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A Modified Benders Decomposition Algorithm for a Last-mile Network with Flexible Delivery Options

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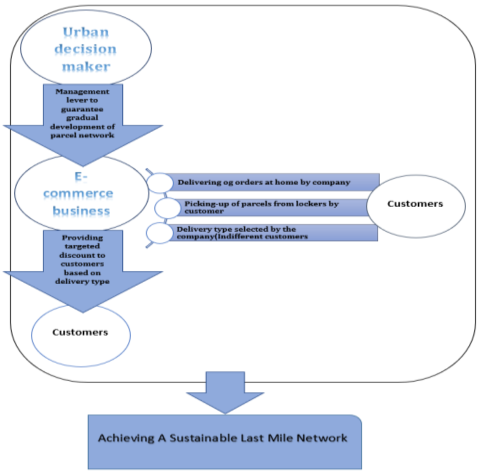
A B S T R A C T

The purpose of this paper is to introduce an integrated and specialized approach to tackling a challenging issue, known as "Last-mile Transportation". This issue, which is classified in terms of the decision-making level at the tactical level, is a model of the operational and application processes of prominent businesses in online commerce in developed countries, while it has attracted little attention from an operation-research aspect. This is a single-echelon network that includes a central distributor, parcel lockers, and customers, allowing customers to take advantage of three flexible product delivery options after purchasing the product. In the first option, the parcels are delivered at the door and in the time window specified by the customer. In the second option, customers pick up the parcels from the desired lockers with a discount, and in the third option, customers leave the choice of delivery type to the company under gaining an attractive discount. Offering online targeted discounts based on a selected option to encourage as many customers as possible for cooperation with the company and the guarantee of gradual development of the parcel-locker network by a management lever are other innovations of this study. To solve this model, a Benders decomposition algorithm has been modified by variable neighborhood search and local branching strategies. The results obtained from the analysis of parameters related to problem innovations indicate the efficiency and validity of this presented model in different scenarios and the proposed solution algorithm in large-sized instances.



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NOMENCLATURE

$G=(N,A)$	A non-directional graph	pc_j	Operational and rental cost for each parcel lockers $j \in N_{PL}$ ($)
$N = N \bigcup N_C \bigcup N_{PL}$	Set of network nodes	ω	Transportation cost per kilometers ($)
$N_{CPL} = N_C \bigcup N_{PL}$	Set of network nodes except central distributors	cv_k	Capacity of vehicle $k \in K$
N_C	Nodes related to the customers	cp_j	Capacity of parcel lockers $j \in N_{PL}$
N_{PL}	Nodes related to the lockers	τ	Sales price of product ($)
K	Set of homogeneous vehicles	t_{ij}	Access time from node *i* to $j \in N$ (h)
M^1, M^2	A very large number	Θ_i	Binary parameter equals 1 if a customer $j \in N$ selects the option of a door-to-door delivery; 0, otherwise (option 1).
c_{ij}	Distance between two nodes $i, j \in N$	$x_{ijk} \in \{0,1\}$	Binary variable equals 1 if vehicle $k \in K$ immediately moves from the node i to $j \in N_C$; 0, otherwise.
pc_j	Operational and rental costs for each parcel lockers $j \in N_{PL}$ ($)	$y_{ij} \in \{0,1\}$	Binary variable equals 1 if customer $i \in N_C$ is assigned to lockers *j*; 0, otherwise
d_i	Demand of cutomer $i \in N_C$	$h_i \in \{0,1\}$	Binary variable equals 1 if customer $i \in N_C$ is provided in the first-option manner (doo- to-door); 0, otherwise
a_i	Lower bound of customer *i*'s time window.	$l_i \in \{0,1\}$	Binary variable equals 1 if customer $i \in N_C$ is provided in the second-option manner (parcel locker); 0, otherwise
b_i	Upper bound of customer *i*'s time window	$z_{uj} \in \{0,1\}$	Binary variable equals 1 if the customer $i \in N_C$ or the parcel locker $j \in N_{PL}$ is provided by the central distributor (depot) $u \in N_u$; 0, otherwise
V_i	Upper bound of customer *i*'s time window	$st_{uk} \geq 0$	Continuous variable indicates when vehicle *k* leaves the central distributor $u \in N_u$
$ls_{ij} \in \{0,1\}$	Maximum capacity of the central distributor (depot) $u \in N_u$	$ft_k \geq 0$	Continuous variable indicates when a vehicle $k \in K$ arrives at the last customer
Ψ	Binary parameter equals 1 if a customer $i \in N_C$ adds locker i to his/her favorite list; 0, otherwise	$dp_{jk} \geq 0$	Continuous variable that indicates amounts of parcels delivered to locker $j \in N_{PL}$ by vehicle $k \in K$
Υ_i	Binary parameter equals 1 if a customer $i \in N_C$ selects the delivery option of picking up from parcel lockers; 0, otherwise (option 2)	$s_{ik} \geq 0$	Continuous variable indicates when vehicle $k \in K$ starts service to node $i \in N_C$
Γ_i	Binary parameter equals 1 1f a customer $i \in N_C$ allows the company to select delivery option type; 0, otherwise (option 3)		

1. INTRODUCTION

The road transport sector is known as one of the largest producers of greenhouse gases [1, 2]. Automated parcel lockers enabling consumers to pick up e-purchased merchandise from allocated 24-hour Locker locations are rising while they suggest the promising better-quality accessibility to merchandise and declined travel of customers and firm vehicles [3]. Since similar to many new emerging technologies, parcel lockers have competitive advantages as well as social and implementation challenges, seeking efficient solutions to create a managerial perspective is a challenging issue for parcel-locker investors as economical beneficiaries and urban policymakers as beneficiaries of managerial urban issues. How and when to deploy and integrate this new technology into the existing traditional network, which on the one hand causes companies to flourish in a competitive global market and on the other hand on a horizon time results in the establishment of the sustainable urban last-mile delivery network, is its main strategic challenges. González-Varona et al. [3] introduced delivery of a last-mile parcel in their model that employs the current newspaper stands as parcel lockers.

