



## Effect of Particle Size, Moisture Content and Density on the Hyperbolic Model Parameters for Non-cohesive Soil

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### ABSTRACT

The hyperbolic non-linear elastic constitutive model for idealization of non-cohesive soil has been commonly used by researchers in numerical modeling of geotechnical problems. The hyperbolic model consists of several parameters such as modulus number ' $K$ ', exponent ' $n$ ', angle of internal friction ' $\varphi$ ' and failure ratio ' $R_f$ ', which are evaluated using laboratory shear test. The parameters ' $K$ ', ' $n$ ' and ' $R_f$ ' are evaluated from transformed stress-strain curve whereas ' $\varphi$ ' is directly evaluated from normal and shear stress. The study on ' $\varphi$ ' for various soil samples have been performed by many researchers whereas the variation of ' $K$ ', ' $n$ ' and ' $R_f$ ' for various soil samples have not been much explored in the literatures. In addition to it, the evaluation procedure of hyperbolic model parameters (HMP) is a very tedious task when samples are in large numbers. Therefore it is necessary to study the variation of HMP for various non-cohesive soil conditions and to propose certain correlations for its evaluation. The HMP are highly dependent on particle size, moisture content and density. Thus in order to study the influence of these factors on HMP, coarse, medium and fine sand as well as fine gravels with varying densities have been taken into consideration. The direct shear tests have been conducted in dry and moist conditions. The HMP have been evaluated for every samples and the effect of particle size, moisture content and density have been studied. It has been found that the influence of particle size is more than that of moisture content and density. Further the correlations have been developed for HMP with respect to particle size, moisture content and density. The correlations are useful in evaluation of HMP.

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## 1. INTRODUCTION

Soil is a complex material having non-linear stress-strain response when subjected to loading [1]. Due to availability of high speed computers and advanced numerical techniques such as finite element method, it is possible to incorporate the non-linearity of soil [2-4]. The non-linearity of soil has been taken into consideration by various constitutive models such as, hypo-elastic models, plasticity models, hyper-elastic models, etc [1, 5, 6]. The performance of these models mainly depends on its parameters [7, 8]. As the numbers of parameter are more, the accuracy of the model is more [9, 10]. Thus every parameter present in the model is having some specific significant contribution on the behavior of soil. These model parameters are depending on many factors such as

type of soil, particle size, water content, density, etc. The influencing factors are varying based on the type of soil. The model parameters are generally showcased strength and stiffness criteria.

In present investigation, the hyperbolic model [11-13] for representation of constitutive behavior of non-cohesive soil has been studied and its parameters have been evaluated. The model is versatile and has been used for many geotechnical applications [14-16]. Hence the appropriate study on its parameters is necessary. This model is helpful in static and quasi-static condition and predicts the load-displacement behavior appropriately [17-19]. The model finds out the tangent modulus ' $E_T$ ' at any stress level using Equation (1). Further, for some specific geotechnical applications, researchers have modified the hyperbolic model [20, 21].

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$$E_T = \left[ 1 - \frac{R_f(1 - \sin \phi)(\sigma_1 - \sigma_3)}{2(C \cos \phi + \sigma_3 \sin \phi)} \right]^2 K.P_a \left( \frac{\sigma_3}{P_a} \right)^n \quad (1)$$

where, ' $P_a$ ' is atmospheric pressure and ' $\sigma_1$ ' & ' $\sigma_3$ ' are major and minor principal stresses

The model has several parameters such as modulus number ' $K$ ', exponent ' $n$ ', failure ratio ' $R_f$ ', angle of internal friction ' $\phi$ ' and cohesion ' $C$ '. These parameters have been evaluated using shear test [22-24]. The parameters ' $K$ ', ' $n$ ' and ' $R_f$ ' have been evaluated from transformed stress-strain curve whereas ' $\phi$ ' (in degrees) has been directly evaluated from normal and shear stress [16, 25]. The researchers have been focused on the variation of strength parameters for various soil conditions whereas the performance of parameters such as ' $K$ ', ' $n$ ' and ' $R_f$ ' for various soil conditions have not been much explored. Hence there is necessity to study the variation of these parameters with respect to various non-cohesive soil conditions. Also, if the samples are in large numbers, the methodology of evaluation of HMP is a very tedious task. Hence there is need to propose certain correlations which will be helpful in predicting the HMP for non-cohesive soil.

The HMP evaluation procedure has been well explained by various researchers [1, 26, 27]. The hyperbolic model is based on the stress-strain curves of drained triaxial compression tests of sands and clays [25, 28]. Its failure criterion is based on the Mohr-Coulomb model. But as far as non-cohesive samples are considered, the undisturbed sample preparation is very difficult. Hence in order to overcome this difficulty, the HMP has been evaluated from direct shear test by performing few modifications [18, 29, 30]. According to Asadi et al. [29] and other researchers [31, 32], the evaluation of HMP from direct shear test has been found in good agreement as compared to triaxial test.

