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A State-of-the-art Model of Location, Inventory, and Pricing Problem in the Closed-loop Supply Chain Network

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ABSTRACT

The main objective of designing the supply chain is to increase profitability. For this reason, a state-of-the-art model of a three-echelon closed-loop supply chain is proposed that consists of the manufacturer, retailer, and collection centers. For the first time, a new separate and autonomous channel is considered in this model for the sale of Reman products aiming at increasing the manufacturer's profitability. In this model, location, inventory, and pricing of the product are also taken into consideration. Lingo software is utilized to solve nonlinear objective functions at a small scale, and metaheuristic genetic algorithms and particle swarm optimization were utilized to solve at a large scale. The research results depict that the state-of-the-art model design of the closed-loop supply chain network is credible and the optimal location of supply chain components, optimal response of product flow, and product price are determined in a proper manner. As a result, the profitability of the whole closed-loop supply chain network increased. Sensitivity analysis of mathematical model depicts that this model shows higher sensitivity to retailer replacement factor, price and purchasing power of collection centers. Also, the genetic algorithm shows better performance on a large scale in terms of response quality. On the other hand, the time required by particle swarm optimization to reach a response is far better than the genetic algorithm. Ultimately, practical suggestions for the managers are presented considering the state-of-the-art model design of a closed-loop supply chain network.

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1. INTRODUCTION

The subject of supply chain and its profitability in different fields are among the highly-focused research subjects in numerous articles. The main focus of this article is to design and propose a state-of-the-art model of a three-echelon closed-loop supply chain (CLSC). The advantages of closed-loop supply chain network design include reduced pollution, increased customer satisfaction, reduced costs, and increased profitability.

Considering the importance of supply chain design and location significance in it, a three-echelon closed-loop supply chain is designed that consists of the manufacturer, retailer, and collection centers. The manufacturer sells economic order quantity to the retailer at an optimal price and then, the retailer sells the products to the customers at his/her optimal prices. When the commodity has no use for the customers, the customer sells them to the collection centers at an optimal price designated by the collection centers. Afterwards, the economic quantity of collected commodities are sold to the manufacturers for salvage at an optimal price of the manufacturer and after this option, Reman products are sold to the customers in a direct manner at the manufacturer's optimal selling price for the Reman products. Such process for resale of the commodity is presented for the first time in the proposed supply chain model. Figure 1 gives an overall view of the proposed closed-loop supply chain.

At each stage/phase, the optimal prices were separately calculated. The economic quantity of order and delivery was determined considering the cost

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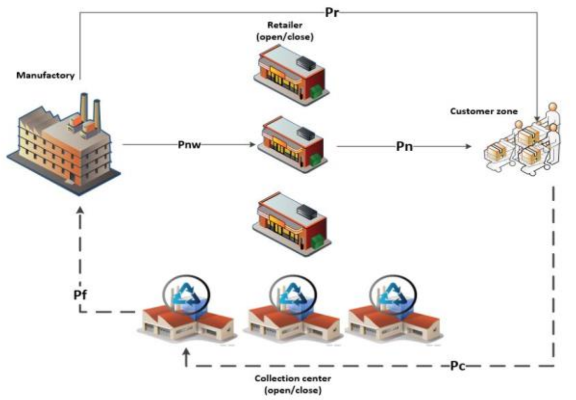


Figure 1. Overall view of proposed CLSC

function of retailers and collection centers. Customer demand is deterministic and depends on prices determined by the retailers and by manufacturers for Reman products. Following the determination of optimal prices and using a mathematical model for the manufacturer, the location of retailers and collection centers is determined in this model. In this problem, inventory costs of retailers and collection centers are also taken into consideration.

2. LITERATURE REVIEW

The closed-loop supply chain is one of the crucial and state-of-the-art subjects in supply chain literature. The main objective of each supply chain is to increase profitability. In this article, therefore, three main topics are taken into consideration for closed-loop supply chain modelling for the first time. These topics include:

- Pricing, inventory, and location
- A separate sales channel for the manufacturer
- The distance-sensitive location between each construct/component

Previous literature has not investigated the above topics simultaneously. However, we intend to investigate their impact on the closed-loop supply chain by applying them simultaneously. In the next section, the previous literature is reviewed.

Pricing of a three-echelon closed-loop supply chain was conducted for perishable products in a research and the echelons of this supply chain consist of the manufacturer, retailer, and customer [1]. In this research, the manufacturer has two options, to either sell the product to customers directly or to sell the product to the retailer. Related to the second option, the return policy is designed in a way that returned product is acquired by the retailer from the customer and then, sold to the manufacturer. In this article, we consider a separate channel for selling manufacturers' products to the customer in a direct manner, with a difference that there is no sale limit. Based on the pricing, the manufacturer can sell a percentage of the product to the retailer and another to the customer. The return policy of both channels is through collection channels.

Researchers investigated the pricing of a single-product closed-loop supply chain that includes retailers, customers, and collection centers [2]. This research considers a direct channel for product sale through the manufacturer to the customer. The supply chain and demand functions in our article are designed similar to this research. The difference is that the present research has investigated the subject of location and inventory and carrying costs. Moreover, the problem is solved for a multi-product scenario.

It was investigated the pricing of a two-echelon supply chain that consists of suppliers and retailers. In the proposed model, there is a direct channel from the supplier to the customer [3]. To solve the problem, the profit function of supplier and retailer is written for both channels and inventory costs are calculated for 6 situations and then, they are compared. However, we propose a three-echelon supply chain where there is a separate channel for product sale to the customer in a direct manner. Moreover, carrying costs are calculated using a single method.

It was investigated the optimal location and price for the retailers [4]. First, they proposed a separate price function in order to achieve the final price for the retailer and calculated the least possible price. Then, the price of commodities is changed randomly and the amount of profit is calculated in each change. After achieving the optimal price, the value is placed in the objective function and the problem is solved for optimal placement and determining the optimal location. Here, pricing and placement/location are also investigated with a difference that the optimal price is calculated for each component of the supply chain and pricing is carried out for the retailer and collection centers with the aid of derivative of the dependent function with respect to the demand. Next, the manufacturer price is calculated with the aid of a mathematical model by taking the profit of supply chain components into consideration. Regarding placement, the optimal location is determined considering the distance from customer to retailer and collection centers.

