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Structural Stiffness Matching Modeling and Active Design Approach for Multiple Stepped Cantilever Beam

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A B S T R A C T

Aiming at the problem that it is difficult to realize the optimal design due to the fuzzy mapping relationship for the structural stiffness of multiple stepped cantilever beam; a stiffness matching modeling and active stiffness design approach was proposed. Firstly, by deriving out the continuous coordination conditions and the load extrapolation expressions of the cantilever joint, the stiffness analytical model and the recursive model were established for multiple cantilever beam segments, and the stiffness influence coefficient of those composition parameters were obtained by the sensitivity analysis. Then, the active stiffness optimization design process was constructed according to the stiffness design level of the stepped cantilever beam, and those implementation procedures were clearly figured out. Finally, the comparison and verification of the stiffness design of the stepped cantilever beam was carried out through numerical simulations, finite element analysis and bench test. The obtained results showed that the established models and the active stiffness design method are reasonable and effective. The stiffness match parameters are easy to meet the stiffness index requirements, and the safety factor is greater than 1; when the number of steps is not more than 5. The relative error between the match stiffness and the test stiffness is less than 15%, which can be reduced to less than 5% by adding redundancy coefficient (1.05, 1.15).

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1. INTRODUCTION

With the gradual transition from the secondary load-bearing structure to the primary load-bearing structure of the multi-step cantilever beam, the rapid calculation and preliminary optimization of its structural stiffness has become a key issue in engineering application [1, 2]. The current stiffness design mainly adopts the empirical coefficient method, which includes the experience designing, stiffness checking and the modifying steps. But it is often blind to a certain extent, and it is unavoidable that insufficient or redundant stiffness occurs. Although some researches have used the optimization design method; since the structural stiffness match models of the multi-step cantilever beam has not yet been found in the relevant literature. It is still challenging to establish the objective function and constraint conditions according to the level of stiffness design, such as the continuous coordination conditions and the load extrapolation expressions are not clear. Therefore, it is of great significance to carry out the active stiffness design research on the stepped cantilever beam. On the basis of mastering the equivalent mapping relationship between the cantilever parameters and the structural stiffness, the deformation of the cantilever beam segments can be efficiently controlled to achieve the designed requirements at one time, which could promote the application of stepped cantilever beam to aerospace, robotics and other fields.

The existing researches [3-5] on the stiffness design of some characteristic structures mainly focus on the active design method or the forward design method. Li et al. [6] proposed the beam-frame model aeroelastic optimization method and the three-dimensional model conversion method for designing the global stiffness of a high aspect ratio wing. Ke et al. [7] established the

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matching relationship between the key parameters of layer scheme and the stiffness of composite leaf spring, and also, its structural layout was designed for the matching stiffness target. Shi et al. [8] put forward a top-down design method for the static and dynamic stiffness of precision horizontal machining centers. By summarizing those researches, it is worth noting that although those stiffness design methods could preliminarily achieve the control of structural deformation, the distribution of structural internal force, etc., but many application restrictions still appear when combining with the stepped cantilever structure. The structural stiffness match model of multiple stepped cantilever beam has not been found in relevant literatures, and the mapping relationship is not clear between the structural stiffness and different scale or characteristic parameters. The existing stiffness design methods are mainly for machine tools and other specific devices [9, 10], the design process should be adjusted for multiple stepped beam.

In order to realize the structural stiffness match and active stiffness design of multi-step cantilever beams, the continuous coordination conditions and the load extrapolation expressions are derived out through the motion and deformation modeling of the cantilever structure, and the stiffness match models shown in section 2 are constructed by using the elastic mechanics theory, of which includes the explicit expression of double stepped beam and the recursive expression of multiple stepped beam. On this basis, the stiffness coefficient of various composition parameters is obtained with the sensitivity analysis. In section 3, the active stiffness design process is constructed according to the stiffness requirements of the stepped cantilever structure, and the key implementation elements are also presented. Finally, the reliability of the established stiffness match models and the rationality of the active design process are verified according to the numerical simulations, the finite element simulations and the bench test of the cantilever beam sample in section 4. Meanwhile, the high-precision and high-confidence application mode of the established active stiffness design method is pointed out, which is helpful to promote the robust design and reliable application of multi-step cantilever beams. In addition, section 1 is the introduction, section 5 is the conclusion, section 6 is the acknowledgment, section 7 is the list of references.

2. STIFFNESS MATCH MODEL

According to the practical application situation, the parametric model of the stepped cantilever structure composed of multiple beam is shown in Figure 1. To simplify the modeling process, principle hypothesizes are as follows: (a) the length, width and height parameter of the rectangle section of n-steps $(n \geq 2)$ beams are L_k, B_k, H_k $(k = 1, \cdots, n)$ respectively; (b) the fixed and the free ends of the whole cantilever structure satisfy the boundary conditions; (c) the combined area $S_1, \cdots, S_{n-1}$ satisfy deformation coordination conditions.

Based on the stiffness index concept [11], the geometric analysis mainly focusses on the maximum allowable deformation under the external load. The stiffness coefficient expression can be expressed as:

$$K = P/w \tag{1}$$

where, P is the external load, which could be the concentrated force, distributed force, moment and torque, etc. w is the maximum deformation, angle, etc.

