Comparison of the solution method results in the 40-unit case study

Method	Cost($)		
	Best	**Average**	**Worst**
GA-API [21]	139864.96	NA	NA
SDE [22]	138157.46	NA	NA
QOTLBO [23]	137329.86	NA	NA
BBO [24]	137026.82	137116.58	137587.82
DE/BBO [24]	136950.77	136966.77	137150.77
ORCCRO [24]	136855.19	136855.19	136855.19
KHA [25]	136670.37	136671.24	136671.86
Proposed	136450.21	136450.21	136450.21

As a significant advantage, we design the MI-ME leveraging advanced deterministic modeling techniques as well as standardized off-the-shelf solvers. On the other side, AI algorithms are mainly devised based on some 'heuristics' and personal experiences. To the best of our knowledge, the algorithms lack compelling evidence for convergence. The large discrepancies between the obtained solutions of the algorithms also suggest their weakness in finding a unique solution and lack of reliable convergence. However, the proposed model and the employed solvers have solid evidence to prove their reliable convergence, as the achieved result confirms the advantage as well.

The optimal scheduling of the generating units in case II, obtained by the proposed method, is illustrated in Table 4. The scheduling results in $136450.21, as the operation cost.

As noted earlier, we also used an NLP solver to refine the solution of the MI-ME. To analyze the share of the MI-ME and the NLP solvers in the improvement of the obtained solutions, we separately reported the solutions obtained in the models in Table 5. Moreover, we also shown the change in the solutions of the MI-ME model after the application of the NLP solver in percentage terms. Mathematically, the change can be computed as follows:

$$Difference(\%) = \frac{Cost_{MI-ME} - Cost_{NLP}}{Cost_{MI-ME}} \times 100 \qquad (21)$$

Comparison of the results of MI-ME and NLP models, in columns two and three of Table 5 in both cases, indicates that the obtained objective functions are close to each other. To put it simply, the difference, as defined in Equation (21), between the obtained objective functions is less than 0.1%, as can be seen from column four of Table 5. Therefore, the MI-ME model converges to one point quite close to the final solution, and then, the NLP solver solely takes a small step forward to improve the solution locally.

To illustrate the change more clearly, we display the generation levels in the obtained solutions of the MI-ME and NLP models for case study I in Figure 4. As the figure shows, the change in generation levels in the MI-ME solution after the application of the NLP solver is negligible, suggesting that the MI-ME solution is placed very close to the final solution.

To show the power of the MI-ME in finding the optimal solution of the problem, the convergence characteristic of the incumbent value in case study II is illustrated in Figure 5. The 40-unit case includes many local minimal complicating finding the globally optimal solution. However, as the figure reveals, it only takes 6 steps to find the optimal solution.

Finally, to demonstrate the effectiveness of the proposed partitioned McCormick againts the classical McCormick, the case studies are also solved using the classical one. Table 6 compares the derived solution of the two McCormick types.

The table shows that in small-sized problems, such as 6-unit case, the classical McCormick performs satisfactory, as it also may find the optimal solution. Nonetheless, as the problem size increases, the classical one fails to find the optimal solution. On the other side, the proposed partitioned McCormick presents a tighter relaxation and can obtain the more optimal solutions consequently.

TABLE 4. The optimal generation scheduling of the proposed method in case II

Units	**Generation (MW)**	**Units**	**Generation (MW)**
P1	114	**P21**	523.2794
P2	114	**P22**	550
P3	120	**P23**	523.2794
P4	179.7331	**P24**	523.2794
P5	87.7999	**P25**	523.2794
P6	140	**P26**	523.2794
P7	300	**P27**	10
P8	300	**P28**	10
P9	290.4802	**P29**	10
P10	279.5997	**P30**	87.7999
P11	243.5997	**P31**	190
P12	94	**P32**	190
P13	484.0392	**P33**	190
P14	484.0392	**P34**	200
P15	484.0392	**P35**	164.7998
P16	484.0392	**P36**	164.7998
P17	489.2794	**P37**	110
P18	489.2794	**P38**	110
P19	511.2794	**P39**	110
P20	511.2794	**P40**	550
The objective function($)	**136450.21**		

TABLE 5. cost obtained in the MI-ME model compared with the NLP model

Case study	**Cost ($)**		
	MI-ME	**NLP**	**Difference (%)**
6-unit	15443.64	15449.89	0.04
40-units	136311.371	136450.21	0.10