In an other paper, it was investigated the location of chain and pricing components in a five-echelon closed-loop supply chain [5]. The modelling objective in this problem is to maximize total supply chain profit considering the optimal location of supply chain components and optimal commodity sale price. Similar to our article, this research investigated the location and pricing, with a difference that our research not only investigates the optimal sale price but also the optimal purchase price of supply chain components. The location of components was determined considering the distance between the customer and components and the carrying and inventory costs are taken into consideration in modelling .

Researchers investigated the pricing for a three-echelon closed-loop supply chain that consists of the manufacturer, supplier, and customer [6]. The chain is designed so that manufacturer can acquire the used product from the customer after selling a new product to the customer and can employ different approaches for the acquired product. In this article, the manufacturer also acquires used products from the customer through an intermediary. However, different approaches are not considered for the manufacturer and the acquired products can be only salvaged and re-supplied to the supply chain.

In 2017, two researchers have been considered the pricing for a two-echelon supply chain that consists of one manufacturer and several retailers [7]. The cost is assumed as a price-sensitive function in which carrying costs and prices, retailer shortage and inventory carrying cost for the manufacturer are taken into account. Two methods namely particle swarm optimization and binary quantum-behaved particle swarm optimization algorithm are utilized for solving the problem. In this article, we also investigated the pricing and inventory, with a difference that carrying costs are only calculated for the retailer and collection centers, and shortage costs are not among the problem premises.

It has been reviewed the pricing of multi-period and multi-product problems in which demand is assumed as price and advertisement sensitive function [8]. Inventory carrying costs are also taken into consideration in the proposed model. Here, however, we have investigated the pricing and inventory matter in a multi-product supply chain and the demand function of supply chain components depends on optimal purchase price, and carrying and location optimal costs. However, advertisement is not taken into consideration in this model.

It investigated the inventory costs in a non-deterministic and multi-product supply chain with the aim of maximizing the chain's profit [9]. In the research, demand is assumed as a price-sensitive function, and differentiation is utilized in order to determine the optimal/economic model quantities. In this article, we also investigated the pricing and inventory costs in a multi-product supply chain with a difference that parameters are deterministic.

Fahimnia et al. [10] presented a five-echelon green and closed-loop supply chain consisting of the manufacturer, wholesaler, customer, collection centers, and salvage centers. The aim of this article is to minimize supply chain costs and determine the optimal price for the level of produced carbon dioxide by the supply chain. The objective of our article is to maximize the profit.

Ahmadi-Javid and Hoseinpour [11] designed a multi-product network consisting of the wholesaler, retailer, distribution center, and customer. The objective is to maximize the profit by taking inventory costs into consideration. The location of the distribution center is determined with the aid of the proposed model and sale price is determined following the model solving process. In our article, all the above steps are carried out in a closed-loop supply chain and a separate channel is taken into consideration to sell the product directly to the customer.

Fattahi et al. [12] investigated the components pricing and placement in a two-echelon supply chain consisting of manufacturer and wholesaler. They proposed a mixed-integer linear programming model for a multi-product supply chain over a multi-period horizon by taking flow capacity and inventory system costs into consideration. The difference between this research with our article is associated with supply chain, network design, and model solution. In our article, the model was solved on a small scale and also, metaheuristic genetic algorithms were also utilized on a large scale. However, we have proposed a single-period model Kaya and Urek [13] investigated the location and pricing decision of products with the aid of a nonlinear model. This study is conducted for a chain store in Turkey. Our article not only attempts to propose a model but also determine the optimal price with the aid of differentiation and concepts such as market share and product flow capacity and a separate sale channel is taken into consideration.

Ahmadzadeh and Vahdani [14] provided a single-product and three-echelon close-looped supply chain under possible conditions of uncertainty. The model objective is to maximize the total supply chain profit by taking inventory shortages costs into consideration. The genetic algorithms (GAs), imperialist competitive algorithm (ICA), and the firefly algorithm (FA) were utilized for solving the model. The comparison between these algorithms depicts that FA achieved a better result over a minimum length of time. Conditions of uncertainty are not taken into consideration on our inventory shortage costs but we have considered a multi-product supply chain. In addition, not only did we solve the model on a small scale, but also the genetic algorithm and particle swarm optimization were utilized for large scales.

In another study, location and pricing in a four-echelon close-looped supply chain has been investigated, including supplier, manufacturer, retailer, and recovery center. This study examines the issue of reducing electronic pollution and cell phone reuse. The mathematical model is a multi-objective problem that involves maximizing the profit. This mathematical model has been prepared using the fuzzy method and the Epsilon Constraints method has been used to solve it. In this study, only pricing and location in the closed-loop supply chain have been considered, and inventory and maintenance costs have not been examined, and a separate channel for direct sales of products has not been considered [11].

Tavakkoli-Moghaddam et al. [15] examined pricing, inventory, and location in a four-echelon closed-chain supply chain, including manufacturer, retailer, and distribution center. In this study, the routing of perishable goods is investigated by considering urban traffic and product expiration time. In this research, an independent channel for selling products directly to customers is not considered.

Another work deals with pricing in a four-echelon close-looped supply chain, including suppliers, manufacturers, retailers, and collection centers. In this mathematical model, profit maximization is discussed [16]. In this study, only the pricing in the closed chain supply chain has been studied and the location, inventory and maintenance costs and separate channels for selling directly to customers have not been studied.

In conclusion, "pricing, inventory, and location", "a separate sale channel for the manufacturer", and "distance sensitive location between the components" are investigated in the previous literature. However, these three topics have not been taken into consideration in a closed-loop supply chain. Hence, these topics are utilized for the first time for modelling and the results are analyzed. A summary of previous literature are presented in Table 1.

3. PROBLEM DEFINITION AND MODELLING

In this article, we proposed a closed-looped supply problem chain that is modelled by taking into account the following topics, simultaneously: "pricing, inventory, and location", "a separate sale channel for the manufacturer", and "distance sensitive location between the components". The three-echelon closed-loop supply chain consists of the manufacturer, retailer, and collection centers .

The manufacturer sells optimal order quantity to the retailer and then, the retailer sells the products to the customers. When the products have no use for the customers, the customer sells them to the collection centers. Afterwards, the optimal quantity of collected products is acquired by the manufacturer for salvage and then, the manufacturer sells the Reman products directly to the customers. Such a process for re-selling the product (a separate sale channel) is proposed for the first time in our closed-loop supply chain model.

Following assumptions are taken into consideration for problem modelling in Figure 1:

1. Each customer can only acquire one product from the retailer.
2. Each customer can sell the used product only to one collection center.
3. The new and Reman products can be replaced.