2. 1. Motion and Deformation Modeling

In the simple-beam framework of elastic mechanics, the bending modeling assumption of rectangular beam can be pointed out as follows: (a) the bending deformation is always in the main plane or cross section with respect to its body coordinate system, the shear and torsional deformation can be ignored; (b) the cross section could keep as plane during the motion, and it is perpendicular to the axis of the deformation beam; (c) the rotational kinetic energy of beam unit can be ignored, and also, the shear deformation potential energy can be ignored compared with bending deformation potential energy.

Taking the uniform beam with rectangle section as the object, the basic models corresponding to one-dimensional bending problem can be expressed as:

$$\kappa = \frac{\mathrm{d}^2 w}{\mathrm{d}x^2}, M = EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}, Q = EI\frac{\mathrm{d}^3 w}{\mathrm{d}x^3}, q = EI\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} \tag{2}$$

where, $w(x)$ is the deflection function of the mid plane, κ is the deformation curvature of the mid plane, M is the bending moment and Q is the transverse shear force on the section, E is the elastic modulus and I is the bending inertia moment, q is the transverse force.

(1) Single cantilever beam

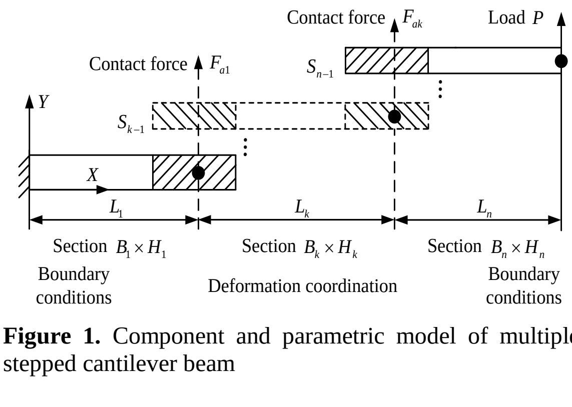

Figure 1. Component and parametric model of multiple stepped cantilever beam

The single cantilever beam can be equivalent to a curved beam element, and the boundary conditions of its fixed end are given by:

$$w(x)\big|_{x=0}=0,\quad \mathrm{d}w(x)/\mathrm{d}x\big|_{x=0}=0 \tag{3}$$

Combining to the concentrated load that located at the free end, the motion model and the maximum deformation can be expressed as follows:

$$\frac{EI}{L^3}\begin{bmatrix}12 & -6L\\ -6L & 4L^2\end{bmatrix}\begin{bmatrix}w_L\\ \theta_L\end{bmatrix}=\begin{bmatrix}P\\ 0\end{bmatrix},\theta_L=\frac{3w_L}{2L},\ w_L=\frac{PL^3}{3EI} \tag{4}$$

Then, the equivalent structural stiffness of the single cantilever beam can be expressed as follows:

$$K_L=3EI/L^3 \tag{5}$$

(2) Double stepped cantilever beam

When the double stepped cantilever beam has a separation trend under the external load, and the combined area is similar with the fixed boundary form, the deflection equation of the first-step beam can be expressed as follows:

$$\begin{cases} w\big|_{x=L_1}=\dfrac{F_aL_1^{\ 3}}{3E_1I_1},\quad \theta\big|_{x=L_1}=\dfrac{F_aL_1^{\ 2}}{2E_1I_1}\\ w(x)=\dfrac{F_ax^2}{6E_1I_1}\left(3L_1-x\right)\end{cases} \tag{6}$$

where, F_a is the contact force of the combined area.

Through the fixed constraint of the combined area, the motion model of the second-step beam can be expressed as follows:

$$E_2I_2\frac{\mathrm{d}^2W}{\mathrm{d}X^2}=\hat{P}\left(L_1+L_2-X\right),\quad X\in\left[L_1,L_1+L_2\right] \tag{7}$$

By using the direct differential method, the deflection equation can be derived and expressed as follows:

$$W(X)=\frac{\hat{P}}{E_2I_2}\left[-\frac{X^3}{6}+\frac{\left(L_1+L_2\right)X^2}{2}+\tilde{C}_1X+\tilde{C}_2\right] \tag{8}$$

After substituting the deformation coordination conditions of the displacement and the section rotation into Equation (8), it can be simplified and given by the following expression:

$$W(X)\big|_{X=L_1}=w\big|_{x=L_1},\quad \frac{\partial W}{\partial X}\bigg|_{X=L_1}=\theta\big|_{x=L_1} \tag{9}$$

$$W_2\left(L_1+L_2\right)=\frac{F_a}{3E_1I_1}L_1^{\ 3}+\frac{\hat{P}L_2^{\ 3}}{3E_2I_2}+\frac{F_a}{2E_1I_1}L_1^{\ 2}L_2 \tag{10}$$

where, $W_2\left(L_1+L_2\right)$ is the maximum deformation.

