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Utilizing the Modified Popovics Model in Study of Effect of Water to Cement Ratio, Size and Shape of Aggregate in Concrete Behavior

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ABSTRACT

Three parameters, size, shape of aggregate, and water to cement ratio, play important role on concrete behavior. To study the effect of these parameters, two types of aggregates were used, rounded (river) and sharped corners (broken). The maximum sizes of aggregates were chosen to be 9.5, 12.5, 19 and 25 mm for water to cement ratio were 0.35, 0.42, 0.54 and 0.76. In this investigation, the total of 32 mixed designs were made. The stress-strain tests were performed on the entire samples, and the results were compared with the Popovics model. To further evaluate the analysis, three criteria, correlation coefficient, variation coefficient, and percentage of change in energy absorption were demonstarted. Analysis showed that there is significant differences between the Popovics model and our experimental results. The Modified Popovics model was introduced for better understanding the concrete behavior in compression. The proposed model covered a wide range of the parameters concerned in this investigation. The Modified Popovics model was comapred with several models such as the Popovics, Hognestad, Thorenfeldt, and Tsai and the results showed that modified approach has a better clarification for behavior of concrete in compression. Moreover, the results indicated that these models were more accurate for prediction of concrete behavior with rounded aggregates in comparison to sharped aggregates.

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1. INTRODUCTION

Concrete is a mixture of cementitious material, aggregate, and water. Variety of gravel shapes and water to cement ratio affect the behavior of concrete.

The aggregate geometry influences required cement paste, placement factors (workability and pumpability), mechanical properties, and seismic parameters. Rounded aggregates are desirable because they joggle in the mixing and handling process. Aggregate can also contain flat or elongated shapes, and it is possible a thin, flat particle is oriented in the hardened concrete due to external stress and change in concrete strength [1–9]. Many researchers were trying to investigate the effect of gravel's size and shape on concrete behavior [10, 11]. Ogundipe et al. [12] and Yu et al. [13] studied the role of coarse aggregate size on concrete behavior in compression. The results of their experimental work were stated that compressive strength increases by raising of coarse aggregate size up to the specified limit.

The water to cement ratio of concrete is important from the aspect of durability, impermeability and strength. Too high water to cement ratio, may cause inadequate structural capability and not provide a durable protective environment for the steel reinforcement, permitting rapid carbonation and subsequent loss of the protective alkaline environment for the steel [14].

Rational analysis and the design of reinforced concrete structures are based on the prediction of stress–strain concrete relationship. Hognestad [15], Smith and Young [16], Desayi and Krishnan [17], Kent and Park [18], Sargin et al. [19], Popovics [20], Wang et al. [21], Carreira and Chu [22], Thorenfeldt et al. [23], Tsai [24], Hsu and Hsu [25], Almusallam and Alsayed [26], Attard

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and Setunge [27], Kumar [28], Lokuge et al. [29], Tasnimi [30], and Lokuge et al. [31], Nematzadeh and Hasan-Nattaj [32], Al-Tikrite et al. [33], and Peng et al. [34] have proposed a number of empirical expressions for the stress–strain curve of concrete in the past.

The accuracy and reliability of stress–strain curve of concrete are dependent on two main parameters: testing circumstance and concrete properties. Testing circumstance includes reliability of the instruments, shape and size of the specimen, strain rate, and the type of strain gauge. Concrete characteristics depend on many interrelated variables such as water to cement ratio, the mechanical and physical properties of cement and aggregate, and the age of specimen when is tested.

The Popovics model [20] is one of the model which is used to study concrete behavior in compression. This model will be explained in the following section.

1. 1. Popovics Stress-Strain Model of Unconfined Concrete

Figure 1 represents the Popovics model [20] which proposes a single equation, is used to describe unconfined concrete stress-strain behavior as given by Equation (1).

A major appeal of this model is that, it only requires three parameters to control the entire pre and post peak behavior.

The parameters define the curve are: ε_c, concrete strain, f_c, concrete stress, f'_c, concrete compressive strength, and ε'_c, concrete strain at f'_c.

$$\frac{f_c}{f'_c} = \frac{n(\frac{\varepsilon_c}{\varepsilon'_c})}{(n-1)+(\frac{\varepsilon_c}{\varepsilon'_c})^n} \tag{1}$$

In the equation, n, can be expressed as an approximate function of the compressive strength of normal weight concrete as:

$$\mathrm{n} = 0.4 \times 10^{-3} f'_c(psi) + 1 \tag{2}$$

The Popovics equation works well for most normal strength concrete ($f'c<55$ MPa), but for higher strength concrete, it lacks the necessary control over the slope of the post-peak branch.