In terms of a conceptual model of the parcel-lockers network, Iwan et al. [4] concentrated on the study of the serviceability and the productivity of the parcel lockers for Polish InPost Company and forecasted the efficient utilization of this sort of solution. Lemke et al. [5] studied on evaluation of this machine's serviceability according to the InPost system's instances. Gatta et al. [6] aimed to estimate the customers' willingness and the service providers to employ an alternative delivering/ picking-up option (e.g., a parcel locker) and to consequently forecast the demand amounts resulting from this willingness in the last-mile B2C e-commerce condition. Tang et al. [7] defined the service quality of the parcel lockers in five cases named service price, service reliability, accessibility, the ability of fault handling, and service diversity. Based on these cases, they surveyed lockers via the service-quality theory and gained the satisfaction of the customer. Maximally motivating the customers both economically and socially to travel to locker positions and thoroughly exploiting the operational potentials of all these facilities, which not only result in not wasting the public and urban resources but also promotes the service level to citizens, are other challenging factors in a tactical level, clearly not observed in mentioned studies at least in terms of an operations research (OR) approach.

In terms of the mathematical model of a parcel-lockers system, Deutsch and Golany [8] for the first time developed a parcel locker system as a promising contribution for the last-mile concept while simultaneously maximizing the revenue resulting from product sales and minimizing costs driven by discounts on the delivery for customers traveling to pick up their packages, facilities' fixed and operational costs and the loss of potential consumers not eager to pick up. Although they considered a discount for customers picking up their parcels from parcel lockers, this discount lacked the necessary flexibility to gradually encourage customers. Lin et al. [9] proposed a parcel locker location problem, in which the company intends to present the locker service under locating locker facilities to attract clienteles. Maximizing the profit (i.e., calculating the revenue and the cost of facilities) is the objective, under the interaction of other delivery modes. To approximate the income, they used the threshold Luce's model. Pan et al. [10] offered a model for designing combined delivery networks in urban zones under applying parcel lockers. The model solved by a hybrid algorithm deals with a two-level structure. the lower level is a multi-depot capacitated vehicle routing problem and the upper level is a parcel network flow problem.

Unlike the previous studies that customers prefer to collect their packages from a predetermined locker, Orenstein et al. [11] presented a logistic model for the delivery of small parcels to a set of parcel lockers so that customers can collect their packages from a parcel locker or a set of ones having the same preference. To minimize operating costs, properties of mobile lockers named the location and the route are combined as a model being non-linear integer by Anderluh et al. [12]. A modified genetic algorithm (GA) is presented to determine optimal distributor locations, the number of lockers required by each distributor, and mobile lockers' scheduling and routing at the same time. Zhou et al. [13] provided a location-routing problem being a bi-level multi-sided terminal under customer's pickup services and a real-time home delivery solved by a hybrid GA and simulated annealing (SA) method, named self-adaptive SGA. Considering time constraints that incorporates important aspect related to same-day delivery logistics, Voccia et al. [14] provided a pickup and delivery problem being multi-vehicle dynamic. To decide more knowledgeably, their solution approach merges information about future demand into decisions of routing. Also, they analyzed the outcome that examines when it is useful for a fleet to wait in a warehouse. Based on issues of postal transportation, Sitek and Wikarek [15] presented a hybrid approach on pick-up and alternative delivery point combined with the capacitated vehicle routing problem while providing innovations such as time window, the distinction of delivery point types about their capacity, and probability of collecting the things for the period of the carrying out of delivery routes.

Based on the mentioned literature of the Last-mile area and the best of our knowledge, the research gaps are identified as follows:

- Despite the potential of exact solution algorithms to achieve optimal solutions, due to complexity and lack of timely convergence in many problems with specific structures, these approaches are not attracted enough researchers' attention in urban logistics issues. Therefore, in the literature related to the parcel lockers network, exact algorithms developed by an efficient strategy to reduce the convergence time of the master problem are not observed.
- Via the parcel locker, proposing a single delivery option can make customers feel that their right about free choice is ignored. So, a flexible delivery system with multiple options, which gradually conduct customers toward parcel lockers or other similar ones by its natural potentials (e.g., discount and 24-hour accessibly) can enable company managers gradually to reach the ultimate last-mile delivery network while following their customer-orientation policy. Despite the importance of this issue, implementing this philosophy in the last-mile delivery concept is not comprehensively observed in the academic literature.
- For the consistency and durability of a last-mile delivery network, discounts can be one of the most effective tools in creating a desire and motivation in customers to accept social responsibility. The proportion between the discounts of the delivery options offered to the customer has a significant impact on the network's customer orientations. Therefore, a lack of this proportion can lead to significant deviations from the goals for a long time. Despite conceptual studies, researches based on a mathematical framework under an integrative perspective in the literature are very rare.

In this paper, a Last-mile network with flexible delivery options (LNFDO) is specially and practically surveyed and modeled. The proposed model could be a good platform for leading businesses in online commerce as well as urban policymakers who want to not only promote the level of online customer service but also in a gradual process, increase customers' social responsibility sense to pick up their parcels from lockers. In the following, section 2 is dedicated to defining the problem concept and model. Section 3 proposes a solution approach based on the modified Benders decomposition to obtain optimal solutions. Section 4 illustrates numerical results achieved from the presented model and proposed solution approach. Finally, Section 5 is relevant to research conclusions.

2. PROBLEM DEFINITION

2. 1. Network Description The considered LNFDO is a development of a vehicle routing problem with a time window. An outstanding difference between the two is to offer three types of delivery options to customers to achieve a gradual evolution from a traditional network to a modern urban logistic network. According to Figure 1, after completing the purchase process that is a simulated environment of an assumptive business application to clarify our contributions, customers can choose one of the three options to receive the product.