TABLE 1. Summary of previous literature

Authors names & Published dates	*Sup*	*CLSC*	*Inventory*	*location*	*pricing*	*Ssc*	*Model*
Barry Alan Pasternack (1985) [1]	●	●			Yes	●	LP
Plastria Vanhaverbeke (2008) [4]				●	Yes		MIP
Jiawang Xu and Yunlong Zhu (2011) [6]	●	●	●		Yes		NLP
Fahimnia et al. (2013) [10]	●	●	●	●	Yes		MINLP
Onur Kaya , Busra Urek (2015) [13]	●	●	●	●	Yes		MINLP
Mohammad Fattahi et al. (2015) [12]	●		●	●	Yes		MINLP
Ahmadi-Javid, Hoseinpour (2015) [11]	●		●	●	Yes		MINLP
Shu-San Gan et al. (2016) [2]	●	●			Yes	●	NLP
Chen and Sarker.(2016) [7]	●		●		Yes		NLP
Ahmadzadeh, Vahdani (2017) [14]	●	●	●	●	Yes		MINLP
R. Ghasemy Yaghin (2017) [8]	●		●		Yes		NLP
M. A. Edalatpour, S. M. Al-e-Hashem (2019) [9]			●		Yes		NLP
Atabaki et al. (2019) [5]	●	●		●	Yes		MILP
Raaid Batarfia, et al. (2019) [3]	●		●		Yes	●	NLP
Ahmadi, Hassanzadeh Amin (2019) [17]	●	●		●	Yes		MIP
Tavakkoli-Moghaddam et al. (2021) [15]	●	●	●	●	Yes		MINLP
Ziari,Sheikh Sajadieh (2022) [16]	●	●			Yes		LP
This WORK	●	●	●	●	Yes	●	MINLP

4. The manufacturer sees no difference between the retailers.
5. The manufacturer sees no difference between the customers.
6. Location decision of retailers and collection centers is sensitive to their distance to the customer and construction costs .
7. Collection centers and retailers bear carrying and inventory costs that follow the inventory model of economic order quantity.

3. 1. Sets, Parameters, Variables, And Functions

The sets of retailers, customers, collection centers, and products are presented in Table 2 and mathematical model parameters are given in Table 3. Moreover, mathematical functions used in the mathematical model and problem variable are presented in Tables 4 and 5, respectively.

TABLE 2. Sets

I: Retailers	*i*=1, 2, …, *I*
J: Customers	*j*=1, 2, …, *J*
L: Collection centers	*l*=1, 2, …, *L*
H: Commodities	*h*=1, 2, …, *H*

TABLE 3. Parameters

$K_{(h,j)}$	Potential demand for products *h* by customers *j* (retailers)
$k'_{(h,j)}$	Potential demand for products *h* by customers *j* (manufacturer)
k_i	Sensitivity coefficient of selling price of products by retailer *i*
k'	Sensitivity coefficient of selling price of products by the manufacturer
b_1	The replacement rate of the product that the retailer sells
b_2	The replacement rate of the product that the manufacturer sells
$h_{(h,i)}$	Holding cost of products *h* by retailer *i*
$h_{(h,l)}$	holding cost of products *h* by collection center *l*
A_i	Fixed cost of ordering products by retailer *i*
A_l	Fixed cost of ordering products by collection centers
$\varphi_{(h,j)}$	Percentage of consumer products *h* that collection centers purchase from customer *j*
$C_{(h,l)}$	The cost of collecting products *h* by the collection center *l*
Crw_h	Cost per unit of raw materials for the purchasing of products *h*
Cm_h	Cost of production of product *h* by the manufacturer
Cr_h	Cost of reproducing production of product *h* by the manufacturer
F_i	Cost of constructing retail *i*
E_l	Cost of constructing collection center *l*
C_{ij}	Customer distance *j* from retailer *i*
D_{lj}	Distance of customer *j* from the collection center *l*
S_i	Construction cost of retailer *i* (depends on the distance of the retailers)
S_l	Construction cost of collection center *l* (depends on the distance of the retailers)
u	Constant value of demand function

TABLE 4. Mathematical functions

$Dr_{(h,i,j)}$	Retail demand function *i* for product *h* (by customer *j*)
$Dm_{(h,j)}$	Manufacturer demand function for product *h* (by customer *j*)
$Dc_{(h,l,j)}$	Collection center *l* demand function for product *h* (by customer *j*)
Π_{Ri}	Retail profit function *i*
Π_{cl}	Collection center profit function *l*
Π_m	Manufacturer profit function

TABLE 5. Variables

$Pnw_{(h,i)}$	Sale price (Manufacturer) of new product *h* to retailer *i*
$Pn_{(h,i,j)}$	Sale price (retailer *i*) of new product *h* to customer *j*
$Pc_{(h,l,j)}$	Purchase price of used product *h* by the collection center *l* from the customer *j*
$Pf_{(h,l)}$	Purchase price of used product *h* by the manufacture frim the collection center *l*
$Pr_{(h,j)}$	Sale price of recycled pruduct *h* by the manufacturer to the customer *j*
$Qm_{(h,j)}$	The amount of recycled products flow *h* that is sent from the manufacturer to the customer *j*
$Qr_{(h,i)}$	The amount of product flow *h* that is sent from the manufacturer to the retailer *i*
$Qc_{(h,l)}$	The amount of used product *h* that is sent from the customer to the collection center *l*
$D1_{(h,i,j)}$	Market share of product *h* for retailer *i* to customer *j*
$D2_{(h,l,j)}$	Market share of second-hand product *h* from the customer *j* for the collection center *l* to the manufacturer
$D3_{(h,j)}$	Market share of recycled product *h* to customer *j*
$Rp_{(h,j)}$	Amount of second-hand product h sold by customer *j* to the collection center
x_{ij}	= 1 if customer *j* buys a product from retailer *i*, otherwise = 0
w_{lj}	= 1 if customer *j* sells product to the collection center, otherwise = 0
y_i	= 1 if the retailer *i* is constructed, otherwise = 0
θ_l	= 1 if the collection center *l* is constructed, otherwise = 0

3. 2. Modelling Here, the problem is presented with the aid of a mathematical model. Utilized parameters and sets are defined in Tables 1 to 4. This mathematical model is an objective function. For problem modelling, demand functions of the retailer, collection centers, and manufacturer are defined firstly. Next, profit functions for both retailers and collection centers are presented and the optimal sale and purchase prices are achieved through differentiation. Ultimately, manufacturer profit is computed and optimal values are achieved by solving this model.