From the literatures, it has also been reviewed that, the strength parameters of non-cohesive soil depends on particle size, moisture content and density. Thus the stiffness parameters are also vary according to the strength parameters. Hence the same factors have been taken into consideration in present study for studying the variation of ' $K$ ', ' $n$ ' and ' $R_f$ ' for non-cohesive soil samples [22, 33]. The performance of ' $K$ ', ' $n$ ' and ' $R_f$ ' for various soil conditions is a novel contribution of present study. The HMP have been evaluated for coarse, medium and fine sand as well as fine gravels. The tests have been performed for dry and optimum moisture content (OMC) conditions. The density for each sample has also been evaluated. Total 13 no. of direct shear tests have been performed for 04 normal stress conditions. Hence, based on the experimentations, the effect of particle size, density and moisture content on the HMP has been studied in this paper. Also on the basis of analysis result, correlations have been formed for evaluation of HMP.

## 2. METHODOLOGY

Effect of particle size, density and moisture content on the HMP has been evaluated for the test samples given in Table 1. The HMP has been evaluated for each sample considering dry and moist conditions. After analyzing the samples, correlations are developed to evaluate the HMP directly from particle size, density and moisture content. The methodology has been applied to non-cohesive soil mass only.

### 2. 1. Materials

The non-cohesive soil samples have been taken into consideration for evaluation of HMP using direct shear test. The sieve analysis, rodded density test and OMC test has been carried out on all the samples under consideration. Table 1 shows the material samples as well as ' $D_{50}$ ' (mean particle size) value, optimum moisture content and rodded density. The material samples have been chosen in such a way that, all gradations of sand and fine gravels will be covered in experimentation. The pictorial representation of samples under consideration is given in Table 2.

### 2. 2. Evaluation of Hyperbolic Model Parameters

Researchers have suggested the methodology for determination of HMP [26, 29]. In this paper, a similar methodology has been adopted.

The direct shear test has been performed on sand and fine gravel samples. The sand has been tested by small box shear test apparatus whereas the fine gravels have been tested by large box shear test apparatus. In order to get familiar with determination of HMP, an example of normal sand sample has been demonstrated here. The direct shear test on normal sand sample has been performed for four normal stress conditions as shown in Figure 1. The graph between shear stress and tangential displacement is plotted as shown in Figure 1.

The plot in Figure 1 has been transferred to transformed stress-strain curve as shown in Figure 2. In Figure 2, the Y- intercept denotes ' $a$ ' and slope of lines denotes ' $b$ '. Thus from Figure 2, the values for ' $a$ ' and ' $b$ ' are determined. The failure ratio is found as ' $R_f = \tau_f / \tau_{ult}$ ' (whereas ' $\tau_f$ ' is shear stress at failure and ' $\tau_{ult}$ ' is ultimate shear stress) thus ' $1/b$ ' gives the value of ' $\tau_{ult}$ ' (Figure 2 represents as the linear regression of all normal stress values thus the shear stress obtained from it is said as ultimate) and ' $\tau_f$ ' is obtained from Figure 1.









From Figure 2, ' $1/a$ ' gives the value of initial tangent modulus. Thus graph between initial tangent modulus Vs normal stress is plotted as shown in Figure 3. The Y- intercept corresponding to unit normal stress gives the value of modulus number ' $K$ ' and slope of the plot gives the value of exponent ' $n$ '.

Finally a graph between shear stress and normal stress is plotted as shown in Figure 4. The slope of plot gives the value of angle of internal friction ' $\phi$ ' and Y- intercept gives the value of cohesion ' $C$ '.

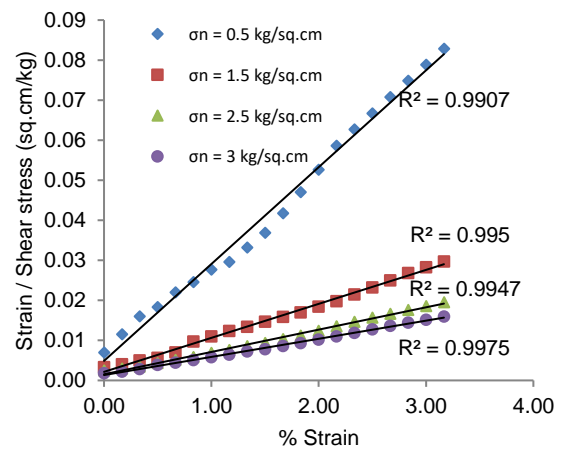
**TABLE 1.** Material Samples under consideration for evaluation of HMP

Sr. No.	Sample	Abbreviation	Particle Size $D_{50}$ (mm)	Moisture Content (%)	Density ( $g/cm^3$ )
1	Fine Sand	A	0.31	0	1.72
2	Sand R 425 u – P 600 u	B	0.51	0	1.69
3	Normal Sand	C	0.68	0	1.78
4	Sand R 600 u – P 1.18mm	D	0.86	0	1.66
5	Sand R 1.18 – P 4.75mm	E	1.76	0	1.61
6	Gravel R 4.75 mm - P 10 mm	F	7.46	0	1.61
7	Gravel R 10 mm – P 12.5 mm	G	11.01	0	1.57
8	Gravel R 12.5 mm – P 20 mm	H	13.52	0	1.54
9	Moist Normal Sand	I	0.68	11.98	1.99
10	Moist Fine Sand	J	0.31	12.35	1.94
11	Moist sand R 425 u – P 600 u	K	0.51	12.72	1.89
12	Moist Sand R 600 u – P 1.18mm	L	0.86	13.05	1.85
13	Moist sand R 1.18 – P 4.75mm	M	1.76	13.48	1.80