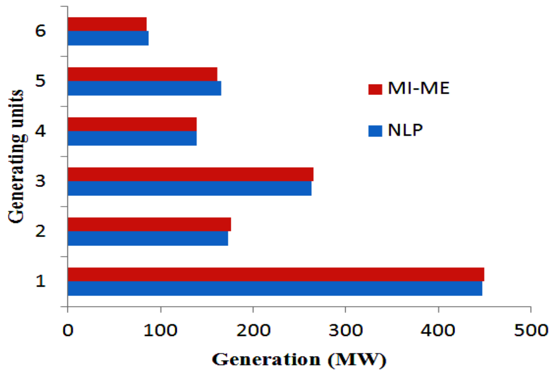


Figure 4. Generation levels in solutions of MI-ME and NLP models

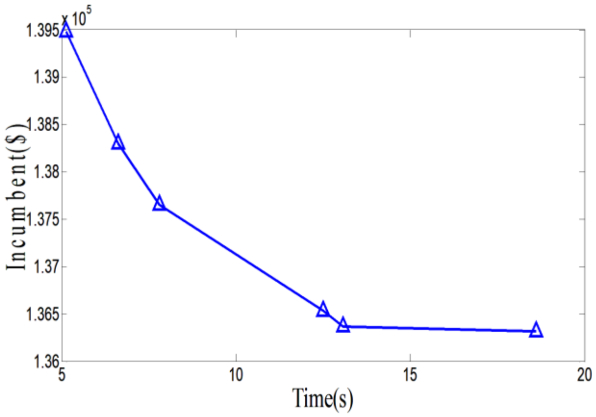


Figure 5. The convergence characteristic of the incumbent value in 40-unit case

TABLE 6. Comparison of the classical and partitioned McCormick relaxations in the considered case studies

McCormick type	Case study	
	6-unit	40-units
The classical McCormick	15449.89	136617.10
The proposed partitioned McCormick	15449.89	136450.21

5. CONCLUSION

To represent a tractable formulation of Kron's formula for transmission loss computation, we have proposed a novel viewpoint on the problem based on the McCormick relaxation of bilinear terms. To this end, we employ an enhanced McCormick envelope that tightly captures the loss equation. Comparison of the obtained solutions of the MI-ME model with the earlier work results on the adopted case studies shows the advantage of the proposed method to find the more optimal solutions. Furthermore, as the presented model relies on deterministic mechanisms for searching the solution space using matured solvers, it exhibits a highly reliable convergence behavior. Finally, the presented model can easily be employed without difficult trial and error procedures usually used to tune the parameters in AI algorithms.

6. REFERENCES

1. Valinejad, J., Oladi, Z., Barforoushi, T. and Parvania, M., "Stochastic unit commitment in the presence of demand response program under uncertainties", ***International Journal of Engineering, Transactions B: Applications***, Vol. 30, No. 8, (2017), 1134-1143. doi: 10.5829/ije.2017.30.08b.04.

2. Lotfi, M.M., "Short-term price-based unit commitment of hydrothermal gencos: A pre-emptive goal programming approach", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 26, No. 9, (2013), 1017-1030. doi: 10.5829/idosi.ije.2013.26.09c.09.

3. Shakouri G, H., Amin Naseri, M. and Neshat, N., "A game theoretic approach for sustainable power systems planning in transition", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 30, No. 3, (2017), 393-402. doi: 10.5829/idosi.ije.2017.30.03c.09.

4. Sharifzadeh, H., "Sharp formulations of nonconvex piecewise linear functions to solve the economic dispatch problem with valve-point effects", ***International Journal of Electrical Power & Energy Systems***, Vol. 127, (2021), 106603. doi: 10.1016/j.ijepes.2020.106603.

5. Bhattiprolu, P.A. and Conejo, A.J., "Multi-period ac/dc transmission expansion planning including shunt compensation", ***IEEE Transactions on Power Systems***, Vol. 37, No. 3, (2021), 2164-2176. doi: 10.1109/TPWRS.2021.3118704.

6. Saleh, S.A. and Chowdhury, M.R., "Survivability analysis of impacts of load-side activities on power systems", ***IEEE Transactions on Industry Applications***, Vol. 58, No. 2, (2022), 1869-1878. doi: 10.1109/TIA.2022.3146102.