Demand functions for retailers (1), manufacturer (2), and collection centers (3) are given as follows for customer j:

$$Dr_{(h,i,j)} = K_{(h,j)} - k_i Pn_{(h,i,j)} + b_1 Pr_{(h\ j)} \tag{1}$$

$$Dm_{(h\ j)} = K'_{(h\ j)} - k'\ Pr_{(h\ j)} + b_2 Pn_{(h\ i\ j)} \tag{2}$$

$$Dc_{(h\ l\ j)} = \varphi_{(h\ j)} Pc_{(h\ l\ j)}{}^{u}(K_{(h\ j)} - k_i Pn_{(h\ i\ j)} + b_1\ Pr_{(h\ j)}) \tag{3}$$

Profit function for the retailers is given as follows by taking inventory carrying costs into consideration (which follows the EOQ method):

$$Max\Pi_{Ri} = Dr_{(h\ i\ j)}\left(Pn_{(h\ i\ j)} - Pnw_{(h\ i)}\right) - \left(\frac{h_{(h\ i)}Qr_{(h\ i)}}{2} + \frac{A_i Dr_{(h\ i\ j)}}{Qr_{(h\ i)}}\right) \tag{4}$$

This is a concave function. Hence, the optimal value $Pn_{(h,i,j)}$ can be achieved through differentiation:

$$\frac{d\Pi_{Ri}}{dPn_{(h\ i\ j)}} = 0 \rightarrow Pn^*_{(h\ i\ j)} = \frac{K_{(h\ j)} + b_1\ Pr_{(h\ j)} + k_i Pnw_{(h\ i)} + (\frac{k_i A_i}{Qr_{(h\ i)}})}{2k_i} \tag{5}$$

Profit function for the collection centers is given as follows by taking inventory carrying costs into consideration (which follows the EOQ method):

$$Max\Pi_{cl} = Dc_{(h\ l\ j)}\left(Pf_{(h\ l)} - Pc_{(h\ l\ j)} - c_{(h\ l)}\right) - \left(\frac{h'_{(h\ l)}Qc_{(h\ l)}}{2} + \frac{Dc_{(h\ l\ j)}A_l}{Qc_{(h\ l)}}\right) \tag{6}$$

This is a concave function. Hence, the optimal value $Pc_{(h,l,j)}$ can be achieved through differentiation:

$$\frac{d\Pi_{cl}}{dPc_{(h\ l\ j)}} = 0 \rightarrow\ Pc^*_{(h\ l\ j)} = \frac{-A_l u + Pf_{(h\ l)}Qc_{(h\ l)}u - c_{(h\ l)}Qc_{(h\ l)}u}{Qc_{(h\ l)} + Qc_{(h\ l)}u} \tag{7}$$

The manufacturer profit model is an objective function. The first part includes the revenue achieved by new product sales to the constructed retailers. The second part is the revenue achieved by Reman product sales to the customers. The third part of the objective function is the purchase price of used products from collection centers. The fourth and fifth part of this function denotes the construction costs of retailers and collection centers. Lastly, the sixth and seventh part includes the costs caused by the distance between customers and constructed retailers and collection centers:

$$Max\Pi_m = \sum_{h\in H\ i\in I} Qr_{(h\ i)}(Pnw_{(h\ i)} - Crw_h - Cm_h) + \sum_{h\in H\ j\in J} Qm_{(h\ j)}(Pr_{(h\ j)} - Cr_h) - \sum_{h\in H\ l\in L} Qc_{(h\ l)}Pf_{(h\ l)} - \sum_{i\in I} F_i y_i - \sum_{l\in L} E_l \theta_l - \sum_{i\in I\ j\in J} S_i C_{(i\ j)} x_{(i\ j)} - \sum_{l\in L\ j\in J} S_l D_{(l\ j)} w_{(l\ j)} \tag{8}$$

Retailers' market share is included in constraint (9):

$$D1_{(h\ i\ j)} = K_{(h\ j)} - k_i Pn_{(h\ i\ j)} + b_1 Pr_{(h\ j)} \quad \forall h\ i\ j \tag{9}$$

Constraint (10) illustrates the optimal sale price of retailers for the customers that can be achieved through the differentiation of retailers' objective function:

$$Pn^*_{(h\ i\ j)} = \frac{K_{(h\ j)} + b_1\ Pr_{(h\ j)} + k_i Pnw_{(h\ i)} + (\frac{k_i A_i}{Qr_{(h\ i)}})}{2k_i} \tag{10}$$

Constraint (11) illustrates that retailers should purchase less or equal to their market share from the manufacturer for sale.

$$Qr_{(h\ i)} \le \sum_{j\in J} x_{(i\ j)} D1_{(h\ i\ j)} \quad \forall\ i\ h \tag{11}$$

Constraint (12) illustrates that customers should purchase products only from the constructed retailers:

$$\sum_{j\in J} x_{i\ j} \le m y_i \quad \forall i \tag{12}$$

Constraint (13) shows that each customer should purchase the needed product from only one retailer:

$$\sum_{i\in I} x_{i\ j} = 1 \quad \forall j \tag{13}$$

Constraint (14) expresses that the product sale price by retailers should be at least 10% higher than the product purchase price (minimum retailers' profit: 10%):

$$Pn_{(h\ i\ j)} \ge 1{,}1 Pnw_{(h\ i)} \quad \forall h\ i\ j \tag{14}$$

Constraint (15) illustrates the ratio of purchased products by the customers willing to sell to the collection centers:

$$p_{(h\ j)} = \varphi_{(h\ j)} \sum_{i\in I} Dr_{(h\ i\ j)} x_{(i\ j)} \quad \forall h\ j\ R \tag{15}$$

Constraint (16) illustrates the market share of purchasing used products for the collection centers:

$$D2^u_{(h\ l\ j} = PD2Rp_{(h\ j)} \quad \forall h\ l\ j \tag{16}$$

Constraint (17) illustrates that each customer can sell their used product only to one collection center:

$$\sum_{l\in L} w_{l\ j} = 1 \quad \forall j \tag{17}$$

Constraint (18) shows that customer can only sell their products to the constructed collection centers:

$$\sum_{j\in J} w_{l\ j} \le n\theta_l \quad \forall l \tag{18}$$

Constraint (19) is the upper bound on the used product quantity that is sold by the collection centers to the manufacturer. This quantity should be less than or equal to the purchase market share of used products for the collection centers:

$$Qc_{(h\ l)} \leq \sum_{j \in J} w_{(l\ j)} D2_{(h\ l\ j)} \qquad \forall h\ l \tag{19}$$