Based on an option selected and other subsequent details, the company offers the customer online and dynamically a discount (i.e., the maximum of which belongs to the third option and the minimum to the first option). In the first option, customers want to receive their parcels in front of their home, in which they must enter their desired time window, which is a continuous sub-period of the company office hours. The larger the interval, the greater the discount since companies can make more flexible planning. In the second option, customers announce the company to pick up the orders from parcel-locker positions, in which they must select the desired positions where lockers are located. The

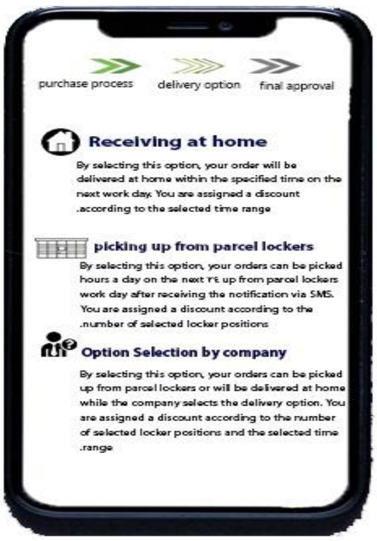

Figure 1. Simulated environment of a business application

greater the number of the selected positions, the more discount is allocated to a customer, and in option three, the customer is indifferent to the first two options and allows the company to choose how to deliver the parcels such a way that gaining the most discount. The model assumtions are as follows:

- It is a single period.
- It is a single product.
- It is a single echelon.
- It is a single sourcing.
- It is a homogeneous vehicle fleet.
- It is deterministic.
- The order of all customers must be met.

2. 2. Modelling of Customer's Discount Parameters

α, β $(\alpha < \beta)$ are maximum discounts on delivery cost, which a customer can gain, receiving goods in front of their home and from designated parcel lockers respectively. Parameters η_k, μ_k, ζ_k which are discounts offered online to customers by a company on their webpages are relevant to delivery options 1 to 3 calculated by respectively:

$$\mu_k = \frac{b_k - a_k}{\varphi - \phi} \times \alpha \quad (1)$$

$$\eta_k = \frac{|\Lambda_k|}{\chi} \times \beta \quad (2)$$

$$\chi = \frac{|N_C|}{|N_{PL}|} \quad (3)$$

$$\xi_k = \theta \times \max\{\eta_k, \mu_k\} \quad (4)$$

Equation (1) indicates the amount of discount offered to customer *k* selecting the door-to-door delivery option under upper and lower bounds of a time window (a_k and b_k). Interval [ϕ, φ] is related to the company's working hours. Equations (2) and (3) indicate the amount of discount offered to customer *k* selecting the picking-up option (parcel lockers), which Λ_k is a set of desired positions of lockers. N_C and N_{PL} are a customer set and set of all parcel-locker positions, respectively. Equation (4) indicates the amount of discount offered to customer *k* selecting option 3, in which the company chooses how to take delivery of goods (θ), is a factor in the range of 1 and 1.1 to encourage customers to cooperate with the company as far as possible.

2. 3. Mathematical Model

(*LNFDO*)

$$Min\ TC(\{p\}) = \left[\begin{array}{l} \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} \omega\, c_{ij} x_{ijk} + pc \sum_{j\in N}\sum_{k\in K} dp_{jk} \\ +(\sum_{i\in N_C} \mu_i d_i + \sum_{i\in N_C} \eta_i d_i + \sum_{i\in N_C} \xi_i d_i) \times \tau \end{array} \right] \quad (5)$$

$$Max\ CP(\{p\}) = \Psi \tag{6}$$

$$\sum_{k\in K}\sum_{j\in N} x_{ijk} = h_i \qquad \forall i \in N_C \tag{7}$$

$$\sum_{k\in K}\sum_{j\in N} x_{ijk} \geq inl_i \qquad \forall i \in N_{PL} \tag{8}$$

$$\sum_{i\in N_C} y_{ij} \leq M_1 \times inl_j \qquad \forall j \in N_{PL} \tag{9}$$

$$\sum_{j\in N_{CPL}} x_{0jk} \leq 1 \qquad \forall k \in K \tag{10}$$

$$\sum_{j\in N_{CPL}} x_{i0k} \leq 1 \qquad \forall k \in K \tag{11}$$

$$\sum_{\substack{j\in N\\ i\neq j}} x_{ijk} = \sum_{\substack{j\in N\\ i\neq j}} x_{jik} \qquad \forall k \in K, i \in N \tag{12}$$

$$h_i + l_i = 1 \qquad \forall i \in N_C \tag{13}$$

$$h_i \geq 1-(1-\mu_i)\times M \qquad \forall i \in N_C \tag{14}$$

$$l_i \geq 1-(1-\eta_k)\times M \qquad \forall i \in N_C \tag{15}$$

$$\sum_{j\in PL} ls_{ij} y_{ij} = l_i \qquad \forall i \in N_C \tag{16}$$

$$\sum_{k\in K} dp_{jk} = \sum_{i\in N_C} y_{ij}\, d_i \qquad \forall j \in N_{PL} \tag{17}$$

$$\sum_{i\in N} d_i \sum_{\substack{j\in N\\ i\neq j}} x_{ijk} + \sum_{i\in N}\sum_{\substack{j\in N_{LC}\\ i\neq j}} dp_{jk} x_{ijk} \leq cv_k \qquad \forall k \in K \tag{18}$$

$$\sum_{k\in K} dp_{jk} \leq cp_j \qquad \forall j \in N_{PL} \tag{19}$$

$$st_k + t_{0j} - M_{0j}^2(1-x_{0jk}) \leq s_{jk} \qquad \forall k \in K, j \in N_C \tag{20}$$

$$s_{ik} + t_{ij} - M_{ij}^2(1-x_{ijk}) \leq s_{jk} \qquad \forall i, j \in N_C, k \in K \tag{21}$$

$$s_{i0} + t_{i0} - M_{i0}^2(1-x_{i0k}) \leq ft_k \qquad \forall k \in K, i \in N_C \tag{22}$$

$$a_i h_i \leq s_{ik} \leq b_i h_i \qquad \forall k \in K, i \in N_C \tag{23}$$

$$\frac{\sum_{k\in K}\sum_{i\in N_C} \xi_i\, y_{ij}\, d_i}{\sum_{k\in K}\sum_{j\in PL} dp_{jk}} \geq \Psi \tag{24}$$

$$x_{ijk}, y_{ij}, h_i, l_i \in \{0,1\} \qquad s_{ik}, st_k, ft_k \geq 0 \tag{25}$$