Besides, the contact force F_a can be determined with the modified concentrated load [12] method, which could be constructed by the constraint relationship of the combined area and expressed as follows:

$$\begin{aligned} &Q_a\hat{P}+Q_a{}'F_a=0,\ Q_a=\frac{1}{2}\frac{I_1}{I_2}\left[3\frac{L_2}{L_1}-2\right]\\ &Q_a{}'=-\left[1+\frac{I_1}{I_2}\right],\ \ F_a=\frac{1}{2}\frac{I_1}{\left(I_1+I_2\right)}\left[3\frac{L_2}{L_1}-2\right]\hat{P}\end{aligned} \tag{11}$$

Then, the equivalent structural stiffness of double stepped cantilever beam can be expressed as follows:

$$K_{L_1+L_2}=\frac{E_1E_2I_2}{\dfrac{E_2I_2}{\left(I_1+I_2\right)}\left[\dfrac{3}{4}L_1L_2^{\ 2}-\dfrac{1}{3}L_1^{\ 3}\right]+\dfrac{1}{3}E_1L_2^{\ 3}} \tag{12}$$

(3) Multiple stepped cantilever beam

With the same bending modeling assumption, the recursive function can be used to derive the equivalent stiffness of multiple cantilever beam.

Firstly, relative to the reference coordinate system, the motion models and the deflection equations of the n-step $(n\geq 3)$ cantilever beam are given by the following expression:

$$\begin{cases} W_1(X)=F_{a1}\dfrac{X^2\left(3L_1-X\right)}{6E_1I_1},X\in\left[0,L_1\right]\\ E_2I_2\dfrac{\mathrm{d}^2W_2}{\mathrm{d}X^2}=F_{a2}\left(L_1+L_2-X\right),X\in\left[L_1,L_1+L_2\right]\\ E_nI_n\dfrac{\mathrm{d}^2W_n}{\mathrm{d}X^2}=\hat{P}\left(L_1+\cdots+L_n-X\right),\\ \qquad X\in\left[L_1+\cdots+L_{n-1},L_1+\cdots+L_n\right]\end{cases} \tag{13}$$

Secondly, based on the deformation coordination conditions of the combined areas $X=L_k(k=1,\cdots,n-1)$, those $n-1$ contact forces can be also derived from the modified concentrated loads and expressed as follows:

$$\begin{cases} Q_{n-1}\hat{P}+Q'_{n-1}F_{a(n-1)}+Q''_{n-1}F_{a(n-2)}=0\\ Q_kF_{a(k+1)}+Q'_kF_{ak}+Q''_kF_{a(k-1)}=0\\ Q_1F_{a2}+Q'_1F_{a1}=0\end{cases} \tag{14}$$

$$\begin{cases} Q_k=\dfrac{I_k}{2I_{k+1}}\left[\dfrac{3L_{k+1}}{\left(L_1+\cdots+L_k\right)}-2\right]\\ Q'_k=-\left[1+\dfrac{I_k}{I_{k+1}}\right],\ k=1,\cdots,n-1\\ Q''_k=\dfrac{1}{2}\left[\dfrac{I_{k-1}}{I_k}\right]^2\left[3-\dfrac{\left(L_1+\cdots+L_{k-1}\right)}{\left(L_1+\cdots+L_k\right)}\right]\end{cases} \tag{15}$$

$$\begin{bmatrix} Q'_{n-1} & Q''_{n-1} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ Q_{n-2} & Q'_{n-2} & Q''_{n-2} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & Q_{n-3} & Q'_{n-3} & Q''_{n-3} & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \ddots & \cdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & Q_2 & Q'_2 & Q''_2 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & Q_1 & Q'_1 \end{bmatrix} \begin{bmatrix} F_{a(n-1)} \\ F_{a(n-2)} \\ F_{a(n-3)} \\ \vdots \\ F_{a3} \\ F_{a2} \\ F_{a1} \end{bmatrix} = \begin{bmatrix} -Q_{n-1}\hat{P} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Finally, the implicit expression of the equivalent stiffness of the multiple stepped cantilever beam can be expressed as follows:

$$K_{L_1+\cdots+L_n} = \frac{\hat{P}}{W_n\left(L_1+\cdots+L_n\right)} \quad (17)$$

For the stiffness match model given in Equation (17), since the end deflection contains the higher-order term of the external load variable, the constant term in the load extrapolation expression cannot be completely offset. However, when the scale parameters, mechanical parameters and external force parameters of the cantilever structure are determined, the maximum deformation can be obtained recursively, and the stiffness characteristics can be figured out by the slope between the maximum deformation and the external load. In the existing researches, the composite element method is used to establish the overall stiffness matrix of the cantilever beam [3, 5], or the nonlinear characterization test results [13] of stepped cantilever structure are used to establish a fitting model to obtain the linear term and cubic term of stiffness coefficients. Compared with the existing methods, the established models introduce the continuous coordination condition and the load extrapolation relationship of the cantilever joint, which avoids the complex calculation of the overall stiffness matrix and the requirement of the physical model test system, and can be directly applied to the rapid calculation and optimization of the stiffness of the cantilever structure.

2. 2. Stiffness Influence Coefficient After establishing the structural stiffness match model of multiple stepped cantilever beam, the stiffness influence coefficient of different composition parameters can be obtained by using the sensitivity analysis method [14].