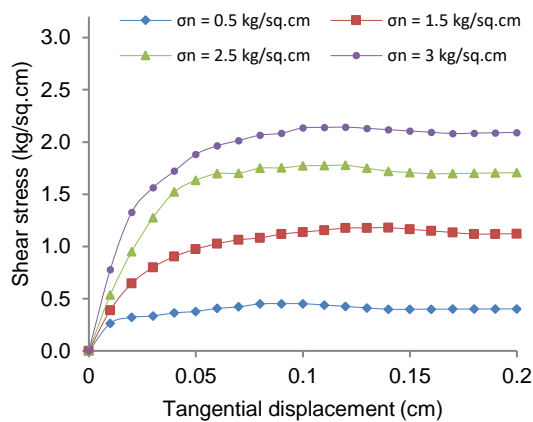
**TABLE 2.** Material Samples Photographs

			
Normal Sand	R 1.18 – P 4.75mm	R 600 u – P 1.18mm	R 425 u – P 600 u
			
Fine R 75 u – P 425 u	R 4.75 mm - P 10 mm	R 10 mm – P 12.5 mm	R 12.5 mm – P 20 mm

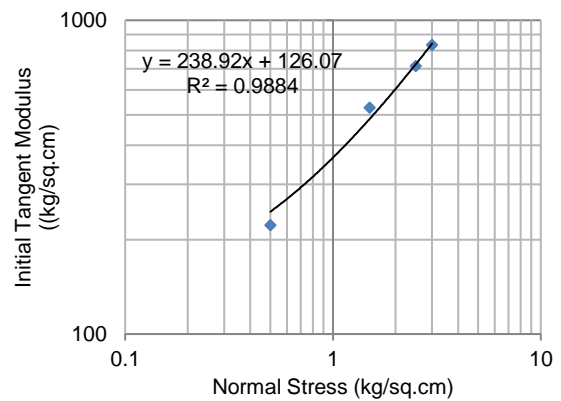
P- Passing; R - Retaining



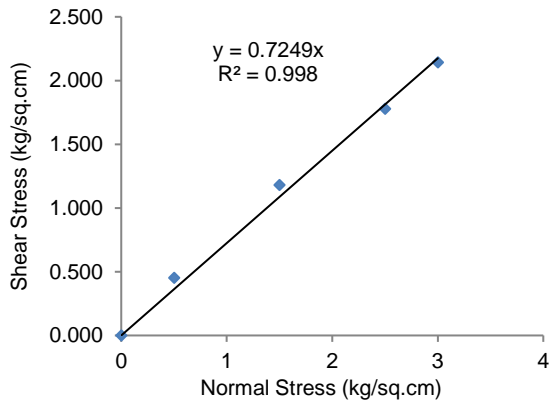
**Figure 2.** Transformed Stress-Strain curve for normal sand



**Figure 1.** Shear Stress Vs Tangential displacement plot for normal sand



**Figure 3.** Initial Tangent Modulus Vs Normal Stress Plot for normal sand



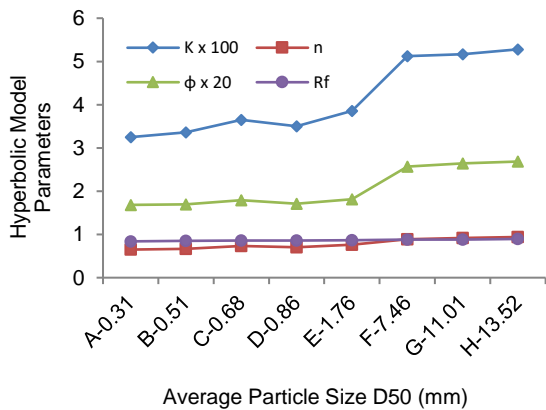
**Figure 4.** Shear Stress Vs Normal Stress Plot for normal sand

Same methodology has been adopted for evaluation of HMP for all the soil samples.

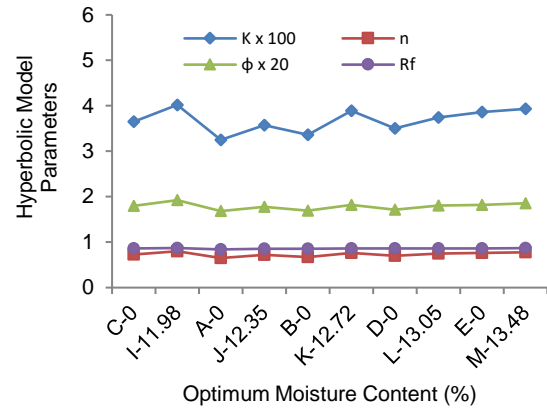
### 3. VARIATION OF HYPERBOLIC MODEL PARAMETERS WITH RESPECT TO PARTICLE SIZE, MOISTURE CONTENT AND DENSITY

The HMP has been determined using the procedure discussed in section 2.2. In order to study the variation of particle size ( $D_{50}$ ), moisture content and density, HMP were plotted against particle size, moisture content and density as shown in Figures 5, 6 and 7, respectively.