7. Su, W., Yu, S.S., Li, H., Iu, H.H.-C. and Fernando, T., "An mpc-based dual-solver optimization method for dc microgrids with simultaneous consideration of operation cost and power loss", ***IEEE Transactions on Power Systems***, Vol. 36, No. 2, (2020), 936-947. doi: 10.1109/TPWRS.2020.3011038.

8. Sharifzadeh, H. and Amjady, N., "A review of metaheuristic algorithms in optimization", ***Journal of Modeling in Engineering***, Vol. 12, No. 38, (2014), 27-43. doi: 10.22075/JME.2017.1677.

9. Zou, D. and Gong, D., "Differential evolution based on migrating variables for the combined heat and power dynamic economic dispatch", ***Energy***, Vol. 238, (2022), 121664. https://doi.org/10.1016/j.energy.2021.121664

10. Niknam, T., Azizipanah-Abarghooee, R. and Aghaei, J., "A new modified teaching-learning algorithm for reserve constrained dynamic economic dispatch", ***IEEE Transactions on Power Systems***, Vol. 28, No. 2, (2012), 749-763. doi: 10.1109/TPWRS.2012.2208273.

11. Ellahi, M., Abbas, G., Satrya, G.B., Usman, M.R. and Gu, J., "A modified hybrid particle swarm optimization with bat algorithm parameter inspired acceleration coefficients for solving eco-friendly and economic dispatch problems", ***IEEE Access***, Vol. 9, No., (2021), 82169-82187. doi: 10.1109/ACCESS.2021.3085819.

12. Braik, M.S., Awadallah, M.A., Al-Betar, M.A., Hammouri, A.I. and Zitar, R.A., "A non-convex economic load dispatch problem using chameleon swarm algorithm with roulette wheel and levy flight methods", ***Applied Intelligence***, (2023), 1-40. https://doi.org/10.1007/s10489-022-04363-w

13. Sutar, M. and Jadhav, H., "An economic/emission dispatch based on a new multi-objective artificial bee colony optimization

algorithm and nsga-ii", ***Evolutionary Intelligence***, (2022), 1-36. https://doi.org/10.1007/s12065-022-00796-x