The first objective (5) is to minimize the total network cost. This cost consists of three terms, in which the first one is the transportation cost of vehicles and the second term is the operational cost plus the rent of lockers valued by players and the third term is discounts assigned to customers based on delivery options selected. The second objective (6) is to maximize the exploitation of lockers as far as possible for customers selecting the third delivery option. Constraint set (7) states that vehicles must visit exactly once a node related to a customer selecting either the first delivery option or the third delivery option that company does the delivery of their goods in front of customers. Constraint set (8) indicates that vehicles must visit exactly once a node related to a parcel locker in either the second or third delivery option in which the company puts goods in a locker. Constraint set (9) determines whether a locker on the time horizon is used or not. Constraint sets (10)-(12) guarantee flow equilibrium in the network. Constraint sets (13)-(16) convert the conceptual nature of the three delivery options into the mathematical nature. In the third option, the order must be either taken to the customer’s home or delivered to the customer by a locker. Constraint set (17) shows that the total inventory of each locker equals the total parcel assigned to this locker. Constraint set (18) confirms that vehicle capacity limitation is met. Constraint set (19) warrants that a locker capacity limitation is met. Constraint sets (20) -(23) guarantees that vehicles arrive at the customer’s home in a predetermined time window. The constraint set (24) controls amounts of the demands covered by parcel lockers via the third delivery option (push lever for gradual development parcel-locker). Constraint set (25) represents the model variables.

3. SOLUTION APPROACH

In this paper, to solve the proposed model, a modified Benders decomposition algorithm enhanced by three well-known strategies is proposed. The Benders decomposition algorithm [16] is a classic approach to combinatorial optimization problems based on the idea of partitioning and gradually producing constraints. This algorithm decomposes an original problem into two simpler sub-model called master and sub-problems. The master problem is a limited model of the original one only consisting of complicated variables and the relevant constraints while the sub-problem is the same original model, whose complicated variables are fixed. This algorithm relies on exchanging solutions obtained between master and sub-problems until achieving an optimized provable solution for the original program. A solution to the master problem provides a gradual development for the dual problem and Benders bounds. Then, these variables are fixed into the dual sub-problem, whose optimal solutions result in cuts of the master problem. These cuts are called optimality cuts if the solution of the master problem becomes feasible for the sub-problem and feasibility cut if not. This process is continued until the gap value between master and dual problems converges on a predefined certain amount. To avoid further elaboration, the interested readers in the details of this algorithm are referred to the study of Benders [16]. The flowchart of the proposed solution approach is demonstrated in Figure 2. To reduce the overall running time of the Benders algorithm, the VNS algorithm and the local branching strategy are used in the first iterations when the solution of the master problem (MP) is not yet of the sufficient quality. The local branching strategy limits the exchange range of the binary variables and the VNS algorithm achieves efficient solutions for the master problem.

The MP is an integer model as shown below. An optimality cut (i.e., Constraint (26)) aims to produce the high quality for the next iteration. The feasibility cut (i.e., Constraint (27)) aims to generate cuts that keep the algorithm in the feasibility space for subsequent iterations.

$$Max\ Z^{Master}$$

s.t.

$$\begin{aligned} Z^{Master} \le \Big[&\sum_{\forall k \in K_{[s_m]}} \Big(-cv_k + \sum_{i \in N} d_i \sum_{\substack{j \in N \\ i \ne j}} x_{ijk}\Big) \bar{B}_k - \sum_{\forall k \in K} \sum_{i \in N_C} b_i h_i \bar{I}_{ik} \\ &- \sum M^3 (1 - x_{ijk}) \bar{D}_{ijk} - cp \sum_{j \in N} \bar{E}_j + \sum_{\forall k \in K} \sum_{i \in N_C} a_i h_i \bar{J}_{ik} \\ &+ \sum_{j \in N_C} \sum_{\forall k \in K} (t_{0j} - M^2_{0j} (1 - x_{0jk})) \bar{F}_{jk} + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \omega\, c_{ij} x_{ijk} \\ &+ \sum_{\forall k \in K} \sum_{j \in N_C} \sum_{i \in N_C} (t_{ij} - M^2_{ij} (1 - x_{ijk})) \bar{G}_{ijk} + \sum_{j \in N_{c[s_m]}} \sum_{i \in N_{c[s_m]}} d_i y_{ij} \bar{A}_j \\ &+ \sum_{\forall k \in K} \sum_{i \in N_C} (t_{i0} - M^2_{i0} (1 - x_{i0k})) \bar{H}_{ik} \Big] \end{aligned} \tag{26}$$

$$\begin{aligned} \Big[&\sum_{\forall k \in K_{[s_m]}} \Big(-cv_k + \sum_{i \in N} d_i \sum_{\substack{j \in N \\ i \ne j}} x_{ijk}\Big) \bar{B}_k + \sum_{j \in N_{c[s_m]}} \sum_{i \in N_{c[s_m]}} d_i y_{ij} \bar{A}_j \\ &+ \sum_{j \in N_c} \sum_{\forall k \in K} (t_{0j} - M^2_{0j} (1 - x_{0jk})) \bar{F}_{jk} + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} \omega\, c_{ij} x_{ijk} \\ &- \sum M^3 (1 - x_{ijk}) \bar{D}_{ijk} - cp \sum_{j \in N} \bar{E}_j + \sum_{\forall k \in K} \sum_{i \in N_c} a_i h_i \bar{J}_{ik} \\ &+ \sum_{\forall k \in K} \sum_{i \in N_c} (t_{i0} - M^2_{i0} (1 - x_{i0k})) \bar{H}_{ik} - \sum_{\forall k \in K} \sum_{i \in N_c} b_i h_i \bar{I}_{ik} \\ &+ \sum_{\forall k \in K} \sum_{j \in N_c} \sum_{i \in N_c} (t_{ij} - M^2_{ij} (1 - x_{ijk})) \bar{G}_{ijk} \Big] \le 0 \end{aligned} \tag{27}$$

Equatons (7) - (16) (28)

3. 1. Variable Neighborhood Search

In this paper these cuts are modified by the variable neighborhood search (VNS) algorithm successful on Vehicles routing problems. When a solution is found, this strategy can be used to obtain a higher-quality solution. As shown before, these new solutions can be used to generate new cuts for the master problem. neighborhood solutions (even with lower quality) may help to generate quality cuts in initial Benders iterations. To implement this strategy, a well-known local search algorithm, called variable neighborhood search [17], is used. This algorithm can be used in Finding an efficient and feasible solution to the master sub-problem.