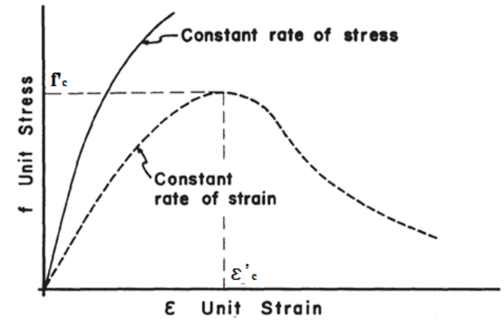


Figure 1. Popovics stress-strain model for unconfined concrete behavior

Besides the Popovics model, there are the other models, Hognestad [15], Thorenfeldt et al. [23] and Tsai [24] which are used for concrete behavior estimation. In this research, these models are used for validation of proposed stress-strain model and explained in the following sections.

1. 2. Hognestad Stress-Strain Model of Unconfined Concrete

Hognestad [15] sugessted a stress-strain relation for unconfined concrete as followed:

$$\mathrm{f_c} = \mathrm{f'_c}\left[\frac{2\varepsilon_c}{\varepsilon'_c} - \left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2\right] \tag{3}$$

The definition of ε_c, f_c, ε'_c, f'_c parameters are similar to the Popovics models explained in section 1.1.

1. 3. Thorenfeldt Stress-Strain Model of Unconfined Concrete

Thorenfeldt et al. [23] modified the Popovics [20] equation to adjust the descending branch of the concrete stress-strain equation. The Thorenfeldt et al. [23] sugessted the following relation for the unconfined concrete:

$$\frac{\mathrm{f_c}}{\mathrm{f'_c}} = \frac{\mathrm{n}(\frac{\varepsilon_c}{\varepsilon'_c})}{(\mathrm{n}-1)+(\frac{\varepsilon_c}{\varepsilon'_c})^{\mathrm{nk}}} \tag{4}$$

In Equation (4) ‘k’, takes a value of 1 for values of $(\varepsilon_c/\varepsilon'_c)<1$ and values greater than 1 for $(\varepsilon_c/\varepsilon'_c)>1$. Thus by adjusting the value of ‘k’ the post-peak branch of the stress-strain equation can be made steeper. This method can be illustrated for high-strength concrete where the post-peak branch becomes steeper with a raise in the concrete strength.

1. 4. Tsai Stress-Strain Model of Unconfined Concrete

Tsai [24] presented a generalized form of the Popovics [20] relation, which has greater control over the post-peak branch of the stress-strain equation. Tsai’s relation includes two additional parameters, one to control the ascending and a second to control the post-peak behavior of the stress-strain curve. The sugessted stress-strain equation for unconfined concrete by Tsai is shown below:

$$\mathrm{y} = \frac{\mathrm{mx}}{1+\left(\mathrm{m}-\frac{\mathrm{n}}{\mathrm{n}-1}\right)\mathrm{x}+\frac{\mathrm{x^n}}{\mathrm{n}-1}} \tag{5}$$

where $y=f_c/f'_c$= the ratio of the concrete stress to the ultimate strength, $x=\varepsilon_c/\varepsilon'_c$= the ratio of concrete strain to the strain at y=1, $m=E_0/E_c$= the ratio of initial tangent modulus to secant modulus at y=1, 'n'=a factor to control the steepness rate of the descending portion of the stress-strain equation. The following expressions were expressed for the factors 'm' and 'n'.

$$\mathrm{m} = 1 + \frac{17.9}{\mathrm{f'_c}} \tag{6}$$

$$\mathrm{n} = \frac{\mathrm{f'_c}}{6.68} - 1.85 > 1 \tag{7}$$

For assessment, 32 mix designs are considered to check the compatibility of the Popovics model with the stress-strain experimental data in compression. The samples cover a wide range of size and shape of aggregate, and water to cement ratio.

The following section explains the outline of experimental programs.

2. EXPERIMENTAL PROGRAM

In this section, the material types, mix design, preparation and curing are described, respectively.

2. 1. Material Types The ordinary Portland cement is used in the specimens. They are made according to ASTM C150 [35] standard. The gravel and sand aggregates are of river type in accordance with ASTM C33 [36] standard. The sand sizes range from 0 to 4.75 mm with apparent weight of 2650 kg/m^3 in SSD (Saturated Surface Dry) state with 24-hour water absorption of 1.5%, and additionally, the super-plasticizer of P10-3R type is used based on ASTM C494 [37]. Gravels, rounded and sharped types, are in four different sizes with maximum diameters of 9.5, 12.5, 19, and 25 mm, as shown in Figure 2.

2. 2. Mix Design In this study, 32 mix designs are used and summarized in Table 1. Three samples are built for each mix design and as a result, three stress-strain plots are obtained from three experiments, the plots are averaged out to a single stress-strain curve and it is considered as an averaged plot.

In Table 1, the codes designate the following:

Character WC followed by the numbers 1-4 are: water to cement ratio (W/C) with 0.76, 0.54, 0.42 and 0.35, respectively, GN and GB followed by the numbers 1-4 stand for Gravel of Natural (rounded corners), Gravel of Broken (sharped corners), and the maximum size of coarse aggregate 9.5, 12.5, 19, and 25 mm, respectively.