Taking the double stepped cantilever beam as the object, the stiffness match model that given in Equation (12) can be expressed as follows:

$$I_\alpha = \frac{1}{12}B_\alpha H_\alpha^{3}, \quad \Delta L = \frac{1}{16}L_1^{2}L_2 - \frac{1}{36}L_1^{3}$$
$$K_{L_1+L_2} = \frac{E_1E_2B_2H_2^{3}}{\dfrac{E_2B_2H_2^{3}}{\left(B_1H_1^{3}+B_2H_2^{3}\right)}\Delta L + \dfrac{1}{36}E_1L_2^{3}} \quad (18)$$

$$\bar{K} = \psi_B \cdot B_1 \ \text{ or } \ \bar{K} = \psi_E \cdot E_1$$
$$\bar{K} = \psi_H \cdot H_1^{3} \ \text{ or } \ \bar{K} = \psi_D \cdot L_1^{-3} \quad (19)$$

$$\lambda_B = \frac{B_2}{B_1}, \ \lambda_e = \frac{E_2}{E_1}, \ \lambda_h = \frac{H_2}{H_1}, \ \lambda_L = \frac{L_2}{L_1}$$
$$\psi_B = \frac{\lambda_B E_1E_2H_2^{3}}{\dfrac{\lambda_B E_2H_2^{3}}{\left(H_1^{3}+\lambda_B H_2^{3}\right)}\Delta L + \dfrac{1}{36}E_1L_2^{3}}$$
$$\psi_E = \frac{\lambda_e B_2H_2^{3}}{\dfrac{\lambda_e B_2H_2^{3}}{\left(B_1H_1^{3}+B_2H_2^{3}\right)}\Delta L + \dfrac{1}{36}L_2^{3}} \quad (20)$$
$$\psi_H = \frac{E_1E_2B_2\lambda_h^{3}}{\dfrac{E_2B_2\lambda_h^{3}}{\left(B_1+B_2\lambda_h^{3}\right)}\Delta L + \dfrac{1}{36}E_1L_2^{3}}$$
$$\psi_D = \frac{36E_1E_2B_2H_2^{3}}{\dfrac{E_2B_2H_2^{3}}{\left(B_1H_1^{3}+B_2H_2^{3}\right)}\left[\dfrac{9}{4}\lambda_L - 1\right] + E_1\lambda_L^{3}}$$

It can be found that when the scale and mechanical parameters of each beam are proportional, the structural stiffness is positively linear with the width *B* and the elastic modulus *E*, and also it is positively cubic with the height *H*, but it is negatively cubic with the effective length *L*.

In addition, Equation (18) can be transformed with the slender ratio η and given by the following equations:

$$L_j = \eta_j H_j, \quad \Delta\eta = \frac{1}{16}\eta_2 H_2 - \frac{1}{36}\eta_1 H_1$$
$$K_{L_1+L_2} = \frac{E_1E_2B_2}{\dfrac{E_2B_2\eta_1^{2}H_1^{2}}{\left(B_1H_1^{3}+B_2H_2^{3}\right)}\Delta\eta + \dfrac{1}{36}E_1\eta_2^{3}} \quad (21)$$

$$\frac{\partial K_{L_1+L_2}}{\partial \eta_1} = -C_\eta H_\eta \eta_1\left[\frac{1}{8}\eta_2H_2 - \frac{1}{12}\eta_1H_1\right]$$
$$\frac{\partial K_{L_1+L_2}}{\partial \eta_2} = -C_\eta\left[\frac{1}{16}H_\eta\eta_1^{2}H_2 + \frac{1}{12}E_1\eta_2^{2}\right]$$
$$C_\eta = \frac{E_1E_2B_2}{\left[H_\eta\eta_1^{2}\Delta\eta + \dfrac{1}{36}E_1\eta_2^{3}\right]^2} \quad (22)$$
$$H_\eta = \frac{E_2B_2H_1^{2}}{B_1H_1^{3}+B_2H_2^{3}}$$

It can be concluded that when the scale constraint $\left(L_2/8 - L_1/12\right) < 0$ is satisfied, the structural stiffness has a positive correlation with the slender ratio η_1 of the first-step beam. Otherwise, it would have a negative correlation. Under the arbitrary constraint conditions, the structural stiffness has a negative correlation with slender ratio η_2 of second-step beam.

Based on Equation (17), the stiffness influence coefficient of those composition parameters of multiple stepped beam can be also obtained by using the sensitivity analysis and the chain derivation method, while those analytical expressions are omitted.

2. ACTIVE STIFFNESS OPTIMIZATION DESIGN

Combined to detailed decomposition of structural stiffness requirements, such as the bending parameter of gun barrel and the critical deformation parameter of ballistic missile body, etc., the active stiffness optimization design flowchart is constructed and shown in Figure 2. Omitting most of the match modeling process, the key to the implementation procedures for stiffness optimization [15] of multi-step cantilever beam is shown in Figure 3.

(1) Selecting and implementing the optimal design method. In general, the optimization design includes design variables, objective functions, constraints and algorithms. Besides, both mature multi-objective optimization algorithms and the improved algorithm [16] can be used. The implementation process can be achieved by the self programming and the mature software, such as ANSYS optimization module, UG parametric module, etc.

(2) Presenting and applying the stiffness design parameters. The materialization process of the stiffness index involves the structural form, the control of the structure weight and so on. It is necessary to determine the most ideal cantilever structural parameters within the optional configuration system.

(3) Checking the difference between the actual stiffness characteristics and the stiffness index. For the preliminary design parameters, it is necessary to figure out the stiffness error and the impact on the overall stiffness performance. And then, the local corrections could be carried out to modify the actual stiffness characteristics of multiple stepped cantilever beam.