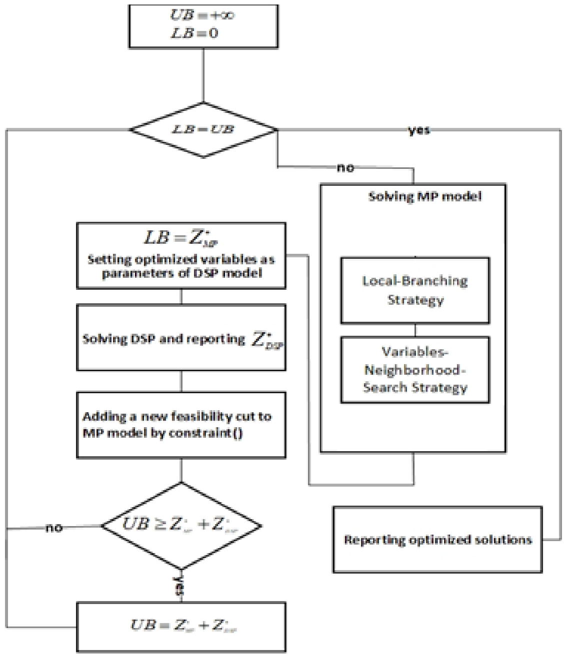

Figure 2. Proposed modified Benders decomposition flowchart

The VNS algorithm is used in the first iterations when the solution of the MP is not yet of the sufficient quality to reduce the total running time of the Benders algorithm. Therefore, the solution space of this algorithm is the space of the MP.

Defining symbols of *D*, *C*, and *L* as distributors, customers, and parcel lockers, respectively, Figure 3 shows a solution representation of the VNS algorithm, indicating how to assign customers to vehicles and lockers. Figures 4 to 7 show the VNS operators and the associated sequences for moving from one neighbourhood to another, referred to as the node-entering, node-swapping, edge-swapping, and block-swapping operators, respectively.

3. 2. Local-branching Strategy

The main idea of this strategy is that when the Master problem is solved, a certain number of variables that in the last iteration were equal to 1 are allowed to exchange their value to 0 [18]. Assuming that there is an initial feasible solution to the Master problem (MP), the set $\bar{S} := \{ j \in \beta : x_j = 1 \}$ is

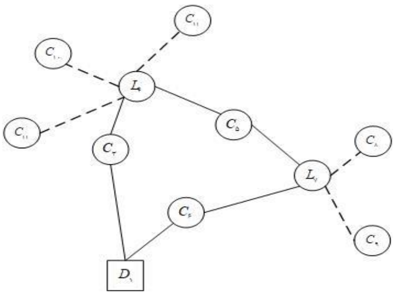

Figure 3. Customers allocation to lockers and vehicles

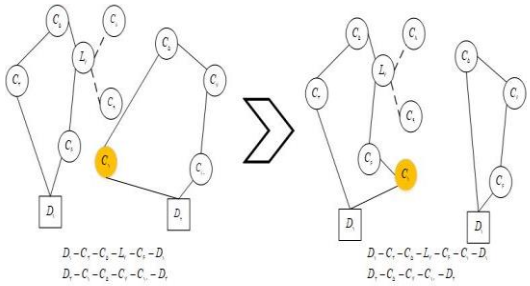

Figure 4. Example of a node-entering operator

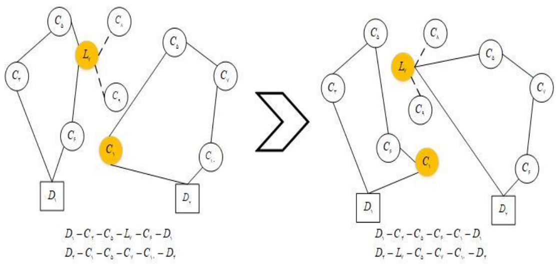

Figure 5. Example of a node-swapping operator

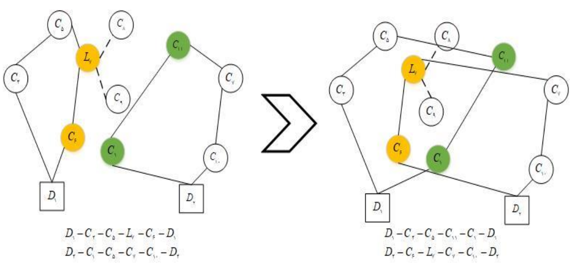

Figure 6. Example of an edge-swapping operator

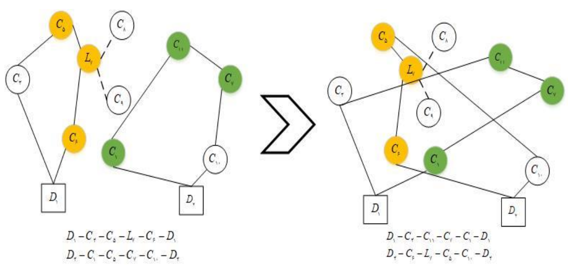

Figure 7. Example of a block-swapping operator

defined as variables being equal to 1. setting the certain positive integer number k in the range of 10 to 20, the neighborhood $k-opt$ of the variable $\overline{x}$, shown by $\Gamma(\overline{x},k)$, is defined as solutions satisfying Constraint (29).

$$\Delta(x,\overline{x})=\sum_{j\in S}(1-x_j)+\sum_{j\in\beta\setminus S}x_j\leq k \tag{29}$$

Constraint (29) limits the sum of the variables exchanging their values from zero to one and one to zero. Local-branching constraints can be used as a benchmark within a counting scheme for MP. In other words, according to the current solution, the solution space associated with the current node can be separated by the new left and right-branch node of the local-branching method. The parameter of the neighborhood size (k) that can be produced as the largest radius of local searching for two new nodes, named right- and left-handed problem that is probably much easier to solve than its parent problem. According to Figure 8, this idea resulted from the fact that the neighborhood associated with the left and right branches $\Gamma(\overline{x},k),\Gamma(\overline{x},k+1)$ must be small enough to optimize in a short time, and large enough to achieve a better solution. We consider the mentioned manners, such as Constraint (29) for all binary variables of the proposed model ($x_{ijk}, y_{ij}, h_i, l_i$).