From Figure 5, it is observed that, as the ' $D_{50}$ ' value is increasing, the HMP such as ' $K$ ', ' $\phi$ ', ' $n$ ' are also increasing. Due to increasing in particle size, the friction in between the particles increases; thus, the angle of internal friction also increasing. Similarly, as friction is increased, the initial tangent modulus also tends to increase. Hence the value of ' $K$ ' and ' $n$ ' has increased with an increase in particle size. The variation of ' $R_f$ ' with



**Figure 5.** Variation of Particle Size on HMP

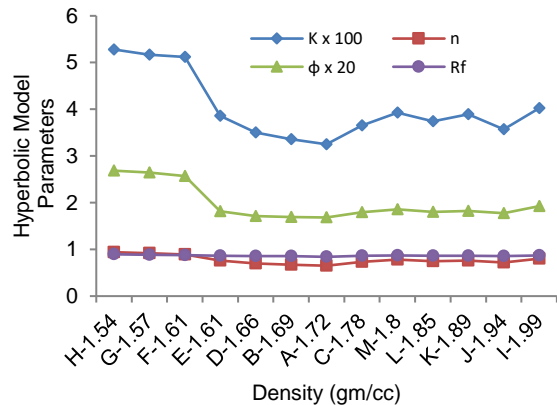


**Figure 6.** Variation of Moisture Content on HMP

respect to particle size is very insignificant. The parameters ' $K$ ', ' $\phi$ ', and ' $n$ ' are dependent on each other; thus, the graphical variation is also similar.

From Figure 6, the variation of HMP with respect to dry and moist sand is observed (sample identification is given in Table 1). It is seen that, the sample with OMC slightly gets dense as compared to dry samples. The tests have been carried out on sand samples only because OMC is not feasible for gravel type of samples. As the samples are getting densified due to OMC, the values of ' $K$ ', ' $\phi$ ' and ' $n$ ' are getting slightly increased as compared to dry samples. Also it is observed that, as particle size increases, the denseness decreases due to lubrication. Thus the ' $K$ ', ' $\phi$ ' and ' $n$ ' is slightly reduced as compared to fine particles. The variation of ' $R_f$ ' is again very less in spite of variation in moisture content.

Figure 7 illustrates the effect of density on HMP. The density of moist sand is slightly more than that of dry sand, thus the value of parameters such as ' $K$ ', ' $\phi$ ', and ' $n$ ' are slightly more than that of dry sand. Whereas for fine gravels, the density is less still the HMP are higher than that of the sand. This has happened due to production



**Figure 7.** Variation of Density on HMP

of excessive friction between gravel particles. Again ' $R_f$ ' is not much varying as per increase in density. The parameter ' $R_f$ ' is a ratio, thus according to experimentation conditions its value is calculated. Hence ' $R_f$ ' slightly looks independent of particle size, moisture content and density.

**3. 1. Sensitivity Analysis** The sensitivity analysis has been carried out to study the influence of various input parameters (such as particle size, moisture content and density) on HMP. The analysis is based on the sensitivity index ( $I$ ). The sensitivity index is a ratio of relative change in the output parameter to relative change in the input parameter [34]. The sensitivity index has been calculated using Lenhart et al. [35] equation. (Equation (2)).

$$I = \frac{X_0(Y_2 - Y_1)}{Y_0(X_2 - X_1)} \quad (2)$$

where,  $X_0$  – central value of input parameter,  $Y_0$  – central value of output parameter at  $X_0$ ,  $X_1 = X_0 - \Delta X$  ( $\Delta X$  – difference in input parameter),  $X_2 = X_0 + \Delta X$  and  $Y_1$  and  $Y_2$  – corresponding to  $X_1$  and  $X_2$ .

Lenhart et al. [35] has also specified the classification of sensitivity index as stated in Table 3.

Based on Equation (2), the sensitivity index was calculated for every input parameter of every HMP. The sensitivity of the input parameters was assessed as stated in Table 3. Table 4 summarized the sensitivity index of each input parameter for HMP with its significance.

From Table 4 and Figures 5 to 7, it has also been found that, the particle size has the maximum influence on HMP than those of density or moisture content. The parameter ' $K$ ' and ' $n$ ' primarily depend on angle of internal friction for non-cohesive soil. The effect of moisture content and density on HMP is only due to sample densification. The sensitivity of input parameters for ' $R_f$ ' is low.