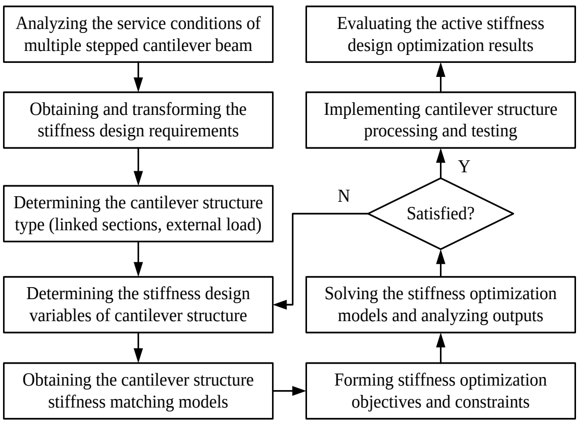

Figure 2. Active stiffness design flowchart

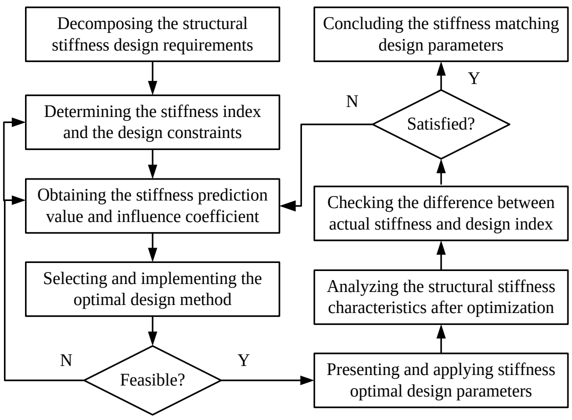

Figure 3. Key procedures of stiffness optimization

4. SIMULATION ANALYSIS AND TEST VERIFICATION

4. 1. Numerical Simulations and Analysis Based on the equivalent stiffness given in Equations (12), (18) and (21), the structural stiffness match results of double stepped cantilever beam are easy to obtain and shown in Figure 4. Besides, the benchmark parameters are as follows: the length, width and height of first-step beam is set to $64\times12\times6$ mm respectively. Its elastic modulus is set to 2.0E+05 MPa ; the corresponding scale parameters and elastic modulus of the second-step beam is $60\times12\times5$ mm, 2.0E+05 MPa .

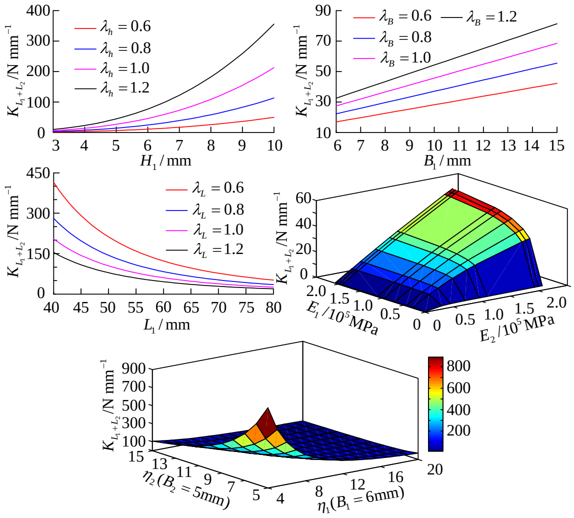

Figure 4. Stiffness match results of double stepped cantilever beam

It can be seen that when the structural parameters of first-step beam are used as the benchmark, the structural stiffness of the double stepped beam has a non-uniform mapping with different composition conditions, of which includes the non-linear relationship with the height and the length, as well as the linear rule with the width and the elastic modulus. From the viewpoint of sensitivity analysis, the stiffness growth rate is large with the height and its proportion coefficient, and it is very small with the elastic modulus. Without considering the proportion coefficient, the sensitivity is arranged from high to low in terms of height, length, width and elastic modulus, which means that it is feasible to change structural stiffness by increasing or decreasing the height conveniently. From the perspective of nonlinear characterization, the change trend of cantilever structure stiffness with scale parameters is a smooth curve, but there is a local jump relative to the change trend of elastic modulus and proportional coefficient. In addition, the structural stiffness also has a non-linear relationship with the slender ratio, and the sensitivity of the slender ratio of second-step beam is larger than that of the first-step beam, the stiffness variation tends to be smooth with an increase in the slender ratio.

Then, the double stepped beam is still taken as the object, and the active stiffness design simulation is carried out sequentially. It may be assumed that the material properties do not change, the effective length and the working load corresponded to the layout mode do not change. At this moment, the optimization model under the lightweight requirement is stated in the following.

The constant parameters are given by $E = 2.0\text{E+05 MPa}, \rho = 7850\ \text{kg m}^{-3}, L_1 = 108\ \text{mm}$, $L_2 = 78\ \text{mm}, \tilde{P} = 75\ \text{N}$, and the design variables are given by $B_1 B_2, H_1 H_2$, the maximum deformation should be less than 2.5 mm. Among them, the minimization objective functions that contain the weight, the scale and the stiffness constraints are:

$$\min:\begin{cases} g_1 = \rho\left(B_1H_1L_1 + B_2H_2L_2\right) \\ g_2 = B_2 - B_1,\ \ g_2 \le 0 \\ g_3 \in \left[-30,\ 0\right] \\ g_3 = \dfrac{\tilde{P}}{\left[f\right]} - \dfrac{36E^2B_2H_1^2H_2^3}{36H_\eta H_2^3\Delta L + EH_1^2L_2^3} \end{cases} \tag{26}$$

$$s.t.\ \begin{cases} 10\text{mm} \le B_1 \le 20\text{mm},\ 10\text{mm} \le B_2 \le 16\text{mm} \\ 3\text{mm} \le H_1 \le 21\text{mm},\ \ 3\text{mm} \le H_2 \le 15\text{mm} \end{cases}$$

The multi-objective optimization model given in Equation (26) can be solved through the NSGA-II algorithm [17], where the population number is set to 200 and the maximum evolution algebra is set to 200 times. The optimization design results are shown in Figure 5 and Table 1.