4. RESULTS

4. 1. Results Driven by the Proposed Model

To solve the proposed models, the parameters are generated based on uniform distribution [19], as shown in Table 1. To demonstrate the efficiency of the proposed model, a network is considered consisting of 2 parcel-lockers and 8, 6, and 5 customers, who have selected the first, second, and third delivery options, respectively. The Pareto solutions to this problem are first shown in Figure 9. As

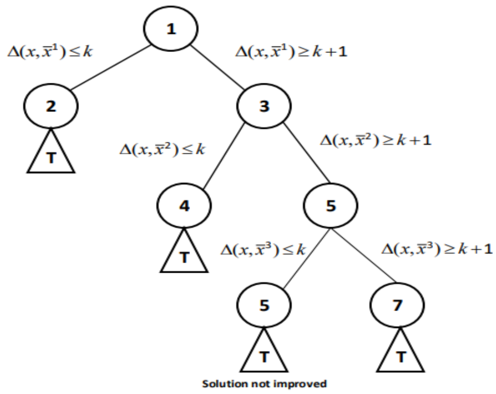

Figure 8. Local branching algorithm

TABLE 1. Generated parameters

Parameters	Value	Parameters	Value
d_i	Uniform(1, 6)	a_i	Uniform(8,12)
c_{ij}	Uniform(10, 50)	b_i	Uniform(9,16)
V_i	Uniform(50,100)	cv_k	Uniform(20,100)
pc_j	Uniform(10,15)	τ	Uniform(100,140)

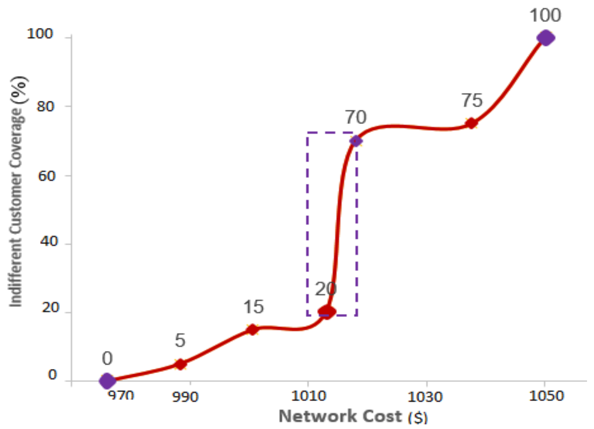

Figure 9. Pareto solution

can be seen, the two axes conflict with each other meaning that the costs of the network are increased and vice versa to increase the coverage of more indifferent customers. By increasing the coverage of indifferent customers from 20 to 70, a deeper gap compared to neighboring solutions is seen. It indicates that in this situation, with a low cost, a large increase in coverage of indifferent customers is created.

Figure 10 corresponds to a Pareto solution from Figure 9 with a cost of 1050 and 100% coverage. In this solution, all indifferent customers are assigned by the company to parcel lockers based on their selections. In this network, to achieve 100% coverage, an 8% increase in network costs is observed. By cpmparision with experts' opinion, the feasiblity of the proposed model is confirmed.

As previously explained, the applied discount of the indifferent customers is calculated from $\zeta_k=\theta\times\max\{\mu_k$ and $\eta_k\}$, where μ_k and η_k are the reference discount for the selection of the first and second delivery options, and by default, θ is between 1 and 1.1. In Figure 11, the current value is considered as the evaluation index, which is indicated by θ=1 and θ=1.1 means a ten percent increase in the discount coefficient, and θ=0.9 represents a ten percent decrease, and so on for other values. This sensitivity analysis is performed for five coefficients of the discount with values of 0.8, 0.9, 1, 1.1, and 1.2. As it

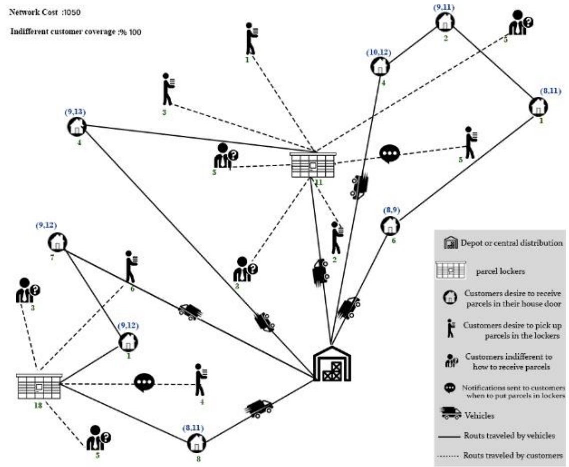
Figure 10. Network structure for a cost of 1050 and 100% coverage

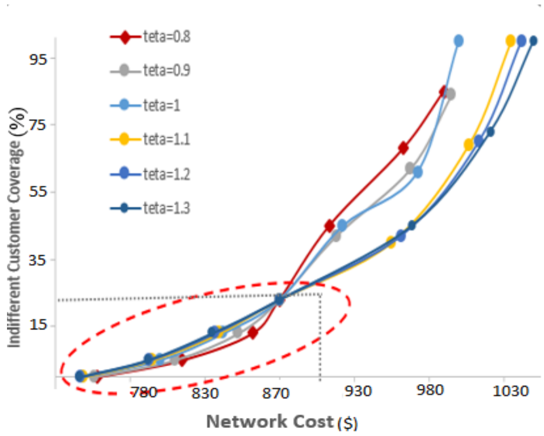

Figure 11. Sensitivity analysis based on a discount coefficient

is obvious, these five graphs can be classified into two classes of behavior, which are the coefficients (1.1, 1.2) in one class and the coefficients (0.8, 0.9, 1) in the other class. The important point is that with up to 20% coverage, all graphs are approximately tangent to each other, which means that cost imposed on the network due to discounts offered to the customer is balanced by saving cost resulting from the elimination of shipping operations. According to this, by increasing this coefficient, although it increases the cost of the network, it increases the potential of the network to cover more indifferent customers by parcel lockers.