Constraint (20) shows that the sale quantity of the manufacturer's Reman product cannot be more than the product quantity that is purchased from collection centers:

$$\sum_{j \in J} Qm_{(h\ j)} \leq \sum_{l \in L} \theta_l Qc_{(h\ l)} \qquad \forall h \tag{20}$$

Constraint (21) illustrates the sale market share of Reman products for the customers:

$$D3_{(h\ j)} = K'_{(h\ j)} - k'\, Pr_{(h\ j)} + b_2 \sum_{i \in I} Pn_{(h\ i\ j)} x_{(i\ j)} \qquad \forall h\ j \tag{21}$$

Constraint (22) shows that sold quantity of Reman product to the customers should be less than or equal to the manufacturer' sale market share:

$$Qm_{(h\ j)} \leq D3_{(h\ j)} \qquad \forall h\ j \tag{22}$$

Constraint (23) guarantees that collection centers will purchase the used products from the customers at an optimal purchase price (optimal purchase price can be achieved through differentiation of profit function by the collection centers):

$$Pc^*_{(h\ l\ j)} = \frac{-A_l u + Pf_{(h\ l)} Qc_{(h\ l)} u - c_{(h\ l)} Qc_{(h\ l)} u}{Qc_{(h\ l)} + Qc_{(h\ l)} u} \tag{23}$$

Constraint (24) illustrates that the sale price of Reman products should be at least 30% less than the new product price:

$$0{,}7 Pn_{(h\ i\ j)} \geq Pr_{(h\ j)} \qquad \forall h\ i\ j \tag{24}$$

4. VALIDATION

4. 1. Model Solving at a Small Scale

Firstly, the proposed problem is solved with the aid of the accurate method at a small scale, both for multi-product and single product situations. The problem dimensions are presented in Table 6. Problem parameters are also generated randomly for single product and multi-product (with two products) situations. The problem is solved for all the customers by taking $\varphi_{(h,j)} = 10\%$ into consideration.

The problem is solved with the aid of the accurate method and Lingo software both for multi-product and single product situations. In Table 7, the optimal prices for both multi-product and single product situations are presented. In Table 8, economic order and delivery quantity are presented for each stage and sale/purchase market share are presented for both multi-product and single product situation.

TABLE 6. Specification of exact solution

Example	Retaile (*i*)	Customer b(*j*)	Collection Center (*l*)	Product (*h*)
1	1	1	1	1
2	1	1	1	2

TABLE 7. Optimal prices

Variables	Single product	Multi product	
pnw^*	314.6748	314.6748	270.6984
Pn^*	413.11	413.11	354.8172
Pc^*	22.95323	22.95323	19.62221
Pf^*	88.91764	88.91764	77.40262
Pr^*	289.1770	289.1770	248.3721
z^*	18368.16	31702.42	

TABLE 8. Optimal quantities of products

Variables	Single product	Multi product	
*Qr**	49.1158	49.1158	41.94021
*D1**	49.1158	49.1158	41.94021
*D2**	17.2012	17.2012	13.79525
*Qc**	17.2012	17.2012	13.79525
*Qm**	17.2012	17.2012	13.79525
*D3**	157.1799	157.1799	131.9336

The results achieved by accurately solving the model illustrates that there is a price balance in the model.

Optimal Sale/purchase prices in supply chain components are designed in a way that the sale price will be higher than the product purchase price and each component will acquire fair profit.

In addition to balanced price, the results illustrate that the proposed problem possesses balanced product flow capacity. Supply chain components acquire the product by taking their market share and current costs into consideration and selling the product in the same quantity.

All the above considerations (balanced price and balanced product flow capacity) show that all the connections in the closed-looped supply chain are designed in a way that all the supply chain components can enjoy the highest profit. In other words, the profit does not belong to a particular part of the chain. The total profit of the network in a multi-product situation is higher than a single-product situation and this also proves the mathematical modelling accuracy.

This non-linear mathematical model cannot be solved for large values with the aid of the accurate method. As a result, metaheuristic algorithms are utilized for solving the large model.

4. 2. Proposed Solution for Larger Scales

Considering the non-linearity and high-complexity of the model, the time required to reach a solution increased for medium and large sizes and therefore, we are forced to utilize metaheuristic algorithms. In this article, metaheuristic genetic algorithms (GAs) and particle swarm optimization (PSO) were utilized for the proposed model.

The genetic algorithm is a metaheuristic algorithm that is inspired by organism reproduction and based on Darwin's theory of natural selection and survival of the fittest. This algorithm is a heuristic optimization approach for finding the best solution (best person). This algorithm is capable of solving complex optimization problems at an acceptable speed and accuracy. The algorithm works firstly by creating the initial population with the aid of a stochastic method. Next, the secondary population is created by using cross-sectional and mutation operations and the algorithm compares the chromosomes constantly from these generations to find the best solution.

The structure of the genetic algorithm in this paper is as follows: first, using the roulette cycle, we select the chromosomes and form the initial population randomly. Then the intersection and mutation operations are performed and a secondary population is formed. The intersection and mutation operations are shown in Figures 2 and 3.

Particle swarm optimization (PSO) is another metaheuristic algorithm. This algorithm is inspired by the motion of bird flocks. In this algorithm, each solution is similar to a bird inside the search space with the following parameters: current position (x_i), velocity (v_i), best-known position (p_i) and the best direction of a bird flock movement (Gbest). Using the following method, the birds' velocity is updated. C_1 and C_2 are the cognitive coefficients with the following values, respectively: 0.5 and 1. r_1 and r_2 denote the random numbers in $[1, 2]$ and w is inertia coefficient with a value equal to 0.3. This recursive relationship is defined in terms of velocity and therefore, the velocity's initial value is considered to be 0:

$$V_i = W.V_{i-1} + C_1.rr_1.(G_{Best} - X_i) + C_2.r_2.(P_{Best} - X_i)$$

In standard particle swarm optimization, the initial population value (quantity) is generated randomly. Pbest values (best-known position) and Gbest (best direction of a bird flock movement) are calculated consistently until the termination condition is realized. The velocity and position of birds are updated with the aid of achieved values at each stage. In Table 9, the solutions by metaheuristic algorithm are compared with the aid of the accurate method. RPD mean index is one of the crucial indices for the performance assessment that is also utilized in this research. This index is defined as follows:

$$RPD = \frac{Alg_{sol} - Best_{sol}}{Best_{sol}} \times 100$$

where Agl_{sol} is the objective function solution in the metaheuristic algorithm and $Best_{sol}$ is the objective function solution in the accurate method.