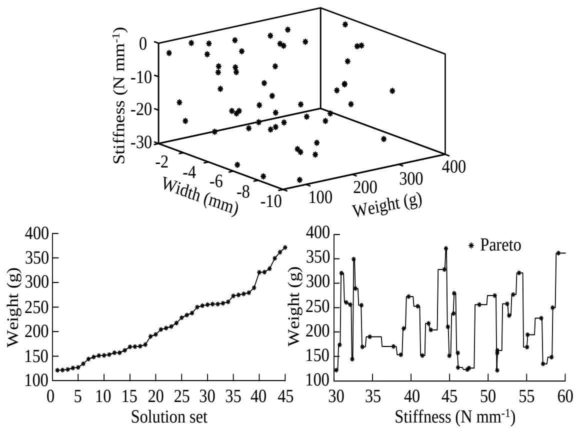

Figure 5. Active stiffness design results of double stepped cantilever beam

TABLE 1. Feasible solutions of the stiffness design

Variable	Pareto-1	Pareto-2	Pareto-3	Pareto-4
B_1 / mm	14.5117	16.6063	15.1705	15.8131
B_2 / mm	13.1158	11.6706	13.7044	12.1791
H_1 / mm	5.4649	8.7993	10.9754	10.3794
H_2 / mm	6.7842	6.3507	5.8456	7.4089
Weight (g)	121.7175	169.2641	190.2119	194.3999
Scale (mm)	-1.3959	-4.9357	-1.4661	-3.6340
Stiffness (N mm^{-1})	30.2928	33.6758	34.6324	55.1330

It should be noted that the Pareto front shown in Figure 5 are in the scale constraint space and the stiffness constraint space, the rest of the non-dominated solutions beyond the limitation range are not listed. The feasible parameters are summarized in Table 1. The optimal selection results by using diversity criteria method [18], of which pareto-1 can be rounded up and the stiffness matching design parameters can be expressed as follows:

$$\begin{aligned} &B_1 = 14.5\ \text{mm},\ B_2 = 14\ \text{mm},\ H_1 = 6\ \text{mm} \\ &H_2 = 6.5\ \text{mm},\ L_1 = 108\ \text{mm},\ L_2 = 78\ \text{mm} \\ &\text{W} = 129.5\ \text{g},\ \left[K\right] = 30.52\ \text{N mm}^{-1} \end{aligned} \tag{27}$$

The Pareto solution set is shown in Figure 5; that indicates that based on the proposed design method, the number of the non-dominant individuals does not have a coincident trend. For the constraints beyond the limitation range, the dominant individuals could reduce and control them within the allowed band. For the allowable constraints, the dominant individuals could make those close to the limitation values. Different stiffness matching results could correspond to the same weight constraint, and different weight distributions may receive the same stiffness constraint of the stepped cantilever structure. Moreover, the diversity screening result of Equation (27) comes from engineering application requirements, while the rounding result closest to the design goal is $B_1 = 14.5$ mm, $B_2 = 13.2$ mm, $H_1 = 5.5$ mm, $H_2 = 6.8$ mm, $W = 122.6$ g, $[K] = 30.27$ N mm^{-1}, but those parameters are not meet the ergonomics requirements.

4. 2. Bench Test Verification Through the stiffness design parameters given in Equation (27), the finite element model and its bench test are constructed respectively. As shown in Figure 6, the materials are set to structural steel, and the boundary conditions are approximate between the verification experiments.

After processing the transient structural analysis results and bench test results with the linear approximation method [11], the comparison verification of the structural stiffness of double stepped beam are shown in Figure 7 and Table 2. For the comparison subjects in Table 2, the stiffness index is consistent with the above mentioned optimization model, the match stiffness is obtained by substituting the round parameters into the established models, the FEM stiffness is determined by fitting the finite element results, and the test stiffness is determined by fitting the test results.