4. 2. Results Derived by the Proposed Solution Method In this subsection, the computational examination is reported in such a way that the proposed solution approach is analyzed by emphasizing and examining the computational efficiency and effectiveness of the outputs. Initially, the impacts of algorithmic enrichments upon the general methodology to solving LNFDO via the Benders decomposition algorithm is surveyed. The codes are written in Python. The master problem and dual sub-problems are solved by the CPLEX library of Python on a Laptop computer under properties of a 2.8-GHz Intel-Core processor and 16-GB RAM. To test the computational improvements of this algorithm, 18 instances are generated.

Table 2 shows the computational outcomes of different instants. The gap percentage of each instance is calculated as (*UB*-*LB*)/*LB*. A gap of 100% implies that a feasible solution is not obtained at the end. the term "Time" is a run-time algorithm displayed in an hour unit. The maximum of the allowed run time to continue iterations is 6 hours, and then, the Benders algorithm is terminated. In the following, after varying the problem size, the efficiency of the enhanced algorithms and the quality of the solutions are evaluated. Given the complexity of the problem, as summarized in Table 2, the performance of the modified algorithm is very promising in terms of the quality of the solution and the computational time obtained. In the class of medium sizes, ILOG CPLEX Optimization Studio is superior to the proposed algorithms; however, in the class of large sizes, all the combinations of the proposed strategies have a relative and absolute advantage, so that the modified Benders decomposition guarantees to obtain an efficient solution close to optimum and optimality gap less than 3.

Figure 12 illustrates how to converge between lower and upper bounds in the manners of classic and modified Benders decomposition algorithm. Regarding the lower bound, up to iteration 9, their growth is almost the same, but after that, the modified algorithm shows more growth than the classic one, and regarding the upper bound, from the beginning of the modified algorithm starts with a significantly lower value than classic one and decreases this value gradually and regularly.

TABLE 2. Properties of the proposed modified Benders decomposition

		CPLEX solver		Classic Benders		Modified Benders	
		Time (h)	Gap (%)	Time (h)	Gap (%)	Time (h)	Gap (%)
Medium	1	0.31	0	0.41	<8	0.29	0
	2	0.38	0	0.45	<7	0.27	0
	3	1.31	0	1.9	<6	0.59	<0.8
	4	1.98	0	2.13	<8	1.11	0
	5	2.56	0	3.1	<5	2.12	0
	6	3.22	0	3.5	<8	2.36	<0.5
large	7	2.83	<1	3.1	<9	1.86	0
	8	3.45	<1	3.48	<11	2.86	0
	9	4.98	<3	3.96	<13	3.76	0
	10	5.89	<3	4.45	<10	4.23	0
	11	>6	>15	4.9	<11	4.46	<1
	12	-	**100**	5.3	<12	5.12	<2
	13	>6	>15	5.6	<14	4.32	<1
	14	>6	>15	5.7	<12	4.11	<1
	15	>6	>15	5.2	<13	4.44	<3
	16	-	**100**	>6	<16	5.23	<3
	17	-	**100**	>6	<12	5.42	<3
	18	-	**100**	>6	<14	5.61	<3

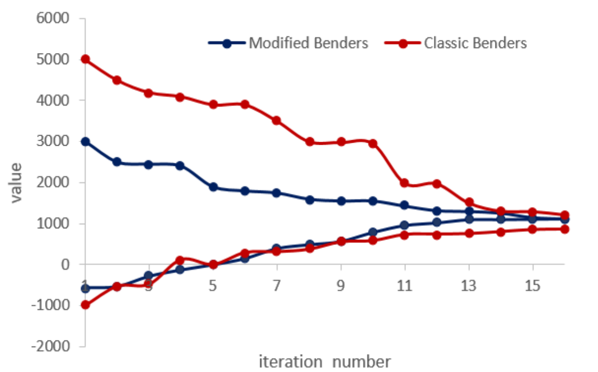

Figure 12. Convergence of the proposed algorithm

4. 3. Discussion and Managerial Insights

As mentioned before, the proposed model has two objective functions. The first one is the network cost consisting of transportation costs, operational costs of parcel lockers, and discounts that the company provides to customers who accept to pick up their product from lockers. The second one is covering indifferent customers by locker facilities.

The companies encourage indifferent customers to pick up their products from the lockers by increasing the discounts, but the total cost of this discount and the operational locker cost should not be significantly greater than the cost of shipping goods in the at-the-door option. A relationship between company's costs and the covering of indifferent customers is not necessarily a linear one. As it is obvious in Figure 9, with only 20% more costs, 50% growth in coverage is gained which from the aspect of the operations research (OR) view can result from significantly expanding of solution space, because of releasing of restrictions such as time windows of vehicles. Given the significant impact of increasing company costs on covering more indifferent customers, the decision-makers should consider these Pareto solutions when determining their strategy and long-term policies in a competitive market and analyze the sensitivity of the model in the adopted policies and its neighborhood to know how to control sudden changes in cost and indifferent customer coverage.

The discounts offered to indifferent customers play an important role in gradually encouraging them to progressively cooperate with the last-mile network. Discounts should be commensurate with the option selected and its parameters specified by the customer. Low discount does not create the necessary motivation and unnecessary discount eliminates the effects of long-term targeted incentives. According to Figure 11, at low percentages of an indifferent customer coverage, the Pareto front for the incremental and decremental discount coefficients is almost tangent to each other, showing the sensitivity of the indifferent customer to the attractiveness of the discounts. In the solutions that the company adopts the cost reduction policy, indifferent customers in all discount coefficients tend not to participate in the cooperation of locker network and instead receive the product at home and in the ones that the company adopts the cost increase policy to develop the network of parcel lockers, a significant non-linear increase in changing of indifferent customers' attitude is obvious. As result, unlike cost-cutting policies, the nonlinear effects of the discount coefficient are more tangible in policies based on the development of a parcel-locker network and the encouragement of more indifferent customers. Thus, in the policy that the company intends to develop a locker network, the sensitivity analysis of discount coefficients to find out the attractiveness of indifferent customers' threshold and control total cost under locker network development is necessary.