According to this table, the values achieved by objective function in the accurate method (Lingo) is close to metaheuristic algorithm solutions. In the first example and second example, there is a difference of less than 0.01% and 0.16%, respectively (according to the RPD index). We conclude that the proposed mathematical model is highly credible.

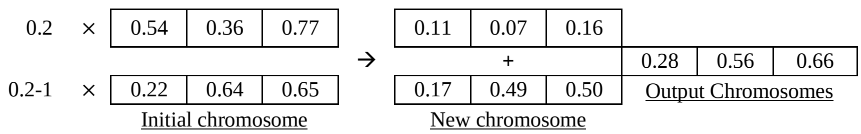


Figure 2. Intersection operations

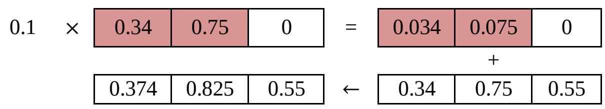


Figure 3. Mutation operations

TABLE 9. Comparing metaheuristics & exact solutions

No.	Lingo	GA	RPD	PSO	RPD
1	18368.16	18366.32	0.01	18366.32	0.01
2	31702.42	31651.77	0.16	31651.77	0.16

However, the accurate method cannot be utilized for problems at a large scale. Hence, we use metaheuristic genetic algorithms (GAs) and particle swarm optimization (PSO). In this regard, 10 different experiments are conducted. The number of retailers is considered to be between 2 and 12, the number of customers between 2 and 9, the number of collection centers between 2 and 10, and the number of products between 2 and 8. The results achieved by using GA and PSO at medium and large scales are presented in Table 10.

According to the achieved results by GA and PSO algorithms, solving the model using the GA algorithm at medium and large scales (according to RPD index) generated far better solutions than the PSO algorithm in terms of quality. However, PSO functions better than GA in terms of the time needed to find the solution.

5. SENSIVITY ANALYSIS

In this section, the impact of the following parameters is investigated on product flow and price and ultimately, supply chain profit: retailer replacement factor (b_1), price sensitivity coefficient in the retailer (k_i), and purchasing power of customers' used product (u).

The sale market share of the retailer and manufacturer for selling the new and used products to the customer is a ratio of each other's sale price. In other words, an increase in the sale price of a retailer's product will cause an increase in the sale market price of the manufacturer at a certain ratio and vice versa. According to Figures (a-1) to (b-1), an increase in retailer replacement factor and in the impact of manufacturer's price on sale market share of the retailer has caused product's sale market share for the retailer to increase and the customers are more inclined to buy a new product from the retailer. As a result, the retailer acquires the product in higher quantities, and product flow increases in all echelons/levels of the closed-loop supply chain network.

An increase in the product purchase volume in different echelons of the closed-loop supply chain will result in an increase in the optimal purchase/sale price of products in all echelons. Another reason for the increase in the optimal price is that retailer's optimal price has a direct connection to the retailer replacement factor. Increasing this factor will cause an increase in the product sale price to the retailer. Hence, purchase/sale price increases in all other echelons. In Figures 4(a-2) and 4(b-2), it is obvious that used product volume, which is sold (online) directly to the customers, increases the sale market share of the manufacturer. Another reason for the increased demand for purchasing used products is the increased sale price of the new products.

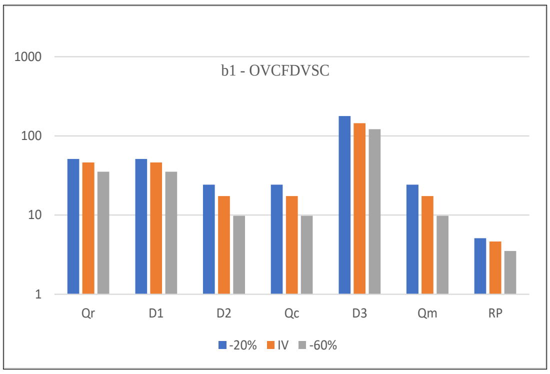
The results of changing sensitivity coefficient parameters in the retailer depict that the retailer reduces the sale price for customer retention if the customer shows higher sensitivity to the product price. Nevertheless, the volume of the acquired product decreases according to retailer function. As a result, all the network echelons reduce the price and the volume of their acquired/sold products. Hence, product flow balance and optimal price are retained (Figures 4c-1, 4d-1, 4c-2, and 4d-2).

According to Figures 4(e-1), 4(f-1), 4(e-2), and 4(f-2), an increase in customers power (purchase price of used product) causes a decline in the product flow and optimal price. Such an event can be attributed to the direct relationship of the market share parameter of collection centers that increases product flow in the whole close-looped supply chain network. Also, due to the inverse relationship between this parameter with product price acquired by the customer, the optimal purchase price of collection centers decrease. Moreover, such an inverse relationship result in the reduction of all optimal prices.

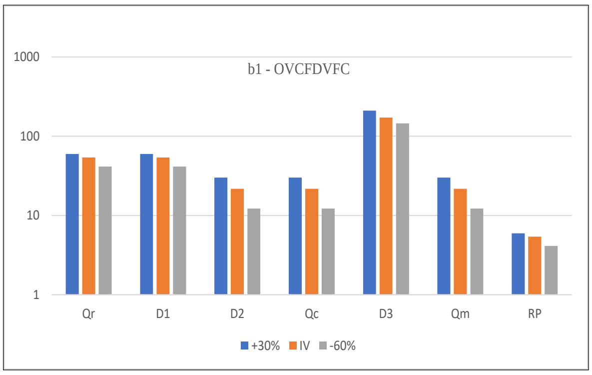
The result of this investigation depicts that increasing retailer replacement factor causes an increase in the optimal product flow and optimal price in all echelons, and it plays a major role in increasing the total profit of