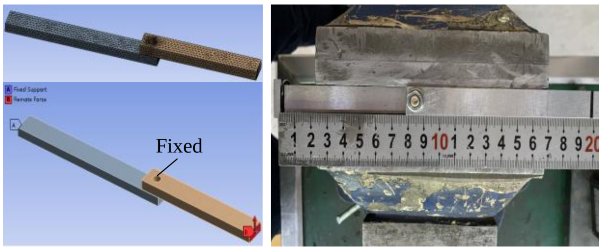

Figure 6. Finite element model and bench test of double stepped cantilever beam

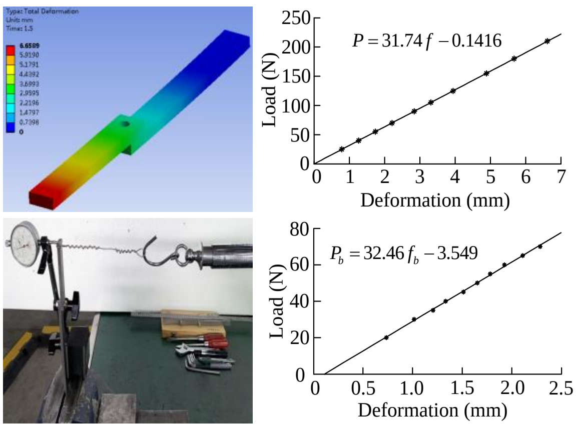

Figure 7. Structural stiffness test of double stepped beam

TABLE 2. Stiffness comparison verification of double stepped cantilever beam

Types	Index	Match	FEM	Test
Value (N mm^{-1})	30	30.52	31.74	32.46
Error (%)	--	+1.73%	+5.80%	+8.20%

Moreover, the stiffness index is used to derive the relative errors.

It is known that when the structural stiffness index is clear to the double stepped beam, its design parameters can be obtained through integrating the active stiffness design and engineering experience, which can ensure that the relative error between the match stiffness and the stiffness index is less than 2%. The relative error of the FEM stiffness is less than 6%, which indicates that the active design parameters are easier to meet the stiffness index. The relative error of the test stiffness is less than 9%, which shows that both the stiffness match models and the active design flowchart are practicable. Meanwhile, the match stiffness is less than the FEM stiffness and the test stiffness, which shows that the established models could meet the stiffness design requirements at one time, the safety factor is greater than 1, which is helpful to improve the reliability of stiffness design.

4. 3. Discussion and Application It can be concluded that the active stiffness optimization design performance of cantilever structure can not entirely consistent with the actual stiffness characteristics when the active design parameters applied to the engineering situation, the reasons are as follows.

(1) The bending modeling assumptions are derived from the simple-beam framework, and the influence of shear deformation is not taken into account. In general, the increase of the slender ratio would reduce the shear factor of beam, thus decreasing its influence on the whole deflection. But for the short beam, the cross section of beam can not keep as plane during the motion process, and the influence of the shear factor may be gradually increasing with the rise of the order of the deflection equations. At the same time, the action area of external force would not completely in the middle surface of the stepped structure, and then the cantilever structure produces torsion, which affects the accuracy of the match models.

(2) When the modified concentrated loads method is used to determine the contact forces of the combined areas, the assumptions are that the adjacent beams only contact at the end part, and the combined areas have the same deflection during the motion process. On the one hand, it is difficult to meet the full contact conditions in the actual application situation, which will lead to the smaller equivalent structural stiffness. On the other hand,

the cantilever joint could also be separated when the external load is large enough. It would cause the cantilever beam systems to gradually degenerate into complete sliding motion, which affects the calculation accuracy of the match models.

To quantitatively analyze the error range and the confidence boundary of stiffness match model, the numerical simulations are adopt to figure out the influence of the slender ratio and the beam' steps, and the relative finite element simulations are conducted. The comparison results are shown in Figure 8. Besides, the width and height of each section is 12×6 mm, the elastic modulus is $2.0\text{E}+05$ MPa, and the range of the slender ratio is set to (5, 20), the number of the steps is set to (3, 5).

It can be known that for the same slender ratio, the increase of the number of steps would reduce its equivalent structure stiffness, but if the slender ratio is large enough, the reduction trend is not obvious, and this situation is also indirectly verified in Figure 3. The verified stiffness based on the finite element simulations are larger than the match stiffness based on the proposed models, and the relative error ranges are $E_{n=3}\in[5\%,\ 8\%]$, $E_{n=4}\in[6\%,\ 11\%]$ and $E_{n=5}\in[7\%,\ 15\%]$ respectively. Also, it can be inferred that the maximum error with the bench test is no more than 15% through analyzing the test results of double stepped beam. The error results shown in Figure 8 are arranged in descending order, where dose not have a consistent one-to-one match between each relative error and each slender ratio.

Furthermore, it can be drawn that when carrying out the active stiffness optimization design of multiple stepped cantilever beam, the redundancy coefficient that belongs to $\sigma\in[1.05,\ 1.15]$ should be added to the stiffness match models, which could avoid the actual stiffness performances exceeding the design requirements too much.

As shown in Figure 9 and Table 3, the bench test of three stepped cantilever beam sample is conducted to verify the reliability of redundancy coefficient. It can be found that when the coefficient is set to 1.05, the relative error between the match stiffness and the FEM stiffness