In a class of large sizes, all the combinations of the proposed strategies had a relative and absolute advantage, so the modified Benders decomposition algorithm guaranteed to obtain an efficient solution close to an optimum and optimality gap of less than 3%. In the class of large sizes, ILOG CPLEX Optimization Studio even could not get a feasible solution in the four instances during the given run time. Thus, because real-world problem dimensions are almost large, the proposed solution approach is suitable for achieving global solutions or very close to it.

5. CONCLUSION

Focusing on parcel-locker systems emerging as a hopeful technology to accelerate the Last-mile logistics in recent years, this study has addressed a Last-mile network with flexible delivery options. Surveying the role of lockers in an integrated network, making this theoretical concept closer to the more practical one, defining management tools to encourage customers to cooperate more with the locker network, and ensuring the gradual development of locker network are the challenges of this investigation. Thus, the multiple delivery options, push lever for the gradual development of the parcel-locker network, online flexible discount to parcel lockers network are the innovations of this study, which have been presented for the first time according to the best of our knowledge. This paper introduces a single-echelon network that includes a central distributor, parcel lockers, and customers, allowing customers to take advantage of three flexible product delivery options after purchasing the product. In the first option, the parcels are delivered at the door and in the time window specified by the customer; In the second option, customers pick up the parcels from the desired lockers with a discount, and in the third option, customers leave the choice of delivery type to the company under gaining an attractive discount. The proposed model could be a good platform for leading businesses in online commerce as well as urban policymakers who want to not only promote the level of online customer service but also in a gradual process, increase customers' social responsibility sense to pick up their parcels from lockers.

To solve this problem, a modified Benders decomposition algorithm enhanced by two efficient strategies (i.e., local branching and variable-neighborhood-search) are presented. The variable-neighborhood-search strategy is used in finding an efficient and feasible solution to the master sub-problem and getting an efficient solution in the local branching problem. In the class of large sizes, all the combinations of the proposed strategies had a relative and absolute advantage, so the modified Benders decomposition algorithm guaranteed to obtain an efficient solution close to an optimum and optimality gap of less than 3%. In the class of large sizes, ILOG CPLEX Optimization Studio even could not get a feasible solution in the four instances during the given run time.

In terms of sensitivity analysis of the model, the relationship between company's costs and coverage of indifferent customers is not necessarily a linear one. According to Figure 9, with only 20% more costs, 50% growth in coverage is gained which from the aspect of operation research view, can result from significantly expanding of solution space, because of releasing of restrictions such as time windows of vehicles. Given the significant impact of increasing company costs on covering more indifferent customers, the decision-makers should consider these Pareto solutions when determining their strategy and long-term policies in a competitive market and analyze the sensitivity of the model in the adopted policies and its neighborhood to know how to control sudden changes in cost and indifferent customer coverage. According to Figure 11, unlike cost-cutting policies, the nonlinear effects of the discount coefficient are more tangible in policies based on the development of a parcel-locker network and the encouragement of more indifferent customers. Thus, in the policy that the company intends to develop a locker network, the sensitivity analysis of discount coefficients to find out the attractiveness of indifferent customers' threshold and control total cost under locker network development is necessary.

For future research, we suggest a simulation-optimization approach to develop this paper based on an online sales company's case study in Iran. Considering the limitations of parcel lockers in this model such as the multi-product, dynamic, robustness, modality and mobility can be a challenging and useful concept to make that more practical.

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Persian Abstract

چکیده

هدف از ارائه این مقاله معرفی رویکردی یکپارچه و تخصصی برای مواجه‌شدن با مسئله‌ای چالش‌برانگیز معروف به «شبکه آخرین گام از توزیع محصول» است. این مسئله که از منظر سطح تصمیم‌گیری در سطح تاکتیکال طبقه‌بندی می‌گردد، یک مدل‌سازی از فرآیندهای عملیاتی و کاربردیِ کسب و کارهای مطرحِ اینترنتیِ کشورهای توسعه‌یافته می‌باشد درحالی‌که از جنبه تحقیق درعملیات مورد توجه زیادی واقع نشده است. این مسئله یک شبک یک سطحی است که دربردارنده توزیع‌کننده مرکزی، کمدهای خودکار تحویل کالا و مشتریان بوده و در آن به مشتریان امکان بهره جستن از سه گزینه منعطف دریافت محصول را پس از خرید محصول، داده می‌شود. در گزینه اول تحویل محصول در درب منزل و در پنجره زمانی تعیین شده توسط مشتری انجام شده؛ در گزینه دوم برداشت محصول توسط مشتریان از کمدهای خودکار مطلوب به همراه تخفیف صورت پذیرفته و در گزینه سوم مشتریان انتخاب توع تحویل را با کسب تخفیف جذاب به شرکت واگذار می‌نمایند. پیشنهاد تخفیفات آنلاین متناسب با گزینه انتخابی بمنظور تشویق هرچه بیشتر مشتریان در همکاری با شرکت و تضمین توسعه تدریجی شبکه کمدهای خودکار توسط یک اهرم مدیریتی از دیگر نوآوری‌های این مدل می‌باشد. برای حل مدل پیشنهادی، یک الگوریتم تجزیه بندرز توسط یک الگوریتم فراابتکاری جستجو با شعاع متغیر شاخه‌زنی محلی اصلاح شده است. نتایج کسب‌شده از تحلیل حساسیت پارامترهای مرتبط با نوآوری‌های مسئله، بیان‌گر کارآمدی و صحّه مدل در سناریوهای مختلف و الگوریتم حل پیشنهادی در مسائلی با ابعاد بزرگ می‌باشد.