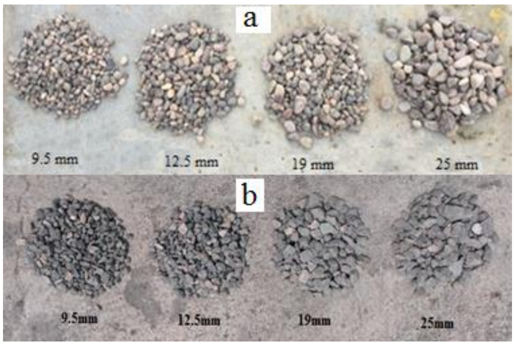


Figure 2. Images demonstrate the size of sieved aggregates; (a): Rounded type; (b): Sharped type

TABLE 1. Mix designs used in the different samples

NO.	Code	Maximum Gravel Size (mm)	Gravel (kg/m^3)	Sand (kg/m^3)	Cement (kg/m^3)	W/C
1	WC1GN1	9.5	1290	898	250	0.76
2	WC1GN2	12.5	1290	898	250	0.76
3	WC1GN3	19	1290	898	250	0.76
4	WC1GN4	25	1290	898	250	0.76
5	WC1GB1	9.5	1290	898	250	0.76
6	WC1GB2	12.5	1290	898	250	0.76
7	WC1GB3	19	1290	898	250	0.76
8	WC1GB4	25	1290	898	250	0.76
9	WC2GN1	9.5	1170	820	350	0.54
10	WC2GN2	12.5	1170	820	350	0.54
11	WC2GN3	19	1170	820	350	0.54
12	WC2GN4	25	1170	820	350	0.54
13	WC2GB1	9.5	1170	820	350	0.54
14	WC2GB2	12.5	1170	820	350	0.54
15	WC2GB3	19	1170	820	350	0.54
16	WC2GB4	25	1170	820	350	0.54
17	WC3GN1	9.5	1090	762	450	0.42
18	WC3GN2	12.5	1090	762	450	0.42
19	WC3GN3	19	1090	762	450	0.42
20	WC3GN4	25	1090	762	450	0.42
21	WC3GB1	9.5	1090	762	450	0.42
22	WC3GB2	12.5	1090	762	450	0.42
23	WC3GB3	19	1090	762	450	0.42
24	WC3GB4	25	1090	762	450	0.42
25	WC4GN1	9.5	947	663	550	0.35
26	WC4GN2	12.5	947	663	550	0.35
27	WC4GN3	19	947	663	550	0.35
28	WC4GN4	25	947	663	550	0.35
29	WC4GB1	9.5	947	663	550	0.35
30	WC4GB2	12.5	947	663	550	0.35
31	WC4GB3	19	947	663	550	0.35
32	WC4GB4	25	947	663	550	0.35

2. 3. Preparation and Curing of Specimens First, concrete is constructed and then inserted into pre-prepared cylindrical molds (with dimensions of 15 cm × 30 cm). They are kept in constant temperature and humidity for 24 hours in order to harden. After 24 hours, the specimens are removed from the molds and are placed into a water pond with temperature of 20 ± 2 °C for curing. The curing time of the samples is equal to 28 days in order to do stress-strain tests.

3. RESULTS AND DISCUSSION

3. 1. Plots of Stress-Strain for Experiment and Popovics Model

The stress-strain tests are performed on all 32 mix design samples. All the plots are analyzed but since there are too many results to be explained, authors discuss only two representative of mix designs, WC2GN1 and WC2GB1. Figures 3a and 3b demonstrate the stress-strain plots according to the tests and Popovics models [20] for two mix designs, WC2GN1 and WC2GB1, respectively.

The following section describes three criteria to evaluate the capability of Popovics model [20] to explain the stress-strain data obtained through experiments.

3. 2. Definition of Criteria for Comparing the Stress-Strain Testing Results with the Popovics Model

These criteria are defined separately, in the next sections which provide the possibility of comparing stresses between behavioral models and the experimental results within the limit of concrete strain.

3. 2. 1. Criterion 1: Correlation Coefficient

Correlation coefficient is a numerical measure, meaning a statistical relationship between two variables (X, Y). These variables (here stresses) are obtained from corresponding strains of the two curves, test and model. These variables (here stresses) are obtained from corresponding strains of the two curves, test and model. The equation for the correlation coefficient [38] can be written as:

$$\rho_{X,Y} = \frac{\sum(X-\bar{X})(Y-\bar{Y})}{\sqrt{\sum(X-\bar{X})^2 \sum(Y-\bar{Y})^2}} \quad (8)$$

where, $\bar{X}$ and $\bar{Y}$ are the means of two variables.