#### 4. CORRELATIONS FOR EVALUATION OF HYPERBOLIC MODEL PARAMETERS

The HMP was calculated for the given samples and plotted against particle size, moisture content and density in section 3. In order to develop a correlation for HMP based on particle size, moisture content and density, the

**TABLE 3.** Classification of sensitivity index

Sr. No.	Sensitivity Index ( $I$ )	Sensitivity
1	$(I) \geq 1$	Very High
2	$1 \geq (I) \geq 0.2$	High
3	$0.2 \geq (I) \geq 0.05$	Medium
4	$0.05 \geq (I) \geq 0$	Low

**TABLE 4.** Sensitivity analysis of various input parameters

Sr. No.	HMP	Input parameters	Sensitivity Index ( $I$ )	Sensitivity
1	$K$	Particle size	0.21	High
		Moisture Content	0.07	Medium
		Density	0.16	Medium
2	$n$	Particle size	0.17	Medium
		Moisture Content	0.08	Medium
		Density	0.14	Medium
3	$\phi$	Particle size	0.21	High
		Moisture Content	0.04	Low
		Density	0.16	Medium
4	$R_f$	Particle size	0.04	Low
		Moisture Content	0.02	Low
		Density	0.03	Low

multivariate regression analysis has been carried out on all set of samples. For regression analysis, HMP is output data whereas particle size, moisture content and density are input data. The 3<sup>rd</sup> degree polynomial was considered for independent variables with 95% confidence level. Further the regression analysis was carried out on all the independent variables and variables having probability more than 5% were removed. In other way, the insignificant variables were excluded from the equation. The significance of coefficient has also been found out by  $F$ -Test and accuracy of input parameters defined with  $T$ -Test. In this way, the independent variables were selected in Equations (3) to (6). Equations (3)-(6) show the correlation for ' $K$ ', ' $n$ ', ' $\phi$ ', and ' $R_f$ ' whereas Figures 8, 9, 10 and Figure 11 show the best fit curves for experimental and predicted values. It has also been observed from Figures 8 to 11 that the values of ' $R^2$ ' are more than 90%, thus the prediction is very appropriate. (Density is denoted by ' $\gamma$ ' and OMC is denoted by ' $w$ ' in Equations (3) to (6))

$$K = 330.27 - 159.223D_{50} + 0.79w + 112.82D_{50}\gamma \quad (3)$$

$$n = 0.6723 - 0.81D_{50} + 0.0029w + 0.13D_{50}\gamma \quad (4)$$

$$\phi = 4.6242 - 15.1085D_{50} - 0.2620w + 16.38\gamma + 10.96\gamma D_{50} \quad (5)$$

$$R_f = 11.974 - 0.6817D_{50} + 1.1422w - 12.6578\gamma + 0.04687D_{50}w - 0.3675w\gamma + 0.4009\gamma D_{50} - 0.0285D_{50}w\gamma + 0.0041D_{50}^2 - 0.0353w^2 + 3.5957\gamma^2 \quad (6)$$

Equations (3) to (6) are useful in predicting the HMP values directly. These equations are applicable to all the non-cohesive soil samples which fall under present study area. Also without conducting the shear test, the HMP can be predicted from these correlations. The correlations are reliable based on the statistical analysis. Hence one

must be confident of using it for further analysis with similar scope of study. In similar way, studies on other soil samples can also be carried out and correlations can be proposed. The methodology present in this paper is useful in carrying out further research work.

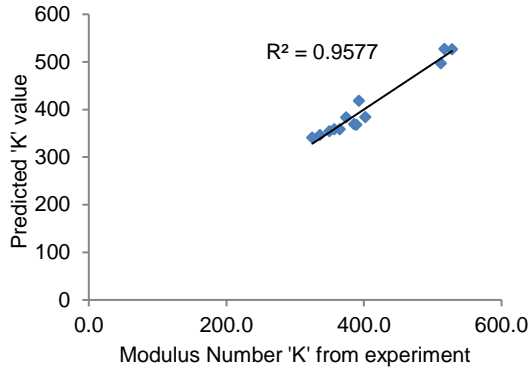


Figure 8. Fitness plot between predicted and experimental values of 'K'

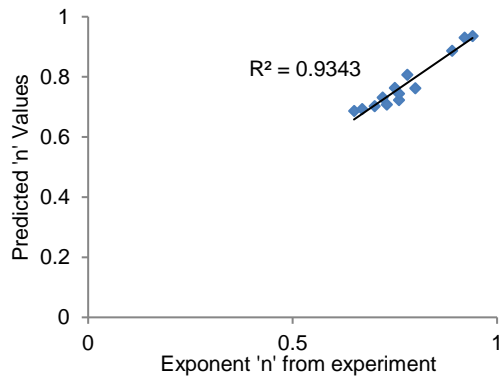


Figure 9. Fitness plot between predicted and experimental values of 'n'

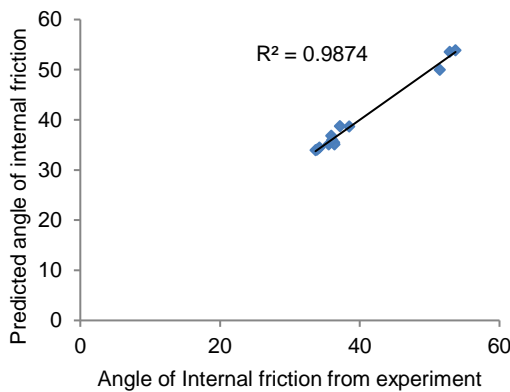


Figure 10. Fitness plot between predicted and experimental values of 'φ'