14. Ali, M.H., El-Rifaie, A.M., Youssef, A.A., Tulsky, V.N. and Tolba, M.A., "Techno-economic strategy for the load dispatch and power flow in power grids using peafowl optimization algorithm", ***Energies***, Vol. 16, No. 2, (2023), 846. https://doi.org/10.3390/en16020846
15. Goudarzi, A., Fahad, S., Ni, J., Ghayoor, F., Siano, P. and Haes Alhelou, H., "A sequential hybridization of etlbo and ipso for solving reserve-constrained combined heat, power and economic dispatch problem", ***IET Generation, Transmission & Distribution***, Vol. 16, No. 10, (2022), 1930-1949. https://doi.org/10.1049/gtd2.12404
16. Iqbal, M.N., Bhatti, A.R., Butt, A.D., Sheikh, Y.A., Paracha, K.N. and Ashique, R.H., "Solution of economic dispatch problem using hybrid multi-verse optimizer", ***Electric Power Systems Research***, Vol. 208, (2022), 107912. doi. https://doi.org/10.1016/j.epsr.2022.107912
17. Beirami, A., Vahidinasab, V., Shafie-khah, M. and Catalão, J.P., "Multiobjective ray optimization algorithm as a solution strategy for solving non-convex problems: A power generation scheduling case study", ***International Journal of Electrical Power & Energy Systems***, Vol. 119, (2020), 105967. https://doi.org/10.1016/j.ijepes.2020.105967
18. Gaing, Z.-L., "Particle swarm optimization to solving the economic dispatch considering the generator constraints", ***IEEE Transactions on Power Systems***, Vol. 18, No. 3, (2003), 1187-1195. doi: 10.1109/TPWRS.2003.814889.
19. Selvakumar, A.I. and Thanushkodi, K., "A new particle swarm optimization solution to nonconvex economic dispatch problems", ***IEEE Transactions on Power Systems***, Vol. 22, No. 1, (2007), 42-51. doi: 10.1109/TPWRS.2006.889132.
20. Elsayed, W.T., Hegazy, Y.G., El-bages, M.S. and Bendary, F.M., "Improved random drift particle swarm optimization with self-adaptive mechanism for solving the power economic dispatch problem", ***IEEE Transactions on Industrial Informatics***, Vol. 13, No. 3, (2017), 1017-1026. doi: 10.1109/TII.2017.2695122.
21. Ciornei, I. and Kyriakides, E., "A ga-api solution for the economic dispatch of generation in power system operation", ***IEEE Transactions on Power Systems***, Vol. 27, No. 1, (2011), 233-242. doi: 10.1109/TPWRS.2011.2168833.
22. Reddy, A.S. and Vaisakh, K., "Shuffled differential evolution for large scale economic dispatch", ***Electric Power Systems Research***, Vol. 96, (2013), 237-245. https://doi.org/10.1016/j.epsr.2012.11.010
23. Roy, P.K. and Bhui, S., "Multi-objective quasi-oppositional teaching learning based optimization for economic emission load dispatch problem", ***International Journal of Electrical Power & Energy Systems***, Vol. 53, (2013), 937-948. https://doi.org/10.1016/j.ijepes.2013.06.015
24. Bhattacharjee, K., Bhattacharya, A. and nee Dey, S.H., "Oppositional real coded chemical reaction optimization for different economic dispatch problems", ***International Journal of Electrical Power & Energy Systems***, Vol. 55, (2014), 378-391. doi. https://doi.org/10.1016/j.ijepes.2013.09.033
25. Mandal, B., Roy, P.K. and Mandal, S., "Economic load dispatch using krill herd algorithm", ***International Journal of Electrical Power & Energy Systems***, Vol. 57, (2014), 1-10. doi. https://doi.org/10.1016/j.ijepes.2013.11.016
26. Sharifzadeh, H., "Solving economic load dispatch by a new hybrid optimization method", ***International Journal of Industrial Electronics Control and Optimization***, Vol. 3, No. 4, (2020), 469-474. https://doi.org/10.22111/ieco.2020.32914.1239
27. Floudas, C.A., "Deterministic global optimization: Theory, methods and applications, Springer Science & Business Media, Vol. 37, (2013).
28. Bergamini, M.L., Aguirre, P. and Grossmann, I., "Logic-based outer approximation for globally optimal synthesis of process networks", ***Computers & Chemical Engineering***, Vol. 29, No. 9, (2005), 1914-1933. https://doi.org/10.1016/j.compchemeng.2005.04.003
29. Castro, P.M., "Tightening piecewise mccormick relaxations for bilinear problems", ***Computers & Chemical Engineering***, Vol. 72, (2015), 300-311. https://doi.org/10.1016/j.compchemeng.2014.03.025

Persian Abstract

چکیده

در سالهای اخیر بعضی محققان از پوش‌های مک‌کرمیخ برای محدب‌سازی بعضی مسائل بهینه‌سازیِ NP-hard دارای عبارات دوسویه استفاده کرده اند. با این حال، تعداد بسیار معدودی از تحقیقات، بر دیگر انواع این پوش جهت بدست آوردن یک ریلکس محدب چفت‌تر بویژه برای کاربردهای عملی تمرکز کرده‌اند. این مقاله یک دیدگاه جدید از رابطه تلفات Kron، با نام رابطه B-matrix نیز شناخته می‌شود، به‌عنوان یک معادله که دارای عبارات دوسویه است ارائه می‌کند. براساس این نگاه، ما رابطه تلفات مذکور را با استفاده از یک ریلکس توسعه‌یافته مک‌کرمیخ به تعدادی قیود خطی تبدیل می‌کنیم. در روش مذکور، دامنه متغیرهای دوسویه به بخش‌های کوچکتری تقسیم می‌شود تا چفتی ریلکس‌سازی بهبود یابد. برای ارزیابی کارایی پوش‌های توسعه‌یافته در توصیف رابطه تلفات Kron، چند سیستم مطالعاتی با عبارات نامحدب مختلف با استفاده از رهیافت مذکور بررسی می‌شوند. یافته‌های شبیه‌سازی‌های عددی حاکی از آن هستند که روش پیشنهادی قادر به نمایش دقیق رابطه تلفات Kron است. همچنین، این روش نسبت به دیگر روش‌های ارائه‌شده در این زمینه موثرتر عمل می‌کند چراکه معمولا به جواب‌های بهینه‌تری همگرا می‌شود.