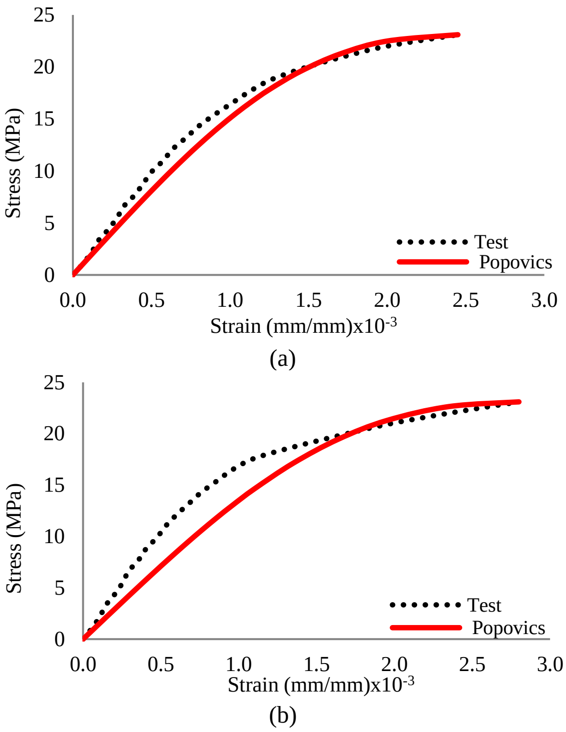


Figure 3. Plots of stress-strain for the test and Popovics model; (a): Mix design, WC2GN1; (b): Mix design, WC2GB1

All values assume in the range from −1 to +1, where +1 indicates the strongest possible agreement and −1 the strongest possible disagreement. If the value of correlation coefficient is close to zero, it is indication of no or weak correlation.

The correlation coefficients between experimental data and the Popovics model for the whole samples (Table 1) are evaluated. These coefficients are reported for rounded and sharped aggregates, separately. The estimated average correlation coefficients between the Popovics model and the experimental data are equal to 0.985 and 0.971 for rounded and sharped aggregates, respectively. They indicate a fairly acceptable correlations, specially for rounded aggregates.

3. 2. 2. Criterion 2: Variation Coefficient

The coefficient of variation (CV) is a statistical measure of the dispersion of data points in a data series around the mean. The coefficient of variation represents the ratio of the standard deviation to the mean. In this research, data points are stresses for corresponding strains. The coefficient of variations are evaluated between experimental data and the Popovics model [20] for the whole samples (Table 1). These coefficients are reported for rounded and sharped aggregates, separately.

The average results show that there are limitations of 1.08% and 1.68% for rounded and sharped aggregates, respectively, in difference between the variation coefficient in the Popovics model [20] and the experimental data which are negligible.

3. 2. 3. Criterion 3: Percentage of Change in Energy Absorption

The percentage of change in energy absorption is defined by the following expression:

$$p = \left|\frac{Area_{exp} - Area_{Popovics}}{Area_{exp}}\right| \times 100 \quad (9)$$

In which: P is percentage of change in energy absorption, $Area_{exp}$ represents area under the stress-strain curve of experimental data, and $Area_{Popovics}$ is area under the Popovics stress-strain curve.

It should be noted that the area under the stress-strain curve implies the absorbed energy in stress-strain behavior. MATLAB software [39] is used for calculating this area for each specified curve. The percentages of change in energy absorption are estimated based on Equation (9) for the entire samples (Table 1). These percentages are reported for rounded and sharped aggregates, separately. The average changes in energy absorption are equal to 7.8% and 11.5%, for rounded and sharped aggregates, respectively which are fairly significant values.

To overcome this difference, a modified relation is proposed regarding Popovics model in the next section.

3. 3. The Modified Popovics Model

The drift among the data in two stress-strain plots, experiments and Popovics model [20], is attributed to the lack of parameters (i.e. size, shape of aggregate, and water to cement ratio) in mathematical formulation of Popovics model [20]. Popivics model only considers compressive strength in establishing the stress-strain curve, whereas parameters such as shape and size of aggregate are effective on integrity of concrete matrix. Moreover, water to cement ratio parameter specifies the effectiveness of cement paste and its cohesion in mixtures. As a result, these parameters are determinative on failure strain and the trend of stress-strain curve.

The Popovics model with new coefficients, is introduced, and is called Modified Popovics model in order to distinguish from the Popovics model.

The modified model is similar to the Popovics model; just the parameter ''m'' is added. The Modified Popovics model is suggested as follows:

$$\frac{f_c}{f_c'} = \frac{nm(\frac{\varepsilon_c}{\varepsilon_c'})}{(n-1)+(\frac{\varepsilon_c}{\varepsilon_c'})^{nm}} \tag{10}$$

“m” is the minor modification coefficient obtained from the following equation:

$$\mathrm{m} = \mathrm{s}\,.\,\mathrm{F}(d_r).\ \mathrm{F}(wc_r) \tag{11}$$

In Equation (11), s is representative for the effect of aggregate geometry (Table 2) and $F(d_r)$ is the aggregate size function in which the independent variable dr is defined as:

$$d_r = \frac{d_i}{d_0} \tag{12}$$

where in, d_i is maximum size of aggregates in mm (i.e. 9.5, 12.5, 19 and 25 mm), d_0: is the base size of aggregate (assumed here 12.5 mm), and $F(wc_r)$ is the water to cement ratio function in which the independent variable wc_r is defined as:

$$wc_r = \frac{wc_0}{wc_i} \tag{13}$$

where in, wc_i is water to cement ratio (i.e. 0.76, 0.54, 0.42 and 0.35) and wc_0 is the base water to cement ratio (assumed here 0.76).