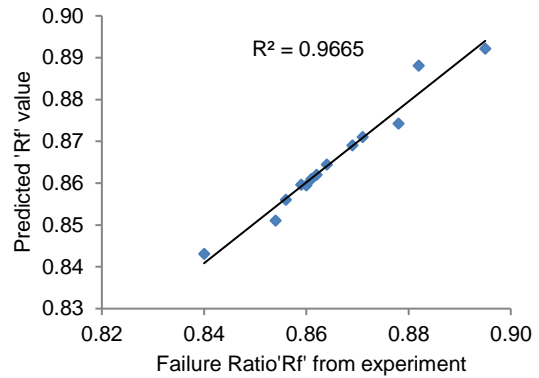


Figure 11. Fitness plot between predicted and experimental values of 'R<sub>f</sub>'

4. 1. Error Analysis

The error analysis has also been performed for examining the accuracy of correlations as shown in Table 5. The values of Mean absolute error (MAE), Mean square error (MSE), Root mean square error (RMSE) and Mean absolute percent error (MAPE) show reliable goodness-of-fit.

TABLE 5. Error Analysis

Parameters	n	MAE	MSE	RMSE	MAPE
K	13	11.90	191.73	13.85	3.04
n	13	0.02	0.00	0.02	2.54
φ	13	0.64	0.66	0.81	1.61
R <sub>f</sub>	13	0.0016	0.0000	0.0024	0.1786

n – total no. of samples

5. CONCLUSIONS

The experimental investigation has been conducted in the present paper to study the effects of particle size, moisture content and density of non-cohesive soil on HMP. The study on variation of 'K', 'n' and 'R<sub>f</sub>' with respect to various non-cohesive soil samples is a novel contribution of present paper. Also looking at the tedious methodology for the evaluation of HMP, the correlations have been proposed. The correlations are generated using multivariate regression analysis. Based on the analysis and correlations presented in this paper, the following conclusions are drawn,

- a. The direct shear test has been successfully implemented for the evaluation of HMP of non-cohesive soil.
- b. The values of 'K', 'n' and 'φ' are increasing with increase in particle size. This is due to an increase in friction between the particles. The sensitivity index of more than 0.2 is observed which means the particle size is highly sensitive.

- c. The non-cohesive soil samples are getting slightly densified due to OMC. Hence the values of ' $K$ ', ' $\phi$ ' and ' $n$ ' are getting slightly increased as compared to dry samples. Whereas further increase in water content, reduces the ' $K$ ', ' $\phi$ ' and ' $n$ ' due to lubrication effect.
- d. The fine gravel is having lesser density than that of sand, still the values of ' $K$ ', ' $n$ ' and ' $\phi$ ' are increasing with respect to sand samples. This is because of production of excessive friction between the particles.
- e. The variation of failure ratio with respect to particle size, moisture content and density is very less. The sensitivity index of less than 0.05 confirms that.
- f. Particle size is having maximum influence on HMP than that of moisture content and density.
- g. The effect of moisture content and density on HMP is only due to densification of soil.
- h. The proposed correlations are useful in predicting the HMP. The correlations are applicable for non-cohesive soil samples which fall under present scope of the study.
- i. The correlations are reliable based on the statistical analysis (such as  $R^2$  value and error analysis).
- j. The methodology proposed in the present study is applicable to other soil samples as well for further study.