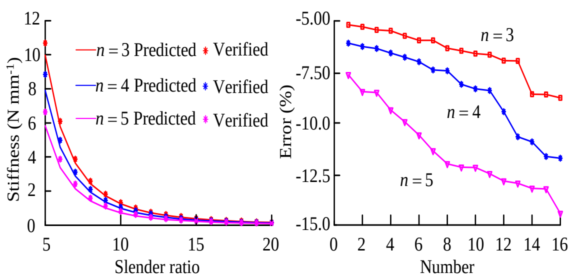

Figure 8. Multifactor analysis of structural stiffness error

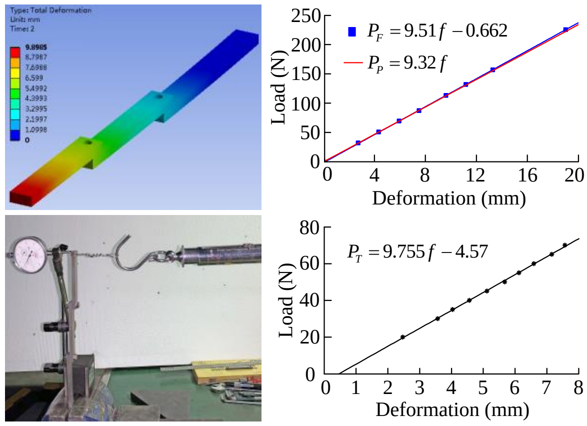

Figure 9. Bench test of three stepped cantilever beam

TABLE 3. Stiffness verification of three stepped beam

Types	Match	FEM	Test
Value (N mm^{-1})	9.32	9.51	9.755
Error (%)	--	+2.04%	+4.67%

can reduce to 2%, the relative error between the test stiffness and the match stiffness can reduce to 5%, which means that the stiffness redundancy can be improved effectively, both the accuracy of the established stiffness models and the feasibility of the presented active stiffness design flowchart are able to achieve the stiffness design of multiple stepped cantilever beam preferably. As the cantilever structure with more than 6 steps (except the stepped shaft structure) does not have engineering practical value, its stiffness change law will not be shown in detail here.

Finally, taking the test results of the double stepped beam and the three stepped beam as the object, and the Iwan model that given in literature [13] is used to process the raw data and generate the linear term of the stiffness coefficient, the comparison results are obtained and shown in Table 4.

It can be seen that under the unified test sample, the Iwan stiffness coefficient is less that the test stiffness, but also greater than the match stiffness. When the redundancy coefficient is not added to the double stepped beam, the relative error between the match stiffness and the Iwan stiffness coefficient is 5.3%, while the relative error reduces to 3.4% when taking in account the redundancy coefficient for the three stepped beam.

TABLE 4. Comparison results of different stiffness design methods

Types	Match	Test [11]	Iwan [13]
Double stepped beam	30.52	32.46	32.14
Three stepped beam	9.32	9.76	9.64

Moreover, the average error between the test stiffness and the Iwan stiffness coefficient is about 1%, which shows that the established models could reduce the demand for the physical model test system and retain high stiffness match accuracy.

3. CONCLUSIONS

This paper provided a structural stiffness matching modeling and active design approach for multiple stepped cantilever beam, and the validity was verified through simulations and bench test.

The stiffness match models were constructed according to the motion and deformation of multiple stepped cantilever beam, which included the analytical model of double stepped beam and recursive model of multiple stepped beam. Meanwhile, the qualitative stiffness influence coefficient of different scale and mechanical parameters were figured out through the sensitivity analysis. Through the stiffness requirements conversion, both the active stiffness optimization design flowchart and its implementation modes are presented for stiffness design of multiple stepped beam.

The simulations and bench test results showed that for the multiple stepped beam with rectangle section, the stiffness influence coefficient are arranged from high to low in terms of height, length, width and elastic modulus. Based on the same active stiffness design parameters, the FEM results are larger than the match results, the stiffness index are easier to achieve during the active stiffness design process. The relative error between the test stiffness and the stiffness index is less than 9%, and the redundancy coefficient that belongs to (1.05, 1.15) can be adopt to avoid the actual stiffness exceeding design requirements overmuch.

The overall results indicated that the proposed method is effectiveness and it is useful to reduce the redundant stiffness and increase the insufficient stiffness of the multiple stepped cantilever beam.

4. ACKNOWLEDGMENT

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Persian Abstract

چکیده

با هدف این مشکل که به دلیل رابطه نقشه‌برداری فازی برای سختی ساختاری تیرهای چند پله‌ای کنسول، دستیابی به طراحی بهینه دشوار است. یک مدل سازی تطبیق سختی و رویکرد طراحی سختی فعال پیشنهاد شده است. ابتدا، با استخراج شرایط هماهنگی پیوسته و عبارات برون یابی بار اتصال کنسول، مدل تحلیلی سختی و مدل بازگشتی برای قطعات تیرهای چندگانه کنسول ایجاد شد و ضریب تأثیر سختی آن پارامترهای ترکیب با تجزیه و تحلیل حساسیت به دست آمد. سپس، فرآیند طراحی بهینه‌سازی سختی فعال با توجه به سطح طراحی سختی تیر کنسول پلکانی ساخته شد و این روش‌های اجرایی به وضوح مشخص شدند. در نهایت، مقایسه و تایید طراحی سختی تیر کنسول پلکانی از طریق شبیه‌سازی عددی، تحلیل اجزای محدود و تست رومیزی انجام شد. نتایج به‌دست‌آمده نشان داد که مدل‌های ایجاد شده و روش طراحی سختی فعال معقول و مؤثر هستند، پارامترهای مطابقت سختی برای برآوردن الزامات شاخص سختی آسان هستند و ضریب ایمنی بیشتر از ۱ است. زمانی که تعداد مراحل بیشتر از ۵ نباشد، خطای نسبی بین سختی مطابقت و سختی تست کمتر از ۱۵ درصد است که با افزودن ضریب افزونگی (۱.۰۵، ۱.۱۵) می‌توان به کمتر از ۵ درصد کاهش داد.