$F(d_r)$ and $F(wc_r)$ functions as well as “s” are obtained by the curve fitting of stress-strain tests data with the Modified Popovics model. This regression is based on the three mentioned criteria in section 3.2. The functions $F(d_r)$, $F(wc_r)$ and the coefficient “s” are obtained as follows:

$$\mathrm{F}(d_r) = 0.12\ d_r + 0.63 \tag{14}$$

$$\mathrm{F}(wc_r) = -0.25\ \mathrm{wc}_r + 1.25 \tag{15}$$

TABLE 2. Calculation of "s" coefficient based on curve fitting

	Rounded corners	**Sharped corners**
“s” value	1.2	1

In the following section, the equation of Modified Popovics model, Equation (10) is plotted for only two mix designs.

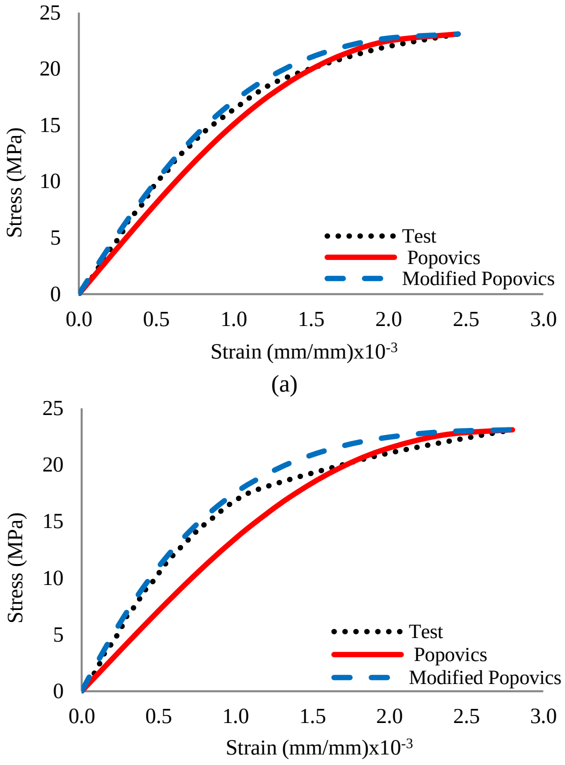
3. 4. Comparison of Stress-Strain Experimental Data with Popovics and Modified Popovics Models

In order to better understand the trend of Equation (10), stress-strain plots are drawn for two representative mix designs, WC2GN1 and WC2GB1 (See Figure 4).

This section focuses on compatibility of stress-strain experimental data with Popovics and Modified Popovics models aided by three criteria, defined in section 3.2 “*Definition of criteria for comparing the stress-strain testing results with the Popovics model*”. The data from stress-strain tests are used to support plotting the Figures 5-7 and 9-11.

3. 4. 1. Correlation Coefficient Criterion for Comparison

In Figure 5, the average correlation coefficients of experimental data are compared with



(a)

(b)

Figure 4. Plots of stress-strain for the test, Popovics and Modified Popovics models; (a): Mix design, WC2GN1; (b): Mix design, WC2GB1

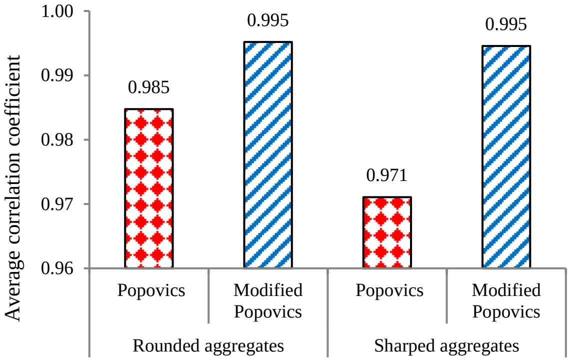


Figure 5. Comparison of average correlation coefficients from experimental data with Popovics and Modified Popovics models

the Popovics and Modified Popovics models for rounded and sharped aggregates, separately.

From this figure, the average correlation coefficients of the Popovics model [20] are equal to 0.985 and 0.971 for rounded and sharped aggregates, respectively, but the coefficient in Modified Popovics model, increases to 0.995 for both types of aggregates. These improvements are not tangible.