## 6. REFERENCES

1. Al-Shayea, N., Abduljawwad, S., Bashir, R., Al-Ghamedy, H. and Asi, I., "Determination of parameters for a hyperbolic model of soils", *Proceedings of the Institution of Civil Engineers-Geotechnical Engineering*, Vol. 156, No. 2, (2003), 105-117, doi: 10.1680/jeng.156.2.105.37526.
2. Horpibulsuk, S. and Miura, N., "Modified hyperbolic stress-strain response: Uncemented and cement stabilized clays", *Reports of the Faculty of Science and Engineering, Saga University*, Vol. 30, No., (2001), 39-47, doi.
3. Farhadi, N., Saffari, H. and Torkzadeh, P., "Evaluation of seismic behavior of steel moment resisting frames considering nonlinear soil-structure interaction", *International Journal of Engineering, Transactions A: Basics*, Vol. 31, No. 7, (2018), 1020-1027, doi: 10.5829/ije.2018.32.07a.03.
4. Sarlak, A., Saeedmonir, H. and Gheyretmand, C., "Numerical and experimental study of soil-structure interaction in structures resting on loose soil using laminar shear box", *International Journal of Engineering, Transactions B: Applications*, Vol. 30, No. 11, (2017), 1654-1663, doi: 10.5829/ije.2017.30.11b.05.
5. Wang, Y. and Stokoe, K., "Development of constitutive models for linear and nonlinear shear modulus and material damping ratio of uncemented soils", *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 148, No. 3, (2022), 04021192, doi: 10.1061/(asce)gt.1943-5606.0002736.
6. Liu, K., Yin, Z.-Y., Chen, W.-B., Feng, W.-Q. and Yin, J.-H., "Nonlinear model for the stress-strain-strength behavior of unsaturated granular materials", *International Journal of Geomechanics*, Vol. 21, No. 7, (2021), 04021103, doi: 10.1061/(asce)gm.1943-5622.0002042.
7. Varga, G. and Czap, Z., "Soil models: Safety factors and settlements", *Periodica Polytechnica Civil Engineering*, Vol. 48, No. 1-2, (2004), 53-63.
8. Rezayibana, B., "The effect of soil type on seismic response of tall telecommunication towers with random vibration analysis", *International Journal of Engineering, Transactions C: Aspects*, Vol. 33, No. 3, (2020), 419-426, doi: 10.5829/ije.2020.33.03c.06.
9. Çakır, Ö. and Coşkun, N., "Theoretical issues with rayleigh surface waves and geoelectrical method used for the inversion of near surface geophysical structure", *Journal of Human, Earth, and Future*, Vol. 2, No. 3, (2021), 183-199, doi: 10.28991/hef-2021-02-03-01.
10. Tarrad, A.H., "3d numerical modeling to evaluate the thermal performance of single and double u-tube ground-coupled heat pump", *HighTech and Innovation Journal*, Vol. 3, No. 2, (2022), 115-129, doi: 10.28991/hij-2022-03-02-01.
11. Brinkgreve, R.B., Selection of soil models and parameters for geotechnical engineering application, in *Soil constitutive models: Evaluation, selection, and calibration*. 2005.69-98.
12. Rahnama, H., "A hyperbolic model for stress-strain behavior of unsaturated soils", in *12th International Conference on Computer Methods and Advances in Geomechanics*, Citeseer. Vol. 2008, (2008).
13. Murugan, R.A. and Stalin, V., "Soil behaviour and hyperbolic model", in *Proc., Indian Geotechnical Conference-2009*, Guntur, India., (2009), 632-636.
14. Banerjee, S. and Malek, S., "Assessment of a hyperbolic model for undrained-cyclic shearing of remolded clay", *Journal of Engineering Mechanics*, Vol. 146, No. 7, (2020), 04020064, doi: 10.1061/(asce)em.1943-7889.0001780.
15. George, T.I. and Dasaka, S.M., "Numerical investigation of soil arching in dense sand", *International Journal of Geomechanics*, Vol. 21, No. 5, (2021), 04021051, doi: 10.1061/(asce)gm.1943-5622.0001971.
16. Viladkar, M., Zedan, A.J. and Saran, S., "Nonlinear elastic analysis of shallow footings subjected to eccentric inclined loads", *Geomechanics and Geoenvironmental Engineering*, Vol. 10, No. 1, (2015), 45-56, doi: 10.1080/17486025.2014.902117.
17. Dong, W., Hu, L., Yu, Y.Z. and Lv, H., "Comparison between duncan and chang's eb model and the generalized plasticity model in the analysis of a high earth-rockfill dam", *Journal of Applied Mathematics*, Vol. 2013, (2013), doi: 10.1155/2013/709430.
18. He, Y. and Chen, X., "The application of improved duncan-chang model in unloading soil", *The Open Civil Engineering Journal*, Vol. 8, No. 1, (2014), doi: 10.2174/1874149501408010410.
19. David, T.K., Krishnamoorthy, R.R. and IB, M.J., "Finite element modelling of soil-structure interaction", *Jurnal Teknologi*, Vol. 76, No. 8, (2015), doi: 10.11113/jt.v76.5625.
20. Gabriel, A.-A., Li, D., Chiochetti, S., Tavelli, M., Peshkov, I., Romenski, E. and Dumbser, M., "A unified first-order hyperbolic model for nonlinear dynamic rupture processes in diffuse fracture zones", *Philosophical Transactions of the Royal Society A*, Vol. 379, No. 2196, (2021), 20200130, doi: 10.1098/rsta.2020.0130.
21. Jia, P., Khoshghalb, A., Chen, C., Zhao, W., Dong, M. and Alipour Esgandani, G., "Modified duncan-chang constitutive model for modeling supported excavations in granular soils", *International Journal of Geomechanics*, Vol. 20, No. 11, (2020), 04020211, doi: 10.1061/(asce)gm.1943-5622.0001848.
22. Arivazhagan, R., Premalatha, K., K. Sabarishri and B. S. Xavier Vedha Rayan, "Influence of silt content on the hyperbolic model parameters of sand", *Journal of Critical Reviews*, Vol. 7, No. 17, (2020), 1647-1661, doi: 10.31838/jcr.07.17.210.
23. Gens, A., Carol, I. and Alonso, E., "An interface element formulation for the analysis of soil-reinforcement interaction", *Computers and Geotechnics*, Vol. 7, No. 1-2, (1989), 133-151, doi: 10.1016/0266-352X(89)90011-6.
24. Dhadse, G.D., Ramtekkar, G. and Bhatt, G., "Finite element modeling of soil structure interaction system with interface: A review", *Archives of Computational Methods in Engineering*,