TABLE 10. Solution for large scales

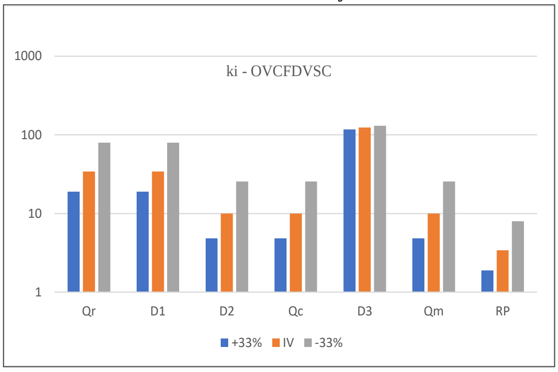
RPD	$TIME_{PSO}$	PSO_{best}	$TIME_{GA}$	GA_{best}	No.
15	27.14	82282.64	41.32	97087.88	1
10	28.18	99110.3	36.39	110784.55	2
13	31.55	226902.66	43.12	263810.89	3
18	27.99	269977.72	44.28	332170.32	4
20	33.90	517905.01	55.22	655377.33	5
3	37.82	1201395.92	57.93	1246158.52	6
25	43.69	1162482.84	69.60	1561082.83	7
19	63.53	2135817.89	159.65	2668196.99	8
40	144.43	3237473.33	370.2	5706453.22	9
37	315.26	3985210.71	424.1	6339803.1	10



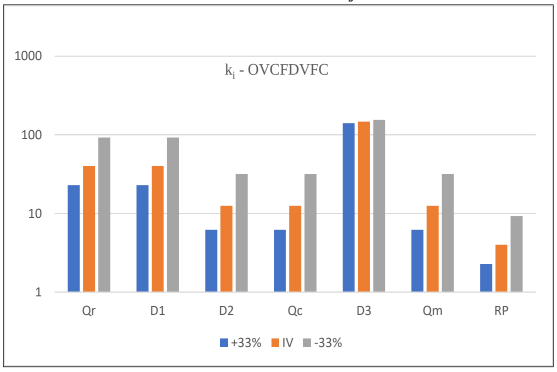
a-1. Optimal values of commodity flow for different values b1 for the first commodity



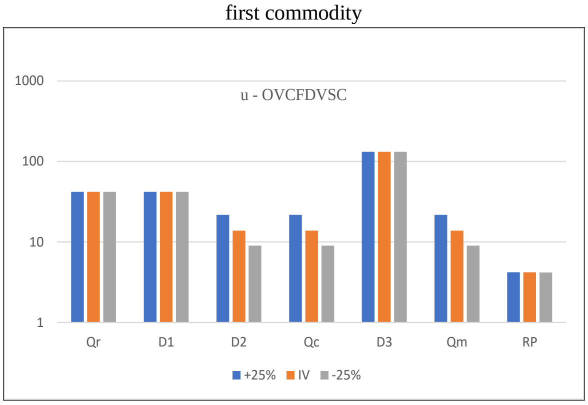
b-1. Optimal values of commodity flow for different values b1 for the second commodity



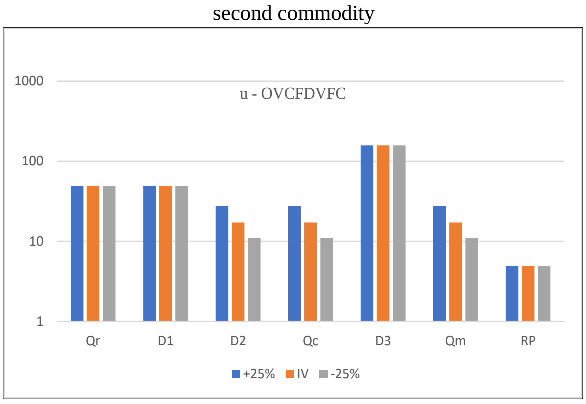
c-1. Optimal values of commodity flow for different values k_i for the first commodity



d-1. Optimal values of commodity flow for different values k_i for the second commodity

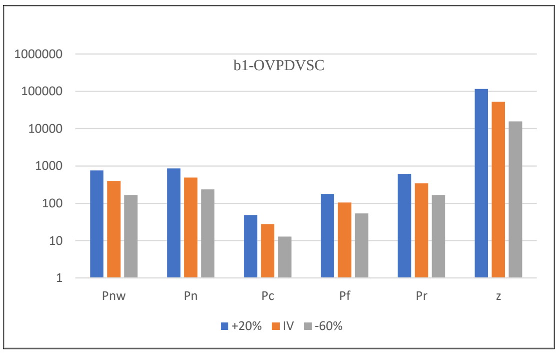


e-1. Optimal values of commodity flow for different values u for the first commodity

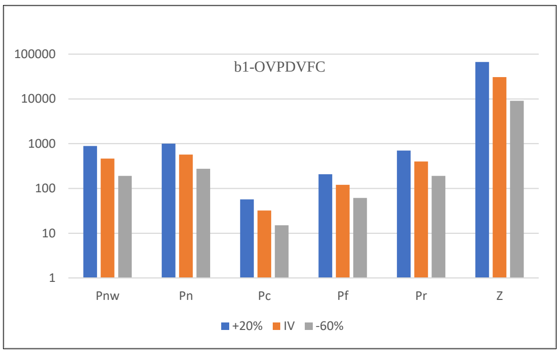


f-1. Optimal values of commodity flow for different values u for the second product

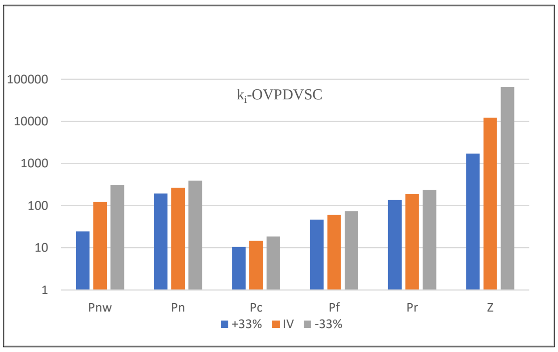
Figures 4. Sensitivity analysis



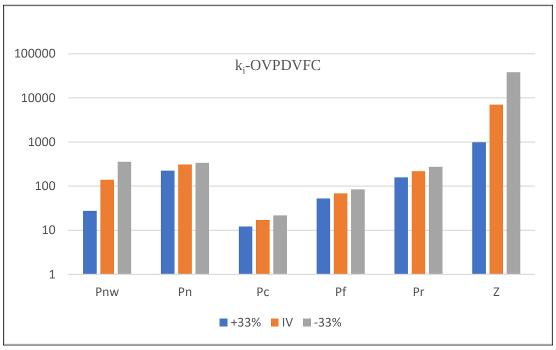
a-2. Optimal values of prices for different values of b1 for the first product