3. 4. 2. Variation Coefficient Criterion for Comparison

Figure 6 shows the comparison of average variation coefficients of experimental data with Popovics and Modified Popovics models for rounded and sharped aggregates, separately.

Figure illustrates the average variation coefficients of the Popovics model are equal to 1.08 % and 1.68%, but the coefficients in Modified Popovics model, reduce to 0.36% and 0.30% for rounded and sharped aggregates, respectively, which are not a significant differences.

3. 4. 3. Percentage of Change in Energy Absorption Criterion for Comparison

In Figure 7, the average changes in energy absorption of experimental data are compared with the Popovics and Modified Popovics models for rounded and sharped aggregates, separately.

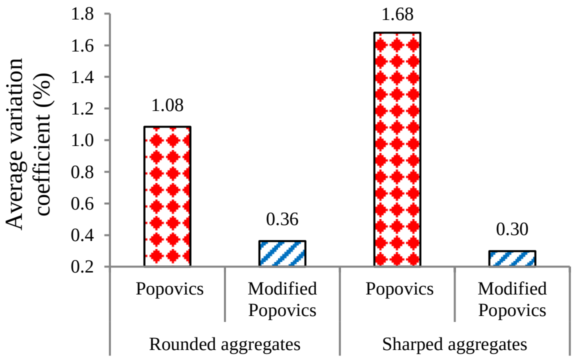


Figure 6. Average variation coefficients from experimental data with Popovics and Modified Popovics models

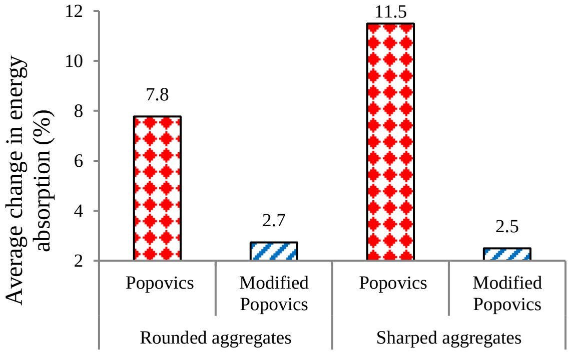


Figure 7. Average changes in energy absorption from experimental data with Popovics and Modified Popovics models

As demonstrated in Figure 7, the average changes in energy absorption of the Popovics model are equal to 7.8% and 11.5% for rounded and sharped aggregates, respectively, but these values decline to 2.7% and 2.5% for the Modified Popovics model, which shows a significant decrease and indicates that the Modified Popovics model has a better acceptable performance in modeling of concrete behavior.

The proposed model leads to more accurate results in comparison to the Popovics model. That is because, the proposed model was extracted and pulled out from the experimental data and the curve fitting. In the following sections, in order to reach an overall approach, the capability of the other models mentioned in "*Introduction*" section is plotted and compared with the Modified Popovics model.

3. 5. The Plots of the Other Models in Comparison with the Modified Popovics Model

The Modified Popovics model is plotted with other models described in the "*Introduction*" section. These models are Hognestad, Thorenfeldt, and Tsai. In Figure 8, for instance, the strain-strain experimental data for two representative mix designs WC2GN1 and WC2GB1 along with the other models, are presented.

3. 6. Validation of the Other Models with the Modified Popovics Model

This section concentrates on comparative study of the Modified Popovics model with the models mentioned in section 1, aided by three criteria as defined in section 3.2, "*Definition of criteria for comparing the stress-strain testing results with the Popovics model*".

3. 6. 1. Correlation Coefficient Criterion for Validation

Experimental data are used to calculate correlation coefficients of Modified Popovics, Hognestad, Thorenfeldt, and Tsai models for rounded

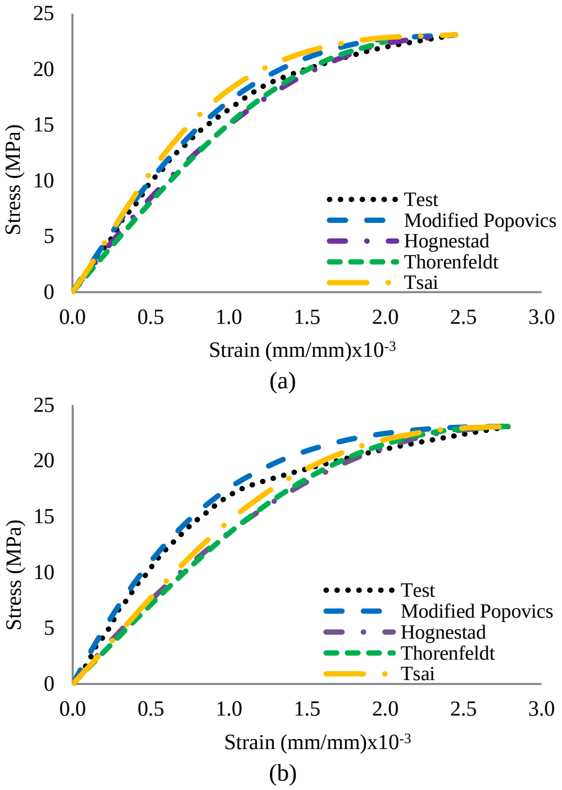


Figure 8. Plots of stress-strain for the test, Modified Popovics, Hognestad, Thorenfeldt, and Tsai models; (a): Mix design, WC2GN1; (b): Mix design, WC2GB1.

and sharped aggregates, separately. The coefficients are averaged out and demonstrated in Figure 9.