- Vol. 28, No. 5, (2021), 3415-3432, doi: 10.1007/s11831-020-09505-2.
25. Viladkar, M., Godbole, P. and Noorzaei, J., "Modelling of interface for soil-structure interaction studies", *Computers & Structures*, Vol. 52, No. 4, (1994), 765-779, doi: 10.1016/0045-7949(94)90358-1.
  26. Abdul-kareem, A.H. and Helal, A., "Estimation of hyperbolic stress-strain parameters for gypseous soils", *International Journal of Civil Engineering*, Vol. 7, (2007), doi: 10.1016/0022-4898(64)90153-3.
  27. Narepalem, S. and Godavarthi, V.R.S.R., "Constitutive behaviour of fine grained soils of vijayawada region", *KSCE Journal of Civil Engineering*, Vol. 23, No. 6, (2019), 2463-2470, doi: 10.1007/s12205-019-0699-5.
  28. Hora, M., "Nonlinear interaction analysis of infilled frame-foundation beam-homogeneous soil system", *Coupled Systems Mechanics*, Vol. 3, No. 3, (2014), 267-289, <http://dx.doi.org/10.12989/csm.2014.3.3.267>
  29. Asadi, M., Shariatmadari, N., Karimpour-Fard, M. and Noorzad, A., "Validation of hyperbolic model by the results of triaxial and direct shear tests of municipal solid waste", *Geotechnical and Geological Engineering*, Vol. 35, No. 5, (2017), 2003-2015, doi: 10.1007/s10706-017-0223-y.
  30. Hormdee, D., Kaikeerati, N. and Angsuwotai, P., "Evaluation on the results of multistage shear test", *Geomate Journal*, Vol. 2, No. 3, (2012), 140-143, doi: 10.21660/2012.3.3m.
  31. Sridharan, A., TG, S.K., Abraham, B.M. and Cyrus, S., "Applicability of hyperbolic method for the prediction of shear strength parameters from multistage direct shear tests".
  32. Adajar, M.A.Q. and Zarco, M.A.H., "Predicting the stress-strain behavior of mine tailing using modified hyperbolic model", *Geomate Journal*, Vol. 10, No. 21, (2016), 1834-1841, doi: 10.21660/2016.21.5127.
  33. Vali, R., "Water table effects on the behaviors of the reinforced marine soil-footing system", *Journal of Human, Earth, and Future*, Vol. 2, No. 3, (2021), 296-305, doi: 10.28991/hef-2021-02-03-09.
  34. Turkane, S.D. and Chouksey, S.K., "Application of response surface method for optimization of stabilizer dosages in soil stabilization", *Innovative Infrastructure Solutions*, Vol. 7, No. 1, (2022), 1-12, doi: 10.1007/s41062-021-00704-9.
  35. Lenhart, T., Eckhardt, K., Fohrer, N. and Frede, H.-G., "Comparison of two different approaches of sensitivity analysis", *Physics and Chemistry of the Earth, Parts A/B/C*, Vol. 27, No. 9-10, (2002), 645-654, doi: 10.1007/BF01743071.

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### Persian Abstract

#### چکیده

مدل ساختاری الاستیک غیرخطی هذلولی برای ایده آل سازی خاک غیر چسبنده معمولاً توسط محققان در مدل سازی عددی مسائل ژئوتکنیکی استفاده شده است. مدل هذلولی شامل چندین پارامتر مانند عدد مدول "K"، توان "n"، زاویه اصطکاک داخلی "φ" و نسبت شکست "Rf" است که با استفاده از آزمون برشی آزمایشگاهی ارزیابی می‌شوند. پارامترهای "n, K" و "Rf" از منحنی تنش-کرنش تبدیل شده ارزیابی می‌شوند در حالی که 'φ' به طور مستقیم از تنش نرمال و برشی ارزیابی می‌شود. مطالعه روی «φ» برای نمونه‌های مختلف خاک توسط بسیاری از محققین انجام شده است، در حالی که تغییرات «n, K» و «Rf» برای نمونه‌های مختلف خاک در مقالات چندین مورد بررسی قرار نگرفته است. علاوه بر آن، فرآیند ارزیابی پارامترهای مدل هذلولی (HMP) زمانی که نمونه‌ها در تعداد زیادی هستند، کاری بسیار خسته کننده است. بنابراین بررسی تغییرات HMP برای شرایط مختلف خاک غیر منسجم و پیشنهاد همبستگی‌های خاصی برای ارزیابی آن ضروری است. HMP به شدت به اندازه ذرات، رطوبت و چگالی وابسته است. بنابراین به منظور بررسی تاثیر این عوامل بر HMP، ماسه درشت، متوسط و ریز و همچنین شن‌های ریز با چگالی‌های متفاوت در نظر گرفته شده است. آزمایش برش مستقیم در شرایط خشک و مرطوب انجام شده است. HMP برای هر نمونه ارزیابی شده و اثر اندازه ذرات، میزان رطوبت و چگالی مورد مطالعه قرار گرفته است. مشخص شده است که تأثیر اندازه ذرات بیشتر از میزان رطوبت و چگالی است. علاوه بر این، همبستگی‌ها برای HMP با توجه به اندازه ذرات، محتوای رطوبت و چگالی توسعه داده شده است. همبستگی‌ها در ارزیابی HMP مفید هستند.

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