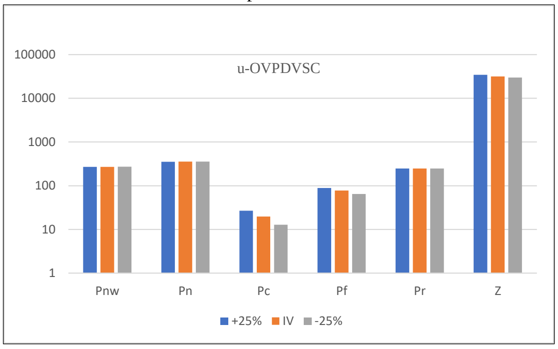
b-2. Optimal values of prices for different values of b1 for the second product



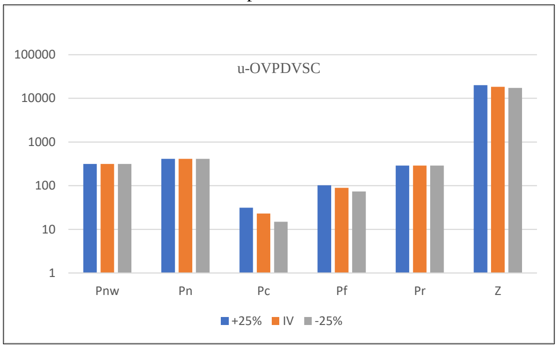
c-2. Optimal values of prices for different values of k_i for the first product



d-2. Optimal values of prices for different values of k_i for the second product



e-2. Optimal values of prices for different values of u for the first product



f-2. Optimal values of prices for different values of u for the second product

Figures 4. Sensitivity analysis

the supply chain. However, consumers' price sensitivity factor can reduce the profit of each component and ultimately, reduce the total profit of the closed-loop supply chain. Customer purchasing power also has a huge impact on the balance between products price and products flow.

6. MANAGERIAL INSIGHTS

In this article, the main objective is to design a supply chain with maximized profit. To achieve such a goal, the following strategies are utilized: optimal location of components, sale-purchase optimal pricing, capability to use used products, and capability to re-sell these products directly (online sale) to the customers.

Due to the importance of location and pricing in the creation or development of an economic activity, such as chain stores, this research is designed so that it can be a great help to the industry by preventing irrecoverable costs and resulting in economic growth. In the present article, the following managerial insights are suggested for more use:

1. Using the designed model (in this paper), the location of supply chain components (e.g., sale/purchase branches of a chain store throughout the country) can be determined by taking the population of each region, customers' distance to the sale branches, and collection centers into account.
2. In this research, determining the purchasing/sale price of products so that the most profit is gained by the chain components. Such a matter is one of the biggest concerns of industry owners. In this research, hence, the optimal price of supply chain components can be determined by taking predictions of customer demand into account, which is sensitive to product inventory costs.
3. Other factors that may help increase industry owners' revenue is the ability to reuse the used products by the customers and also, re-selling these products. In this article, collection centers consider a fair sale/purchase price that encourages the customers to sell their used products and creates a motivation to acquire collected products for the manufacturers. Moreover, a separate channel is taken into account for selling Reman products directly from the manufacturer to the customers without the interference of an intermediary.

7. RESULTS AND FUTURE RECOMMENDATIONS

A small numerical example is proposed in order to investigate the mathematical model accuracy and credibility. Metaheuristic genetic algorithms (GA) and particle swarm optimization (PSO) were utilized to solve the model at a large scale and then, the results were compared with solutions at small scales in the accurate method (through RPD index). Moreover, the performance of both algorithms, PSO and GA, are compared (in terms of time required to reach a solution and RPD index) by changing the number of customers, retailers, collection centers, and the number of products. The result of comparing optimal quantities and time required to reach a solution shows the higher quality of the genetic algorithm compared to particle swarm optimization. In most cases, however, PSO reached a solution over a time, less than GA.

In this model, all the problem and demand parameters are considered deterministic. In future research, the non-deterministic model can be taken into consideration and solved with the aid of algorithms developed for non-deterministic models. In this article, a mathematical model is developed for the problem under the investigation, and carrying and inventory costs are only considered in sale and collection centers through the EOQ method. In the future, different inventory models will be taken into consideration and the best approach will be selected. The following recommendation can be given for future research:

- Considering perishable products or supplements
- Adding route-finding and vehicle type
- Considering multi-objective mathematical models and environmental considerations

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Persian Abstract

چکیده

هدف اصلی از طراحی زنجیره تأمین، افزایش سودآوری است. بدین منظور در این مقاله، یک مدل جدید زنجیره تأمین حلقه بسته‌ی سه سطحی شامل تولیدکننده، خرده‌فروش و مراکز جمع‌آوری ارائه شده است. در این مدل برای اولین بار، با در نظر گرفتن کانالی مجزا و مستقل برای فروش کالاهای بازیافتی، سودآوری تولیدکننده افزایش خواهد یافت. به علاوه، در این مدل زنجیره تأمینِ جدید، مکان‌یابی و موجودی و قیمت‌گذاری کالا نیز در نظر گرفته شده است. سود تولیدکننده با در نظر گرفتن هزینه احداث مراکز خرده‌فروشی و جمع‌آوری کالاهای بازیافتی و در نظر گرفتن ظرفیت جریان کالا و سهم بازار محاسبه خواهد شد. برای حل تابع هدف غیر خطیِ سود تولیدکننده در ابعاد کوچک، از نرم‌افزار لینگو و در ابعاد بزرگ، از الگوریتم‌های فرا ابتکاری ژنتیک و تجمع ذرات استفاده شده است. نتایج حاصل از انجام این پژوهش نشان می‌دهد که طراحی مدل جدید زنجیره تأمین حلقه بسته دارای اعتبار است و مکان‌یابی بهینه‌ی اجزای زنجیره تأمین، پاسخ بهینه‌ی جریان کالاها و قیمت محصولات به درستی انجام شده است. در نتیجه، سودآوری کل شبکه زنجیره تأمین حلقه بسته افزایش یافته است. تحلیل حساسیت مدل ریاضی نیز نشان می‌دهد که این مدل نسبت به پارامترهای ضریب جایگزینی، قیمت و توان قیمت خرید مراکز جمع‌آوری، حساسیت بالایی دارد. همچنین در مقیاس بزرگ، الگوریتم ژنتیک از نظر کیفیت جواب، عملکرد بهتری دارد. اما زمان حل الگوریتم تجمع ذرات بسیار بهتر است. در انتها، با توجه به طراحی مدل جدید زنجیره تأمین حلقه بسته، پیشنهادات کاربردی برای مدیران نیز ارائه شده است.