The maximum value among those values belongs to the Modified Popovics Model, which implies the better estimation of concrete behavior with respect to the other models.

Also, according to this criterion, the figure indicates that the prediction of concrete behavior with rounded aggregates is more precise compared to sharped aggregates for all the models duscussed in this article.

3. 6. 2. Variation Coefficient Criterion for Validation Similarly, in Figure 10, the average variation coefficients of Modified Popovics, Hognestad [15], Thorenfeldt et al. [23] and Tsai [24] models are reported for rounded and sharped aggregates, separately. The minimum value among those values belongs to the Modified Popovics model which shows the better estimation of concrete behavior with respect to the other models.

Once again, this figure illustrates that by considering the entire models, the prediction of concrete behavior with rounded aggregates is more accurate relative to sharped aggregates.

3. 6. 3. Percentage of Change in Energy Absorption Criterion for Validation With the similar method, in Figure 11, the average changes in energy absorption of Modified Popovics, Hognestad, Thorenfeldt and Tsai models are reported for rounded and sharped aggregates, separately. The minimum value among those values belongs to Modified Popovics model that displays the better estimation of concrete behavior with respect to the other models. Lastly, according to this criterion, this figure confirms that considering the entire models, the prediction of concrete behavior with rounded aggregates is more accurate relative to sharped aggregates.

The utilization of the other models, Hognestad, Thorenfeldt, and Tsai, for verification of proposed model illustrates that parameters such as water to cement ratio, shape and size of aggregate have ability to affect the behavior of stress-strain in concrete. This shows that proposed model is accurate regarding to the description of concrete behavior in comparison to the other models used in this research.

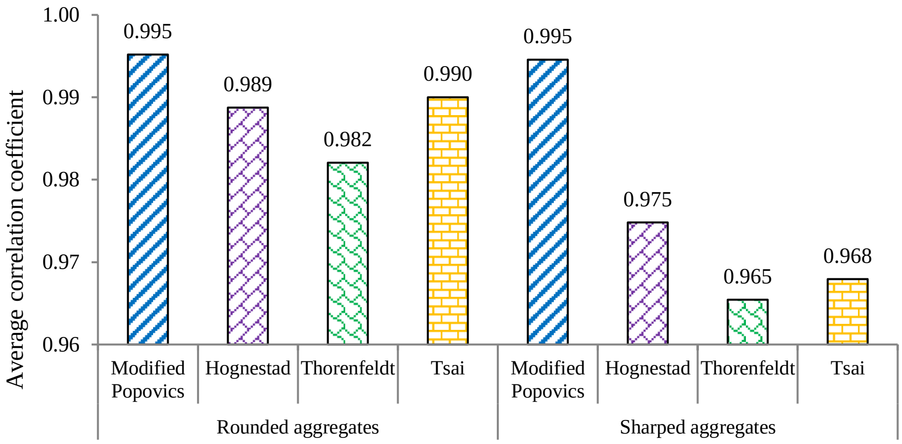


Figure 9. Average correlation coefficients from experimental data with Modified Popovics, Hognestad, Thorenfeldt, and Tsai models

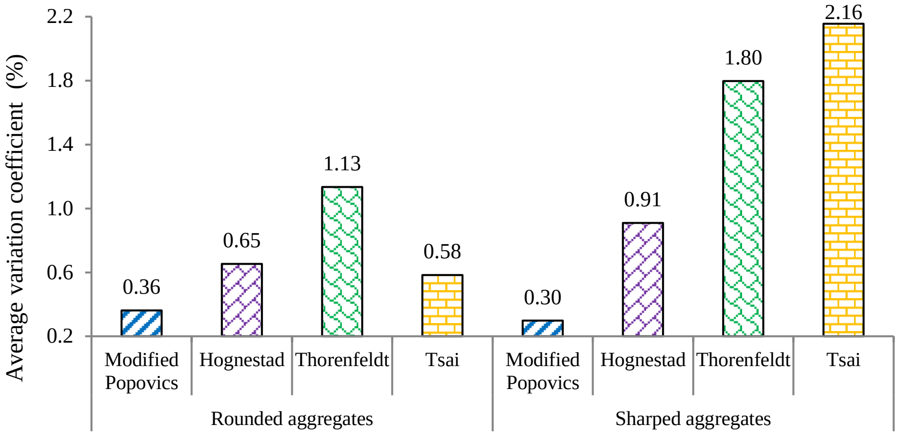


Figure 10. Average variation coefficients from experimental data with Modified Popovics, Hognestad, Thorenfeldt, and Tsai models

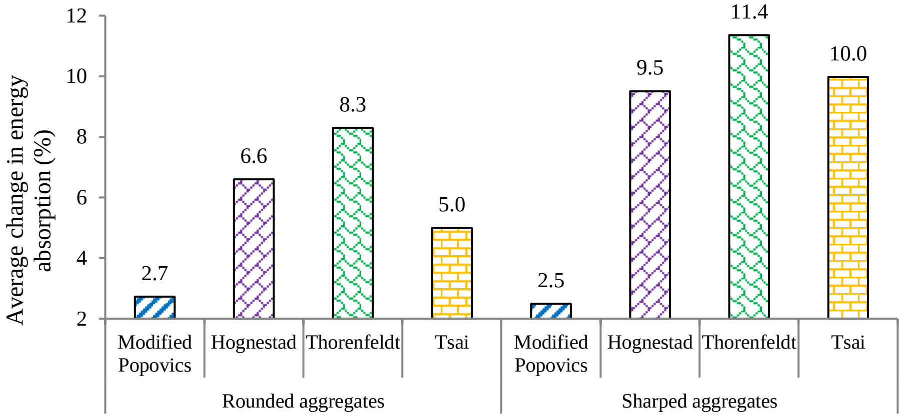


Figure 11. Average changes in energy absorption from experimental data with Modified Popovics, Hognestad, Thorenfeldt, and Tsai models

4. CONCLUSION

In this paper, at first, the Popovics model was compared with the experimental results. The tests considered the effects of concrete characteristics such as size and shape of aggregate, and water to cement ratio.

Then, the Popovics model was modified to have a good fit with the results obtained through experimental tests. For comparison and validation of modified model, three criteria were chosen, correlation coefficient, variation coefficient, and percentage of change in energy absorption.

The following are the summary of the conclusion:

1. The average correlation coefficient between Popovics model and experimental data were 0.985 and 0.971 for rounded and sharped aggregates, respectively, but these values increased to 0.995 for both aggregate types with the Modified model, which did not show any significant improvement with respect to the old values.
2. The average variation coefficient between Popovics model and experimental data were 1.08% and 1.68% but then, these values reduced by 0.36% and 0.30% for rounded and sharped aggregates, respectively regarding the Modified model, which did not apparently indicate any tangible differences.
3. The average change in energy absorption for Popovics model with respect to experimental data were 7.8% and 11.5% but then, these values significantly declined regarding the Modified model by 2.7%, and 2.5% for rounded and sharped aggregates, respectively, which clearly reflected the capability of Modified model.
4. The three criteria confirmed that the prediction of concrete behavior with rounded aggregates is more reliable in comparison to the sharped aggregates for all the models discussed in this article.
5. It was reasonable to conclude that Modified Popovics model expressed clearly the behavior of concrete in compression in comparison to Hognestad, Thorenfeldt, and Tsai models.

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Persian Abstract

چکیده

سه پارامتر، اندازه، شکل سنگدانه و نسبت آب به سیمان، نقش مهمی بر رفتار بتن ایفا می‌کنند. برای مطالعه اثر این پارامترها، دو نوع سنگدانه گردگوشه (رودخانه‌ای) و تیزگوشه (شکسته) استفاده شد. حداکثر اندازه سنگدانه ها ۹/۵، ۱۲/۵، ۱۹ و ۲۵ میلیمتر برای نسبت آب به سیمان ۰/۳۵، ۰/۴۲، ۰/۵۴ و ۰/۷۶ انتخاب شدند. در این بررسی، در مجموع ۳۲ طرح اختلاط ساخته شدند. آزمایش‌های تنش-کرنش بر روی تمام نمونه ها انجام شد و نتایج با مدل پوپوویچ مقایسه شد. برای ارزیابی بیشتر تحلبل انجام شده، سه معیار ضریب همبستگی، ضریب تغییرات و درصد تغییر در جذب انرژی بکار رفت. تحلیل نتایج، اختلاف قابل توجهی بین مدل پوپوویچ و نتایج آزمایشگاهی را نشان داد. مدل پوپوویچ اصلاح شده برای درک بهتر رفتار بتن در فشار معرفی شد. مدل پیشنهادی، طیف گسترده‌ای از پارامترهای مورد بررسی در این تحقیق را پوشش داد. مدل پوپوویچ اصلاح شده با چندین مدل مانند پوپوویچ، هاگنستاد، تورنفلد و سای، مقایسه شد و نتایج نشان داد که رویکرد اصلاحی، وضوح بهتری برای رفتار بتن در فشار دارد. علاوه بر این، نتایج نشان داد که این مدل‌ها برای پیش‌بینی رفتار بتن با سنگدانه‌های گردگوشه در مقایسه با سنگدانه‌های تیزگوشه دقت بهتری دارند.